1. Review questions

a) What two parameters are linked by Poisson’s equation? (4 points)

The electrostatic potential and the charge density

b) What is the physical meaning of the Fermi energy? (4 points)

The Fermi energy is the average energy per particle in the distribution, excluding work or heat.

c) Explain physically why the intrinsic carrier density increases with temperature. (4 points)

The intrinsic carrier density is caused by thermal activation of electrons in the valence band into the conduction band. As the thermal energy is increased by increasing the temperature, the intrinsic carrier density also increases.

d) What is the driving mechanism, which causes diffusion? (4 points)

The random motion of carriers due to the thermal energy causes diffusion.

e) Can the barrier height of a metal-semiconductor junction be negative? Explain. (4 points)

Yes. The barrier height of an n-type M-S junction equals the difference between the workfunction of the metal and the electron affinity of the semiconductor. The barrier height will be therefore negative if the workfunction is smaller than the electron affinity. Similarly for a p-type M-S junction, the barrier height is negative if the workfunction is larger than the sum of the electron affinity and the bandgap energy divided by the electronic charge.

2. Consider a piece of silicon doped with shallow donors (phosphorous). The electron density per unit energy, \( n(E) \), at an energy \( kT \) above the conduction band is \( 10^{23} \text{ cm}^{-3} \text{eV}^{-1} \). Calculate the resistivity of the silicon. (25 points)

<table>
<thead>
<tr>
<th></th>
<th>Arsenic</th>
<th>Phosphorous</th>
<th>Boron</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\text{min}} ) (cm(^2)/V-s)</td>
<td>52.2</td>
<td>68.5</td>
<td>44.9</td>
</tr>
<tr>
<td>( \mu_{\text{max}} ) (cm(^2)/V-s)</td>
<td>1417</td>
<td>1414</td>
<td>470.5</td>
</tr>
<tr>
<td>( N_r ) (cm(^{-3}))</td>
<td>( 9.68 \times 10^{16} )</td>
<td>( 9.2 \times 10^{16} )</td>
<td>( 2.23 \times 10^{17} )</td>
</tr>
</tbody>
</table>
To find the resistivity one first has to find the electron density, which in turn is obtained from the Fermi energy.

\[
n(E) = g_c(E) f(E) = \frac{8\sqrt{2}\pi}{h^3} m^{3/2} \sqrt{E - E_c} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}
\]

Since \( E - E_c \) is known to be \( kT \), one can then solve for \( E_c - E_F = 218 \text{ meV} \)

The electron density is calculated from

\[
n = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = 6.23 \times 10^{15} \text{ cm}^{-3}
\]

with \( N_c = 2.82 \times 10^{19} \text{ cm}^{-3} \)

The mobility is calculated from:

\[
\mu_n = \mu_{\min} + \frac{\mu_{\max} - \mu_{\min}}{1 + \left(\frac{N}{N_c}\right)} = 1241 \text{ cm}^2/\text{V-s}
\]

and the resistivity equals:

\[
\rho = \frac{1}{q\mu_{\max}n} = 807 \text{ m}\Omega\text{-cm}
\]

3. A piece of silicon, uniformly doped with \( 10^{17} \) shallow donors and \( 8 \times 10^{16} \text{ cm}^{-3} \) shallow acceptors and heated to 200 C, is uniformly illuminated with visible light resulting in a 0.4 eV separation between the two quasi-Fermi energies. The minority carrier lifetime is 1 \( \mu \text{s} \).

Calculated the intrinsic carrier density at 200 C. Ignore the temperature dependence of the energy bandgap.

Then calculate the generation rate of electron-hole pairs due to illumination. (30 points)

The intrinsic carrier density calculation requires that one first calculates the effective densities of states:

\[
N_e(473K) = N_e(300K)\left(\frac{473}{300}\right)^{3/2} = 5.59 \times 10^{19} \text{ cm}^{-3}
\]

\[
N_v(473K) = N_v(300K)\left(\frac{473}{300}\right)^{3/2} = 3.62 \times 10^{19} \text{ cm}^{-3}
\]

and the intrinsic carrier density equals:

\[
n_i(473K) = \sqrt{N_e(473K)N_v(473K) \exp\left(-\frac{E_{i}^{\text{F}}}{2kT}\right)} = 4.90 \times 10^{13} \text{ cm}^{-3}
\]

The separation between the two quasi-Fermi energies can be used to find the product of the electron and hole densities:

\[
n_p = n_i \exp\left(\frac{E_{n}^{\text{F}} - E_{i}^{\text{F}}}{kT}\right)n_i \exp\left(\frac{E_{i}^{\text{F}} - E_{p}^{\text{F}}}{kT}\right) = n_i^2 \exp\left(\frac{E_{n}^{\text{F}} - E_{p}^{\text{F}}}{kT}\right) = 4.36 \times 10^{31} \text{ cm}^{-3}
\]

The electron density in thermal equilibrium equals the net doping density:
The hole density in thermal equilibrium is obtained using the mass action law:
\[ p_{n0} = n_i^2/n_0 = 1.2 \times 10^{11} \text{ cm}^{-3} \]
while the hole density can be obtained from:
\[ np = (n_o + \delta n)(p_o + \delta p) = n_o p_o + (n_o + p_o)\delta p + (\delta p)^2 = 4.36 \times 10^{31} \text{ cm}^{-3} \]
where we used the fact that the excess electron density equals the excess hole density. The resulting excess hole density equals \( \delta p = 2.18 \times 10^{15} \text{ cm}^{-3} \)
The optical generation rate equals the net recombination rate, which in turn is obtained from:
\[ U_p = \frac{p_n - p_{n0}}{\tau_p} = 2.18 \times 10^{22} \text{ cm}^{-3} \text{s}^{-1} = G_{opt} \]

4. A metal-semiconductor junction, biased at an unknown voltage, has a maximum electric field of \(-10^5 \text{ V/cm}\) and a capacitance of 1 pF. The semiconductor is n-type gallium arsenide, the built-in potential of the junction is 0.7 V and the diode area is \(10^{-4} \text{ cm}^2\).

Calculate the doping density and the applied voltage. (25 points)

From the capacitance, one finds the depletion layer width
\[ x_d = \frac{\varepsilon_s A}{C} = 1.16 \mu \text{m} \]
The doping density is then obtained from the electric field and depletion layer width as:
\[ N_d = -\frac{\varepsilon_s E(x = 0)}{q x_d} = 6.24 \times 10^{15} \text{ cm}^{-3} \]
The applied voltage equals:
\[ V_a = \phi_i - \frac{|E(x = 0)| x_d}{2} = -5.1 \text{ V} \]