Problems

1. Calculate the wavelength of a photon with a photon energy of 2 eV. Also, calculate the wavelength of an electron with a kinetic energy of 2 eV.

2. Consider a beam of light with a power of 1 Watt and a wavelength of 800 nm. Calculate a) the photon energy of the photons in the beam, b) the frequency of the light wave and c) the number of photons provided by the beam in one second.

3. Show that the spectral density, $u_\omega$ (equation 1.2.4) peaks at $E_{ph} = 2.82 kT$. Note that a numeric iteration is required.

4. Calculate the peak wavelength of blackbody radiation emitted from a human body at a temperature of 37°C.

5. Derive equations (1.2.9) and (1.2.10).

6. What is the width of an infinite quantum well if the second lowest energy of a free electron confined to the well equals 100 meV?

7. Calculate the three lowest possible energies of an electron in a hydrogen atom in units of electron volt. Identify all possible electron energies between the lowest energy and -2 eV.

8. Derive the electric field of a proton with charge $q$ as a function of the distance from the proton using Gauss's law. Integrated the electric field to find the potential $\phi(r)$:

$$\phi(r) = \frac{q}{4\pi \varepsilon_0 r}$$

Treat the proton as a point charge and assume the potential to be zero far away from the proton.

9. Prove that the probability of occupying an energy level below the Fermi energy equals the probability that an energy level above the Fermi energy and equally far away from the Fermi energy is not occupied.

10. The ratio of the wavelengths emitted by two electrons in an infinite quantum well while making the transition from a higher energy level to the lowest possible energy equals two.

a) What are the lowest possible quantum numbers ($n$) of the two higher energy levels, which are consistent with the statement above?

b) What are the energies in electron volt of all three energy levels involved in the transitions? ($L_x = 10 \text{ nm}, m^*/m_0 = 0.067$ and $\varepsilon_s/\varepsilon_0 = 13, m_0 = 9.11 \times 10^{-31} \text{ kg}, \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$)

11. Consider a hollow thin conducting sphere of radius $R = 10 \text{ cm}$, uniformly distributed with a surface charge density of $\sigma = 10^{-9} \text{ coulombs/cm}^2$.

a) Find the field at a radius $r > 10 \text{ cm}$. Explain the vector orientation of the field and the magnitude.
b) Find the field at a radius \( r < 10 \) cm inside the sphere. Explain the vector orientation of the field and the magnitude.

c) Now consider two concentric conducting spherical shells, each coated with a uniformly distributed surface charge density of \( \sigma \), with \( R_1 = R \) and \( R_2 = R/2 \). Find the field at all \( r \).

12. Find the lowest possible energy in a 2 nm quantum well with infinitely high barriers on each side of the well and with a delta function potential positioned in the middle of the quantum well. The integral of the delta function potential equals \( 10^{-10} \) eV-m. Assume that the electron mass equals the free electron mass \((m_0 = 9.1 \times 10^{-31} \text{ kg})\).

13. Consider the potential energy, \( V(x) \), as shown in the figure below, where \( E \) is the particle energy:

![Potential Energy Diagram](image)

a) Find a general solution to the wave equation in region I \((0 < x < L)\) and II \((L < x < 2L)\). Assume that the particle energy is always larger than the potential \( V_0 \).

b) Require that the wavefunction is zero at \( x = 0 \) and \( x = 2L \).

c) Require that the wavefunction and its derivative is continuous at \( x = L \).

d) Derive a transcendental equation from which the possible energies can be obtained.

e) Calculate the lowest possible energy for \( V_0 = 0.1 \) eV, \( L = 1 \) nm and \( m = m_0 \).