Example 1.4 Consider an infinitely long cylinder with charge density $\rho$, dielectric constant $\varepsilon_0$ and radius $r_0$. What is the electric field in and around the cylinder?

Solution

Because of the cylinder symmetry one expects the electric field to be only dependent on the radius, $r$. Applying Gauss's law one finds:

$$\mathbf{E} \cdot \mathbf{A} = \mathbf{E} \cdot 2\pi rL = \frac{Q}{\varepsilon_0} = \frac{\rho \pi r^2 L}{\varepsilon_0} \text{ for } r < r_0$$

and

$$\mathbf{E} \cdot \mathbf{A} = \mathbf{E} \cdot 2\pi rL = \frac{Q}{\varepsilon_0} = \frac{\rho \pi r_0^2 L}{\varepsilon_0} \text{ for } r > r_0$$

where a cylinder with length $L$ was chosen to define the surface $A$, and edge effects were ignored. The electric field then equals:

$$\mathbf{E} (r) = \frac{\rho r}{2\varepsilon_0} \text{ for } r < r_0 \text{ and } \mathbf{E} (r) = \frac{\rho r_0^2}{2\varepsilon_0 r} \text{ for } r > r_0$$

The electric field therefore increases within the cylinder with increasing radius as shown in the figure below. The electric field decreases outside the cylinder with increasing radius.