3. Unipolar devices

3.1 The Metal-Semiconductor (M-S) junction

Metal-to-semiconductor contacts are of great importance since they are present in every semiconductor device. They can behave either as a barrier or as an ohmic contact dependent on the characteristics of the interface. This section discusses the electrostatics of the M-S junction (i.e. the charge, field and potential distribution within the device) as well as the current versus voltage characteristics. The electrostatics are calculated using what is known as the "full depletion approximation" and complemented with a description of the numeric solution. Various approximations are used to obtain closed form expressions under different circumstances.

3.1.1 Electrostatics of the M-S junction

![Flatband energy band diagram of a metal-semiconductor contact](image)

A description of the electrostatics problem starts from the "flatband diagram" shown in Fig.3.1: the energy band diagram for the M-S junction in the absence of charge anywhere.
in the structure. We define the barrier height, $\phi_B$, for a junction containing n-type material as the difference between the metal work function, $\Phi_M$, and the electron affinity, $\chi$. For p-type material it is given by the difference between the valence band edge and the Fermi energy in the metal:

$$\phi_B = \Phi_M - \chi \text{ (n-type)}$$

$$\phi_B = \chi + E_g - \Phi_M \text{ (p-type)} \quad [3.1.1]$$

In addition we define the built-in potential, $\phi_i$, as the difference between the Fermi energy of the metal and that of the semiconductor.

$$\phi_i = \Phi_M - \chi - (E_c - E_{fn})/q \text{ (n-type)}$$

$$\phi_i = \chi + (E_c - E_{fp})/q - \Phi_M \text{ (n-type)} \quad [3.1.2]$$

The traditional derivation of the M-S junction electrostatics is based on the full depletion approximation which assumes that the semiconductor is depleted over a distance $x_d$, called the depletion region. A comparison of the correct solution and the solution obtained when using the full depletion approximation can be found in figures 3.3 through 3.6. Having the semiconductor depleted of free carrier over a distance $x_d$ implies the following charge density for n-type material:

$$\rho = q N_d \quad 0 < x < x_d$$

$$\rho = 0 \quad x_d < x \quad [3.1.3]$$

Using Gauss's law we obtain the maximum field at the interface:

$$\varepsilon_{\text{max}} = - Q_s/\varepsilon_s = - \frac{q N_d x_d}{\varepsilon_s} \quad [3.1.4]$$

and a relation between the applied voltage and the depletion region width is obtained from the electric field:

$$\phi_i - V_a = - \varepsilon_{\text{max}} x_d/2 = \frac{q N_d x_d^2}{2\varepsilon_s} \quad [3.1.5]$$
where the factor of two reflects the triangular field distribution. The above equation can be solved to obtain the depletion region width and the capacitance per unit area:

\[ x_d = \sqrt{\frac{2\varepsilon_s(\phi_i - V_a)}{q N_d}} \quad C = \varepsilon_s x_d = \sqrt{\frac{\varepsilon_s q N_d}{2(\phi_i - V_a)}} \]  

[3.1.6]

### 3.1.2 Exact solution of the M-S junction

In order to correctly assess the error made when using the full depletion approximation we now derive the correct solution by solving Poisson's equation analytically\(^1\). The actual solution for the potential is then obtained by numerically integrating the expression for the electric field. We start from the charge density \(\rho\) in a semiconductor for the general case where electrons, holes, ionized acceptors and ionized donors are present:

\[ \rho(\phi) = q(p + N_d^+ - n - N_a^-) \]  

[3.1.7]

Where \(\phi\) is the potential in the semiconductor. The potential is chosen to be zero deep into the semiconductor. For an n-type semiconductor without acceptors or free holes this can be further reduced to:

\[ \rho(\phi) = q N_d (1 - \exp(\frac{q\phi}{kT})) \]  

[3.1.8]

assuming the semiconductor to be non-degenerate and fully ionized. A similar expression can be obtained for p-type material. Poisson's law can then be rewritten as:

\[ \frac{d^2\phi}{dx^2} = -\frac{\rho(\phi)}{\varepsilon_s} = -\frac{q N_d}{\varepsilon_s} (1 - \exp(\frac{q\phi}{kT})) \]  

[3.1.9]

\(^1\)This derivation follows that of Goodman and Perkins, J. Appl. Phys. 35, p 3351, 1964.
Multiplying both sides with $d\phi/dx$, this equation can be integrated between an arbitrary point $x$ and infinity. The electric field at infinity (deep into the semiconductor) is taken to be zero. The electric field for a given potential is then:

$$
\bar{E}(\phi) = \sqrt{2 \int_{0}^{\phi} \frac{-qNd}{\varepsilon_s} \left(1 - \exp\left(\frac{q\phi}{kT}\right)\right) d\phi = \text{sign}(\phi) \frac{V_t}{L_D} \sqrt{2 \left[\exp\left(\frac{\phi}{V_t}\right) - \frac{\phi}{V_t}\right] - 1}} \quad [3.1.10]
$$

Where the sign function equals +1 or -1 depending on the sign of $\phi$ and $L_D$ is the Debye length given by, $L_D = \sqrt{\frac{\varepsilon_s kT}{q^2 N_d}}$. Equation [3.1.10] is plotted in figure 3.2 using normalized parameters. Depletion occurs for negative potentials while accumulation occurs for positive potentials.

![Normalized Potential vs Normalized Field](image)

**Fig.3.2** Absolute value of the normalized electric field, $|\bar{E}| L_D/V_t$, versus normalized potential, $\phi/V_t$
Applying Gauss's law \((Q = \varepsilon_s \mathcal{E})\) we find the relation between the total charge in the semiconductor region and the total potential across the semiconductor. The capacitance can also be obtained from:

\[
C = \left| \frac{dQ}{dV_a} \right| = \varepsilon_s \left| \frac{d}{dV_a} \left[ \frac{\exp(\phi_s/V_t) - 1}{\sqrt{2[\exp(\phi_s/V_t) - \phi_s/V_t - 1]}} \right] \right| \quad [3.1.11]
\]

where \(\phi_s\) is the potential across the semiconductor and equals \(-\phi_i + V_a\). This expression can be approximated for \(\phi_s < 0\) and \(|\phi_s| >> V_t\) yielding:

\[
C = \frac{\varepsilon_s}{L_D} \frac{1}{\sqrt{2(\phi_i - V_a - V_t)/V_t}} = \frac{\varepsilon_s q N_d}{2(\phi_i - V_a - V_t)} \quad [3.1.12]
\]

This expression equals [3.1.6] as derived using the full depletion approximation, except for the added term in the denominator. This expression yields the capacitance value with a relative accuracy better than 0.3% for \(V_a < \phi_i - 6V_t\).

### 3.1.2 Numeric solution

A numeric solution can be obtained by integrating equation [3.1.10]. The solution to the energy band diagram, the charge density, the electric field and the potential are shown in the figures below: Integration was started four Debye lengths to the right of the edge of the depletion region as obtained using the full depletion approximation. Initial conditions were obtained by assuming the potential at the starting point to be adequately expressed by a solution to the homogenous equation:

\[
\phi = V_t \exp \left(-\frac{(x-x_d)}{L_D}\right) \quad [3.1.13]
\]

Shown are solutions for a gold-silicon M-S junction with \(\Phi_M = 4.75V\), \(\chi = 4.05V\), \(N_d = 10^{16} \text{ cm}^{-3}\) and \(\varepsilon_s/\varepsilon_0 = 11.9\).
Fig. 3.3 Energy band diagram of an M-S junction

Fig. 3.4 Charge density versus position in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.
Fig. 3.5 Electric field versus distance in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

Fig. 3.6 Potential versus distance of an M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

a) Depletion at the Metal-Semiconductor interface
Most metal semiconductor contacts have a depletion region adjacent to the interface. We distinguish between the case where a large potential variation is found across the semiconductor, for which only a small correction is obtained compared to the full depletion approximation, and the case where a small potential variation exists across the semiconductor, for which the full depletion approximation does not apply.

\( \alpha \) large potential approximation

If the potential difference across the semiconductor is larger than the thermal voltage, or \( \phi_s = V_a - \phi_i < 0 \) and \( |V_a - \phi_i| >> kT/q \) we find the effective depletion layer width, \( x_d \), defined as the ratio of the total depletion layer charge to the charge density of the fully ionized donors, to be:

\[
x_d = \frac{Q_d}{qN_d} = L_D \sqrt{\frac{2(\phi_i - V_a - V_t)}{V_t}}
\]  \[3.1.14\]

where \( L_D \) is the extrinsic Debye length of the semiconductor, which is given by:

\[
L_D = \sqrt{\frac{\varepsilon_s kT}{q^2 N_d}}
\]  \[3.1.15\]

The small signal capacitance can be expressed by:

\[
C = \frac{dQ_d}{dV} = \frac{\varepsilon_s}{L_D} \sqrt{\frac{V_t}{2(\phi_i - V_a - V_t)}} = \frac{\varepsilon_s}{x_d}
\]  \[3.1.16\]

where \( Q_d \) is the total charge per unit area in the depletion layer. This result differs from the one obtained by using the full depletion approximation in that the applied voltage is increased by the thermal voltage. However the capacitance is still the ratio of the dielectric constant to the depletion layer width.

\( \beta \) small potential approximation

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If the potential difference across the semiconductor is smaller than the thermal voltage, or 
\( \phi = V_a - \phi_i < 0 \) and \( |V_a - \phi_i| < kT/q \), the depletion layer width is proportional to the Debye length and the applied voltage:

\[
x_d = L_D \frac{\phi_i - V_a}{V_t}
\]

and the capacitance is constant, independent of the applied voltage:

\[
C = \frac{\varepsilon_s}{L_D}
\]

**b) Accumulation at the Metal-Semiconductor interface**

Accumulation occurs at the semiconductor metal interface if the Fermi level of the metal lies between the conduction band edge and the Fermi level in the n-type semiconductor, or \( \Phi_s > \Phi_M > \chi \). A similar condition can be defined for p-type material. Equation [3.1.10] applies for depletion as well as accumulation. However it does not provide a solution for the electric field and potential as a function of position. Instead we start again from the integral formulation of equation [3.1.10] but set the potential equal to zero at the interface and integrate from 0 to \( x \). We also assume that the electron concentration at the surface, \( n_s \) is much larger than the donor concentration. Using this convention, equation [3.1.10] can be rewritten as:

\[
\tilde{\varepsilon}(\phi) = \sqrt{\frac{2 n_s kT}{\varepsilon_s}} \exp\left(\frac{q\phi}{2kT}\right) = -\frac{d\phi}{dx}
\]

integrating this equation again from 0 to \( x \) yields

\[
\sqrt{\frac{2 q n_s kT}{\varepsilon_s}} x = 2 V_t (e^{-\phi/2V_t} - 1)
\]

from which the charge density can be obtained:
\[ \rho(x) = -q \left( n_s e^{\phi/V_t} \right) = \frac{-q n_s}{(1 + \frac{x}{\sqrt{2L_D}})^2} \]  \[3.1.21\]

Integration of the charge density yields the electric field.

\[ \varepsilon(x) = \frac{\sqrt{2} kT}{L_D q} \left( 1 + \frac{x}{\sqrt{2L_D}} \right)^{-1} \]  \[3.1.22\]

The width of the accumulation layer is obtained by solving the expression for the potential for \( x \) with \( \phi(x_d) = \phi_i - V_a \).

\[ x_d = \sqrt{2} L_D (e^{\phi_i - V_a} / 2V_t - 1) \]  \[3.1.23\]

The correct solution can also be obtained by integrating [3.1.10]. A solution for a M-S junction with \( \Phi_M = 4.2V \), \( \chi = 4.05V \), \( N_d = 10^{16} \text{cm}^{-3} \) and \( \varepsilon_s / \varepsilon_0 = 11.9 \) is shown in the figures below.
Fig. 3.7 Charge density, electric field, potential and energy band diagram under accumulation conditions.

3.1.3 Schottky barrier with an interfacial layer
A more elaborate model of the Schottky barrier contains an interfacial layer between the semiconductor and the metal. Typically this layer is a thin oxide layer, with thickness $d$, which naturally forms on the surface of a semiconductor when exposed to air. The analysis of the Schottky diode can now be repeated using the full depletion approximation yielding the following relation between the total applied voltage and the depletion layer width:

$$\phi_i - V_a = \frac{q N_d x_n^2}{2 \varepsilon_s} + \frac{q N_d x_n d}{\varepsilon_{ox}} = \phi_n + \phi_{ox} \quad [3.1.24]$$

from which the depletion layer width can be solved. The capacitance of the structure can be obtained from the series connection of the oxide and semiconductor capacitance:

$$C_j = \frac{1}{d + \frac{x_n}{\varepsilon_{ox}}} = \frac{\varepsilon_s}{L_D} \sqrt{\frac{V_t}{2 (\phi^{*}_i - V_a)}} \quad [3.1.25]$$

with

$$\phi^{*}_i = \phi_i + \left(\frac{d}{\varepsilon_{ox}}\right)^2 \frac{q N_d \varepsilon_s}{2} \quad [3.1.26]$$

This expression is very similar to that of equation [3.1.16] except that the built-in voltage is increased by the oxide layer. The potential $\phi_n$ across the semiconductor can be written as:

$$\phi_n = \phi_i - V_a + \frac{\varepsilon_s q N_d d^2}{\varepsilon_{ox}^2} - \sqrt{\left(\frac{\varepsilon_s q N_d d^2}{\varepsilon_{ox}^2}\right)^2 + \frac{2 \varepsilon_s q N_d d^2}{\varepsilon_{ox}^2} (\phi_i - V_a)} \quad [3.1.27]$$

for zero applied voltage this reduces to:

$$\phi_n = \phi_i + \frac{\varepsilon_s q N_d d^2}{\varepsilon_{ox}^2} - \sqrt{\left(\frac{\varepsilon_s q N_d d^2}{\varepsilon_{ox}^2}\right)^2 + \frac{2 \varepsilon_s q N_d d^2}{\varepsilon_{ox}^2} \phi_i} \quad [3.1.28]$$

instead of simply $\phi_n = \phi_i$ when no oxide is present. This analysis can be interpreted as follows: the interfacial layer reduces the capacitance of the Schottky barrier diode,
although a capacitance measurement will have the same characteristics as an ideal Schottky barrier diode except that the built-in voltage is increased. However the potential across the semiconductor is decreased due to the voltage drop across the oxide layer, so that at low voltage the barrier for electrons flowing into the semiconductor is reduced yielding a higher current that without the oxide. It has been assumed that the interfacial layer forms a very thin tunnel barrier which at low voltages does not restrict the current. As the voltage applied to the Schottky barrier is more positive, the depletion layer width reduces, so that the field in the oxide also reduces and with it the voltage drop across the oxide. The current under forward bias conditions therefore approaches that of the ideal Schottky diode until the tunnel barrier restricts the current flow. This results in a higher ideality factor for Schottky barrier with an interfacial layer. From equations [3.1.24] through [3.1.26] we find that the effect is largest for highly doped semiconductors and interfacial layers with low dielectric constant.

3.1.4 Current across a M-S junction

The current across a metal-semiconductor junction is mainly due to majority carriers. Three distinctly different mechanisms exist: diffusion of carriers from the semiconductor into the metal, thermionic emission of carriers across the Schottky barrier and quantum-mechanical tunneling through the barrier. The diffusion theory assumes that the driving force is distributed over the length of the depletion layer. The thermionic emission theory on the other hand postulates that only energetic carriers, those which have an energy equal to or larger than the conduction band energy at the metal-semiconductor interface, contribute to the current flow. Quantum-mechanical tunneling through the barrier takes into account the wave-nature of the electrons allowing them to penetrate through thin barriers. In a given junction one finds that a combination of all three mechanisms could exist. However typically one finds only one to limit the current, making it the dominant current mechanism.
a) Diffusion theory

This analysis assumes that the depletion layer is large compared to the mean free path, so that the concepts of drift and diffusion are valid. We start from the expression for the total current and then integrate it over the width of the depletion region:

\[ J = q \left( n \mu_n \frac{\partial \phi}{\partial x} + D_n \frac{\partial n}{\partial x} \right) \]  \[ \text{[3.1.29]} \]

which can be rewritten by using \( \frac{\partial \phi}{\partial x} = -\frac{d\phi}{dx} \) and multiplying both sides of the equation with \( \exp(-\phi/V_t) \), yielding:

\[ J \exp(-\phi/V_t) = q D_n \left( n \frac{d\phi}{dx} + \frac{\partial n}{\partial x} \right) \exp(-\phi/V_t) = q D_n \frac{d}{dx} \left[ n \exp(-\phi/V_t) \right] \]  \[ \text{[3.1.30]} \]

Integration of both sides of the equation over the depletion region yields:

\[ J = \frac{q D_n}{\int_0^{x_d} \exp(-\phi/V_t) \, dx} \left[ n \exp(-\phi/V_t) \right] \bigg|_0^{x_d} = q D_n \frac{N_c \exp(-\phi_B/V_t)}{\int_0^{x_d} \exp(-\phi/V_t) \, dx} \left( \exp(V_a/V_t) - 1 \right) \]  \[ \text{[3.1.31]} \]

Where the following values were used for the electron density and the potential:

\[ \begin{array}{ccc}
  x & n(x) & \phi(x) \\
  0 & N_c \exp(-\phi_B/V_t) & -\phi_i + V_a \\
  x_d & N_d = N_c \exp(-\phi_B/V_t) \exp(\phi_i/V_t) & 0 \\
\end{array} \]

and \( \phi^* = \phi + \phi_i - V_a \). The integral in the denominator can be solved using the potential obtained from the full depletion approximation [3.1.5], or:

\[ \phi = - \frac{qN_d}{2\varepsilon_s} (x - x_d)^2 \]  \[ \text{[3.1.32]} \]
so that $\phi^*$ can be written as:

$$\phi^* = \frac{qN_d}{\varepsilon_s} x (x_d - \frac{x}{2}) \equiv \frac{qN_d}{\varepsilon_s} x_{d} = (\phi_i - V_a) \frac{x}{x_d}$$  \[3.1.33\]

where the second term is dropped since the linear term is dominant. Using this approximation one can solve the integral as:

$$\int_{0}^{x_d} \exp(-\phi^*/V_t) \, dx \equiv x_d \frac{(\phi_i - V_a)/V_t}{\varepsilon_s} \quad \[3.1.34\]$$

for $(\phi_i - V_a)>V_t$. This yields the final expression for the current due to diffusion:

$$J = \frac{q D_n N_c}{V_t} \sqrt{\frac{2q(\phi_i - V_a)N_d}{\varepsilon_s}} \exp(-\phi_B/V_t) \left[\exp(V_a/V_t) - 1\right]$$  \[3.1.35\]

**b) Thermionic emission theory**

The thermionic emission theory\(^2\) assumes that electrons which have an energy larger than the top of the barrier will cross the barrier provided they move towards the barrier. The actual shape of the barrier is hereby ignored. The current can be expressed as:

$$J_{\text{right-left}} = \int_{E_C(x = \infty) + q\phi_n}^{\infty} q \nu_x \frac{dn}{dE} dE$$  \[3.1.36\]

For non-degenerately doped material, the density of electrons between $E$ and $E + dE$ is given by: (using [A.1.12] and assuming $E_{fm} < E_c - 3kT$)

$$\frac{dn}{dE} = \delta_{3D}(E) F(E) = \frac{4\pi(2m^* \nu kT)}{h^3} \sqrt{E-E_c} \exp\left[\frac{(E-E_m)}{kT}\right]$$  \[3.1.37\]

\(^2\)see also S.M. Sze "Physics of Semiconductor Devices", Wiley and Sons, second edition, p. 255
Assuming a parabolic conduction band (with constant effective mass \( m^* \)), the carrier energy \( E \) can be related to its velocity \( v \) as:

\[
E - E_c = \frac{1}{2} m^* v^2 = m^* \frac{dv}{2} \quad \frac{dE}{E - E_c} = \sqrt{E - E_c} = v \sqrt{\frac{m^*}{2}} \tag{3.1.38}
\]

Combining [3.1.37] with [3.1.38] yields:

\[
\frac{dn}{dE} \frac{dE}{E - E_c} = 2 \left( \frac{m^*}{h} \right)^3 \exp\left[- \frac{E_c(x = \infty) - E_{fn}}{kT} \right] \exp\left[- \frac{m^* v^2}{2kT} \right] 4\pi v^2 \, dv \tag{3.1.39}
\]

while replacing \( v^2 \) by \( v_x^2 + v_y^2 + v_z^2 \) and \( 4\pi v^2 \, dv \) by \( dv_x \, dv_y \, dv_z \) the current becomes:

\[
J_{r-l} = 2 \left( \frac{m^*}{h} \right)^3 \int_{-\infty}^{\infty} \exp\left[- \frac{m^* v_x^2}{2kT} \right] \, dv_x \int_{-\infty}^{\infty} \exp\left[- \frac{m^* v_y^2}{2kT} \right] \, dv_y \int_{-\infty}^{\infty} \exp\left[- \frac{m^* v_z^2}{2kT} \right] \, dv_z
\]

while setting \( v_x \)

\[
= 2q \left( \frac{m^*}{h} \right)^3 2\pi kT \frac{m^*}{m^*} \exp\left[- \frac{E_c(x = \infty) - E_{fn}}{kT} \right] \exp\left[- \frac{m^* v_0 x^2}{2kT} \right] \frac{kT}{m^*} \tag{3.1.40}
\]

using

\[
\int_{-\infty}^{\infty} \exp\left[- \frac{m^* v_y^2}{2kT} \right] \, dv_y = \int_{-\infty}^{\infty} \exp\left[- \frac{m^* v_z^2}{2kT} \right] \, dv_z = \sqrt{\frac{2\pi kT}{m^*}} \tag{3.1.41}
\]

The velocity \( v_{0x} \) is obtained by setting the kinetic energy equal to the potential across the n-type region:

\[
\frac{1}{2} m^* v_{0x}^2 = q\phi_n \tag{3.1.42}
\]
$v_{OX}$ is the minimal velocity of an electron in the quasi-neutral n-type region, needed to cross the barrier. Using

$$\phi_i - V_a = \phi_B - \frac{1}{q} \{ E_c(x = \infty) - E_{fn} \} = \phi_n,$$

which is only valid for a metal-semiconductor junction\(^3\) one obtains

$$J_{MS} = A^* T^2 e^{-\phi_B / V_t} [e^{V_a / V_t -1}] \quad [3.1.43]$$

where $A^* = \frac{4\pi q m^* k^2}{h^3}$ is the Richardson constant and $\phi_B$ is the Schottky barrier height which equals the difference between the Fermi level in the metal, $E_{fM}$, and the conduction band edge, $E_c$, evaluated at the interface between the metal and the semiconductor. The -1 is added to account for the current flowing from right to left\(^4\). The current flow from right to left is independent of the applied voltage since the barrier is independent of the bandbending\(^5\) in the semiconductor and equal to $\phi_B$. Therefore it can be evaluated at any voltage. For $V_a = 0$ the total current must be zero, yielding the -1 term.

**c) Tunneling across a barrier.**

Quantum mechanical tunneling of carriers through a barrier is an important mechanism for thin barriers. We start from the time independent Schrödinger equation:

\[^3\text{for a n}^+\text{-n junction (section 3.2.3) we will have to modify this expression}\]

\[^4\text{This method assumes that the effective mass of the carriers is the same on both sides of the barrier. This issue is discussed in more detail in section 3.2.3.}\]

\[^5\text{ignoring the Schottky barrier lowering due to image charges}\]
\[ -\frac{\hbar^2}{2m^*} \psi'' + \psi V(x) = E \psi \]  \[3.1.44\]

which can be rewritten as

\[ 2\psi \psi'' = \frac{d(\psi^2)}{dx} = \frac{2m^*(V-E)}{\hbar^2} \frac{d(\psi^2)}{dx} \]  \[3.1.45\]

provided \( V-E \) is independent of position. Integration yields:

\[ \psi' = \pm k \psi \quad \text{with} \quad k = \frac{2m^*[V(x)-E]}{\hbar} \]  \[3.1.46\]

Again assuming \( V \) to be constant this equation can be solved between \( x \) and \( x + dx \)

\[ \psi(x+dx) = \psi(x) e^{-kdx} \]  \[3.1.47\]

The minus sign is chosen since we assume the particle to move from left to right. For a slowly varying potential the amplitude of the wave function at \( x = L \) can be related to the wave function at \( x = 0 \):

\[ \psi(L) = \psi(0) \exp\left(-\int_{0}^{L} \frac{\sqrt{2m^*[V(x)-E]}}{\hbar} dx\right) \]  \[3.1.48\]

This equation is referred to as the WKB approximation\(^6\). From this the tunneling probability, \( \Theta \), can be calculated\(^7\) for a triangular barrier for which \( V(x)-E = q\phi_B(1-\frac{x}{L}) \)

\(^6\)Named after Wigner, Kramers and Brillouin

\(^7\)Using \( \int_{0}^{L} \sqrt{1-\frac{x}{L}} dx = \frac{2L}{3} \)
\[
\Theta = \frac{\psi(L) \psi^*(L)}{\psi(0) \psi^*(0)} = \exp\{-2 \int_0^L \frac{\sqrt{2m^*}}{\hbar} \sqrt{\phi_B (1 - \frac{x}{L})} \, dx\} \quad [3.1.49]
\]

the tunneling probability then becomes

\[
\Theta = \exp\left[-\frac{4 \sqrt{2m^*} (q\phi_B)^{3/2}}{3 \, q \, \hbar \, \varepsilon}\right] \quad [3.1.50]
\]

The tunnel current is proportional to \( \Theta \) and therefore has the same dependence on \( \phi_B \).

### 3.2 The n-n+ junction

#### 3.2.1 The n-n+ homojunction

When contacting semiconductor devices one very often includes highly doped semiconductor layers to lower the contact resistance between the semiconductor and the metal contact. This added layer causes a n-n+ junction within the device. Most often these junctions are ignored in the analysis of devices, in part because of the difficulty treating them correctly, in part because they can simply be ignored. The build-in voltage of a n-n+ junction is given by:

\[
\phi_i = \frac{1}{q} (E_{fn} - E_{fm}) = V_t \ln \frac{N_{d+}}{N_d} \quad [3.2.1]
\]

Which means that the built-in voltage is about 59.4 meV if the doping concentrations differ by a factor 10. It is because of this small built-in voltage that this junction is often ignored. However large ratios in doping concentration do cause significant potential variations.

The influence of the n-n+ junction must be evaluated in conjunction with its current voltage characteristics: if the n-n+ junction is in series with a p-n diode, the issue is whether or not the n-n+ junction affects the operation of the p-n junction in any way.
low current densities one can expect the p-n diode to dominate the current flow, whereas at high current densities the n-n\(^+\) junction could play a role if not designed properly.

For the analysis of the n-n\(^+\) junction we start from a flat band energy band diagram connecting the two regions in absence of an electric field. One can visualize that electrons will flow from the n\(^+\) region and accumulate in the n region. However, since the carrier concentration must be continuous (this is only required in a homojunction), the carrier density in the n region is smaller that the doping concentration of the n\(^+\) region, and the n\(^+\) region is not completely depleted. The full depletion approximation is therefore not applicable. Instead one recognizes the situation to be similar to that of a metal-semiconductor junction: the n\(^+\) is depleted but has a small voltage across the semiconductor as in a Schottky barrier with small voltage applied, whereas the n region is accumulated as in an ohmic contact. A general solution of this structure requires the use of equation [3.1.4].

A simple solution is obtained in the limit where the potential across both regions is smaller than the thermal voltage. The charge in the n-n\(^+\) structure region is then given by:

\[
\rho(x) = \begin{cases} 
\varepsilon_s n \frac{\phi_i - V_a}{L_{Dn} + L_{Dn}^+} e^{x/L_{Dn}} & \text{for } x < 0 \\
\varepsilon_s n^+ \frac{\phi_i - V_a}{L_{Dn}^+ + L_{Dn} + L_{Dn}^+} e^{-x/L_{Dn}^+} & \text{for } x > 0 
\end{cases} 
\]  

where the interface is assumed at x equal zero, and L\(_{Dn}\) and L\(_{Dn}^+\) are the extrinsic Debye lengths in the material. Applying Poisson's equation one finds the potentials to be:

\[
\phi_n(x) = L_{Dn} \frac{\phi_i - V_a}{L_{Dn} + L_{Dn}^+} e^{x/L_{Dn}} \quad \text{for } x < 0 \\
\phi_{n^+}(x) = L_{Dn} \frac{\phi_i - V_a}{L_{Dn}^+ + L_{Dn} + L_{Dn}^+} e^{-x/L_{Dn}^+} \quad \text{for } x > 0 
\]  

[3.2.2]
The solutions for the charge density, electric field, potential and energy band diagram are plotted in the figures below:
Fig. 3.8. Charge, electric field, potential and energy band diagram in a silicon n-n+ structure with $N_d = 10^{16} \text{ cm}^{-3}$, $N_{d^+} = 10^{17} \text{ cm}^{-3}$ and $V_a = 0$. 
3.2.2 The n-n⁺ heterojunction

Consider a n-n⁺ heterojunction including a spacer layer with thickness \( d \) as shown in the figure below.

![Flatband energy diagram of a n-n⁺ heterojunction with a spacer layer with thickness \( d \).](image)

The built-in voltage for a n-n⁺ heterojunction with doping concentrations \( N_d \) and \( N_{d^+} \) is given by:

\[
\phi_i = \frac{1}{q} (E_{fn^+} - E_{fn}) = V_t \ln \left( \frac{N_{d^+}^n N_{cn}}{N_d N_{cn}^+} \right) + \Delta E_c
\]

Where \( N_{cn} \) and \( N_{cn^+} \) are the effective densities of states of the low and high doped region respectively. Unlike a homojunction, the heterojunction can have a built-in voltage which is substantially larger than the thermal voltage. This justifies using the full depletion approximation for the depleted region. For the accumulated region one has to consider the influence of quantization of energy levels because of the confinement of carriers by the electric field and the hetero-interface.

**a) Analysis without quantization**
For the classic case where the material does not become degenerate at the interface one can use \[3.1.4b\] to find the total charge in the accumulation layer:

\[-Q_{acc} = q N_d^+ x_n^+ = \varepsilon_{sn} \frac{V_t}{L_D} \sqrt{2 \left[ \exp\left(\frac{\phi_n}{V_t}\right) - \frac{\phi_n}{V_t} - 1 \right]} \]  \[3.2.5\]

while the potentials and the field can be solved for a given applied voltage using:

\[\phi_{n^+} + \phi_{sp} + \phi_n = \phi_i - V_a \]  \[3.2.6\]

\[\phi_{n^+} = \frac{\varepsilon_{sn} \phi_n^2}{2q\varepsilon_{sn}^+ N_d^+} \]  \[3.2.7\]

\[\phi_{sp} = \frac{\varepsilon_{sn} \phi_n d}{\varepsilon_{ssp}} \]  \[3.2.8\]

The subscript \(sp\) refers to the undoped spacer layer with thickness \(d\), which is located between the two doped regions. These equations can be solved by starting with a certain value of \(\phi_n\) which enables to calculate the electric field, the other potentials and the corresponding applied voltage, \(V_a\).

b) Analysis including quantization

The analysis of a \(n-n^+\) heterojunction including quantized levels is more complicated because the energy levels depend on the potential which can only be calculated if the energy levels are known. A self-consistent calculation is therefore required to obtain a correct solution. An approximate method which also clarifies the steps needed for a correct solution is described below\(^8\).

\(^8\)A similar analysis can also be found in Weisbuch and Vinter, Quantum Semiconductor Structures, pp 40-41, Academic Press, 1991.
Starting from a certain density of electrons per unit area, \( N_s \), which are present in the accumulation layer, one finds the field at the interface:

\[
\varepsilon_n = \frac{qN_s}{\varepsilon_{sn}}
\]  

[3.2.9]

We assume that only the \( \ell = 1 \) energy level is populated with electrons. The minimal energy can be expressed as a function of the electric field using equation\(^9\) [A.1.17]:

\[
E_{1n} = \left( \frac{\hbar^2}{2m^*} \right)^{1/3} \left( \frac{9\pi}{8} q \varepsilon_n \right)^{2/3}
\]  

[3.2.10]

The bandgap discontinuity \( \Delta E_c \) can then be related to the other potentials of the junction using [3.2.4], [3.2.6] and [A.1.8], yielding:

\[
\Delta E_c - V_a = E_{1n} + kT \ln \left[ \exp \left( \frac{\varepsilon_{sn} \varepsilon_n}{qN_{cqw}} \right) - 1 \right] + kT \ln \frac{N_{cn}^+}{N_{d^+}} + q\phi_{n^+} + q\phi_{sp}
\]  

[3.2.11]

where the potentials in turn can be expressed as a function of \( \varepsilon_n \):

\[
\phi_{n^+} = \frac{\varepsilon_{sn} \varepsilon_n^2}{2q\varepsilon_{sn}^+ N_{d^+}}
\]  

[3.2.12]

\[
\phi_{sp} = \frac{\varepsilon_{sn} \varepsilon_n d}{\varepsilon_{ssp}}
\]  

[3.2.13]

These equations can be combined into one transcendental equation as a function of the electric field, \( \varepsilon_n \).

\[
\Delta E_c - V_a = E_{1n} + kT \ln \left[ \exp \left( \frac{\varepsilon_{sn} \varepsilon_n}{qN_{cqw}} \right) - 1 \right] + kT \ln \frac{N_{cn}^+}{N_{d^+}} + \frac{\varepsilon_{sn} \varepsilon_n^2}{2\varepsilon_{sn}^+ N_{d^+}} + \frac{q\varepsilon_{sn} \varepsilon_n d}{\varepsilon_{ssp}}
\]  

[3.2.14]

---

\(^9\)A more detailed quantum mechanical derivation yields the \( 9\pi/8 \) term instead of \( 3\pi/2 \). See for instance F. Stern, Phys. Rev. B 5 p 4891, (1972). The two differ by \( (3/4)^{3/2} = 0.826 \) or 17.5%
Once $\varepsilon_n$ is known all potentials can be obtained.

![Energy band diagram](image)

**Fig. 3.10** Energy band diagram of a Al$_{0.4}$Ga$_{0.6}$As/GaAs $n^+$-$n$ heterostructure with $N_{d^+} = 10^{17}$ cm$^{-3}$, $N_d = 10^{16}$ cm$^{-3}$, $d = 10$ nm and $V_a = 0.15$. Comparison of analysis without quantization (upper curve) to that with quantization (lower curve).

![Electron density plot](image)

**Fig. 3.11** Electron density, $N_s$, in the accumulation region versus applied voltage, $V_a$, with quantization (top curve) and without quantization (bottom curve).
3.2.3 Currents across a $n^+-n$ heterojunction

Current transport across a $n^+-n$ heterojunction is similar to that of a metal-semiconductor junction: Diffusion, thermionic emission as well as tunneling of carriers across the barrier can occur. However to identify the current components one must first identify the potentials $\phi_{n^+}$ and $\phi_n$ by solving the electrostatic problem. From the band diagram one finds that a barrier exists for electrons going from the $n^+$ to the $n$-doped region as well as for electrons going in the opposite direction.

The analysis in the first section discusses the thermionic emission and yields a closed form expression based on a set of specific assumptions. The derivation also illustrates how a more general expression could be obtained. The next section describes the current-voltage characteristics of carriers traversing a depletion region, while the last section discusses how both effects can be combined.

a) Thermionic emission current across a $n^+-n$ heterojunction

The total current due to thermionic emission across the barrier is given by the difference of the current flowing from left to right and the current flowing from right to left. Rather than re-deriving the expression for thermionic emission, we will apply equation [3.1.40] to the $n^+-n$ heterojunction. One complication arises from the fact that the effective mass of the carriers is different on each side of the hetero-junction which would seem to indicate that the Richardson constant is different for carrier flow from left to right compared to the flow from right to left. A more detailed analysis reveals that the difference in effective mass causes a quantum mechanical reflection at the interface, causing carriers with the higher effective mass to be reflected back while carriers with the smaller effective mass are
to first order unaffected\textsuperscript{10}. We therefore use equation [3.1.40] for flow in both directions while using the Richardson constant corresponding to the smaller of the two effective masses, yielding:

\[
J_{HJ} = A^* T^2 \left\{ \exp\left[ -\frac{E_c(x=-\infty) - E_{fn}^+ + q\phi_n^+}{kT} \right] - \exp\left[ -\frac{E_c(x=\infty) - E_{fn} - q\phi_n + \Delta E_c}{kT} \right] \right\} [3.2.15]
\]

where the potentials are related to the applied voltage by\textsuperscript{11}:

\[
\phi_n^+ + \phi_n = \phi_i - V_a
\]  

[3.2.16]

and the built-in voltage is given by:

\[
\phi_i = \frac{\Delta E_c - (E_c(x=-\infty) - E_{fn}^+) + (E_c(x=\infty) - E_{fn})}{q}
\]  

[3.2.17]

Combining these relations yields:

\[
J_{HJ} = A^* T^2 \left\{ \exp\left[ \frac{\phi_n}{V_t} \right] \exp\left[ -\frac{\phi_B^*}{V_t} \right] \left[ e^{V_a/V_t} - 1 \right] \right\} [3.2.18]
\]

where the barrier height \(\phi_{B^*}\) is defined as:

\[
\phi_{B^*} = \frac{\Delta E_c + (E_c(x=\infty) - E_{fn})}{q}
\]  

[3.2.19]


\textsuperscript{11}No spacer layer is assumed in this derivation, but could easily be added if desired.
Assuming full depletion in the n⁺ depletion region and using equation [3.1.10] for the accumulated region, the charge balance between the depletion and accumulation layer takes the following form:

\[
\sqrt{2\varepsilon_{sn}^+ q N_d^+ \phi_{n}^+} = \varepsilon_{sn} V_t \sqrt{2 \left[ \exp\left(\frac{\phi_n}{V_t}\right) - \frac{\phi_n}{V_t} - 1 \right]} \tag{3.2.20}
\]

Combining equations [3.2.20] with [3.2.16] yields a solution for \(\phi_{n}^+\) and \(\phi_n\).

For the special case where \(\varepsilon_{sn}^+ N_d^+ = \varepsilon_{sn} N_d\) and \(\phi_n \gg V_t\) these equations reduce to:

\[
\phi_i - V_a = \phi_{n}^+ + \phi_n = V_t \exp\left(\frac{\phi_n}{V_t}\right) \tag{3.2.21}
\]

The current (given by [3.2.18]) can then be expressed as a function of the applied voltage \(V_a\)

\[
J_{HJ} = \frac{q A^* T \phi_i}{k} \left(1 - \frac{V_a}{\phi_i}\right) e^{-\phi_B^*/V_t} \left[e^{V_a/V_t} - 1 \right] \tag{3.2.22}
\]

Whereas this expression is similar to that of a metal-semiconductor barrier, it differs in that the temperature dependence is somewhat modified and the reverse bias current increases almost linearly with voltage. Under reverse bias the junction can be characterized as a constant resistance, \(R_{HJ}\), which equals:

\[
R_{HJ} = \frac{A J_{HJ}}{V_a} = \frac{A A^* T^2 / V_t e^{-\phi_B^*/V_t}}{e^{V_a/V_t} - 1} \tag{3.2.23}
\]

Where \(A\) is the area of the junction. This shows that the resistance changes exponentially with the barrier height. Grading of the heterojunction is typically used to reduce the spike in the energy band diagram and with it the resistance across the interface.

b) Calculation of the Current and quasi-Fermi level throughout a Depletion Region
We typically assume the quasi-Fermi level to be constant throughout the depletion region. This assumption can be justified for a homojunction but is not necessarily correct for a heterojunction p-n diode.

For a homojunction p-n diode we derived the following expression for the minority carrier density in the quasi-neutral region of a "long" diode

\[ n \equiv n - n_p = n_p e^{V_d/V_t} - 1) e^{-x/L_n} \quad [3.2.24] \]

so that the maximum change in the quasi-Fermi level, which occurs at the edge of the depletion region, equals:

\[ \frac{dF_n}{dx} = \frac{d(kT \ln n/n_i)}{dx} \approx \frac{kT}{L_n} \quad [3.2.25] \]

so that the change of the quasi-Fermi level can be ignored if the depletion region width is smaller than the diffusion length as is typically the case in silicon p-n diodes.

For a hetero-junction p-n diode one can not assume that the quasi-Fermi level is continuous, especially when the minority carriers enter a narrow bandgap region in which the recombination rate is so high that the current is limited by the drift/diffusion current in the depletion region located in the wide bandgap semiconductor.

The current density can be calculated from:

\[ J_n = q \mu_n n \frac{\partial}{\partial x} + q D_n \frac{dn}{dx} \quad [3.2.26] \]

Assuming the field to be constant throughout the depletion region one finds for a constant current density the following expression for the carrier density at the interface:

\[ n = \frac{J}{q \mu_n \frac{\partial}{\partial x} + N_d e^{-\frac{\xi}{N_t}}} \quad [3.2.27] \]
While for zero current one finds, for an arbitrary field
\[
0 = N_d \exp\left(-\int_{-\infty}^{0} \frac{\partial(x)}{V_t} dx\right) = N_d \exp(-\phi_n/V_t) \tag{3.2.28}
\]

Combining the two expressions we postulate the following expression for the carrier density:
\[
n = \frac{J}{q \mu_n \varepsilon_{\max}} + N_d e^{-\phi_n/V_t} \tag{3.2.29}
\]
The carrier density can also be expressed as a function of the total change in the quasi-Fermi level across the depletion region, \(\Delta E_{fn}\):
\[
n = N_d e^{-\phi_n/V_t} e^{-\Delta E_{fn}/kT} \tag{3.2.30}
\]
which yields the following expressions for the current density due to drift/diffusion:
\[
J = -q \mu_n \varepsilon_{\max} N_d e^{-\phi_n/V_t} (1 - e^{-\Delta E_{fn}/kT}) \tag{3.2.31}
\]
where is \(\varepsilon_{\max}\) the field at the heterojunction interface. If \(\Delta E_{fn}\) equals the applied voltage, as is the case for an n\(^{+}\)-n heterostructure, this expression equals:
\[
J = q \mu_n \varepsilon_{\max} N_d e^{-\phi_n/V_t} (e^{-V_a/V_t} - 1) \tag{3.2.32}
\]
which reduces for a MS junction to:
\[
J_{dd} = J_{thermionic} \mu_n \varepsilon_{\max} /v_R \tag{3.2.33}
\]
so that thermionic emission dominates for $v_R << \mu_n n_{\text{max}}$ or when the drift velocity is larger than the Richardson velocity.\(^\text{12}\)

c) Calculation of the current due to thermionic emission and drift/diffusion

The calculation of the current through a $n^+\text{-}n$ junction due to thermionic emission and drift/diffusion becomes straightforward once one realizes that the total applied voltage equals the sum of the quasi-Fermi level variation, $\Delta E_{fn}$, across each region. For this analysis we therefore rewrite the current expressions as a function of $\Delta E_{fn}$, while applying the expression for the drift/diffusion current to the $n^+$ material.

\[
J_{\text{thermionic}} = A^* T^2 \exp\left[\frac{\phi_n}{V_t}\right] e^{-\phi_B^*/V_t} \left[e^{\Delta E_{fn1}/kT} - 1\right]
\]

\[
J_{\text{drift/diffusion}} = - q \mu_{n+} n_{\text{max}} N_{cn+} e^{-\phi_B^*/V_t} \left(e^{\Delta E_{fn2}/kT} - 1\right) \quad [3.2.34]
\]

\(^{12}\)It should be noted here that the drift/diffusion model is not valid anymore as the drift velocity of the carriers approaches the thermal velocity.
3.3 Currents through insulators

Current mechanisms through materials which do not contain free carriers can be distinctly different from those in doped semiconductors or metals. The following section discusses Fowler-Nordheim Tunneling, Poole-Frenkel emission, Space charge effects as well as Ballistic transport.

3.3.1 Fowler-Nordheim tunneling

Fowler-Nordheim tunneling has been studied extensively in Metal-Oxide-Semiconductor structures where it has been shown to be the dominant current mechanism, especially for thick oxides. The basic idea is that quantum mechanical tunneling from the adjacent conductor into the insulator limits the current through the structure. Once the carriers have tunneled into the insulator they are free to move within the valence or conduction band of the insulator. The calculation of the current is based on the WKB approximation (as derived for the Schottky barrier diode in section 3.1.4) yielding the following relation between the current density, $J_{FN}$, and the electric field in the oxide, $\varepsilon_{ox}$:

$$J_{FN} = C_{FN} \varepsilon_{ox}^2 \exp\left[-\frac{4}{3} \frac{\sqrt{2 m^*_{ox}}}{q} \frac{(q \phi_B)^{3/2}}{\varepsilon_{ox}}\right] \tag{3.3.1}$$

where $\phi_B$ is the barrier height at the conductor/insulator interface in Volt, as shown in the figure below for electron tunneling from highly$^{13}$ n-type doped silicon into the silicon dioxide.

To check for this current mechanism, experimental I-V characteristics are typically plotted as $\ln(J_{FN}/\varepsilon_{ox}^2)$ versus $1/\varepsilon_{ox}$, a so-called Fowler-Nordheim plot. Provided the effective

$^{13}$This condition is added to eliminate additional complexity caused by bandbending at the interface.
mass of the insulator is known (for SiO$_2$, $m_{ox}^* = 0.42 m_0$) one can then fit the experimental data to a straight line yielding a value for the barrier height.

It is this type of measurement which has yielded experimental values for the conduction band difference between silicon and silicon-dioxide. The same method could also be used to determine heterojunction energy band off-sets provided Fowler-Nordheim tunneling is indeed the dominant current mechanism$^{14}$. It is important to stress that carriers must tunnel through the insulator which requires:

$$\varepsilon_{ox} d \geq \phi_B \quad [3.3.2]$$

which is typically the case for thick oxides and high electric fields.

3.3.2 Poole-Frenkel emission

The expression for Fowler-Nordheim tunneling implies that carriers are free to move through the insulator. Whereas this is indeed the case in thermally grown silicon-dioxide it is frequently not so in deposited insulators which contain a high density of structural defects. Silicon nitride (Si$_3$N$_4$) is an example of such material. The structural defects cause additional energy states close to the bandedge called traps. These traps restrict the current flow because of a capture and emission process, thereby becoming the dominant current mechanism. The current is a simple drift current described by

$$J = q n \mu \varepsilon_N \quad [3.3.3]$$

\[14\] This condition would required very large energy band discontinuities.
while the carrier density depends exponentially on the depth of the trap which is corrected for the electric field\textsuperscript{15}.

\[ n = n_0 \exp\left[- \frac{q}{kT} (\phi_B - \sqrt{\frac{q \varepsilon_N}{\pi \varepsilon_N}}) \right] \]  

[3.3.4]

The total current then equals:

\[ J_{PF} = q n_0 \mu N \exp\left[- \frac{q}{kT} (\phi_B - \sqrt{\frac{q \varepsilon_N}{\pi \varepsilon_N}}) \right] \]  

[3.3.5]

The existence of a large density of shallow\textsuperscript{16} traps in CVD silicon nitride makes Poole-Frenkel emission\textsuperscript{17} a frequently observed and well characterized mechanism.

3.3.3 Space charge limited current

Both Fowler-Nordheim tunneling and Poole-Frenkel emission mechanism yield very low current densities with correspondingly low carrier densities. For structures where carriers can readily enter the insulator and freely flow through the insulator one finds that the resulting current and carrier densities are much higher. The density of free carriers causes

\textsuperscript{15}This correction is equivalent to the Schottky barrier lowering due to the presence of an electric field.

\textsuperscript{16}deep traps also exist in silicon nitride. While these easily capture carriers, they are too deep to allow emission even in the presence of large fields. This causes a fixed charge in the silicon nitride which remains when the applied bias is removed. This charge trapping mechanism is used in non-volatile MNOS memory devices.

a field gradient which limits the current density. This situation occurs in lowly doped semiconductors and vacuum tubes. Starting from an expression for the drift current and Gauss’s law (where we assume that the insulator contains no free carriers if no current flows)

\[ J = q p \mu \varepsilon \]  

\[ \frac{d\varepsilon}{dx} = \frac{q p}{\varepsilon} \]  

we can eliminate the carrier density, \( p \), yielding:

\[ \frac{J}{\varepsilon \mu} = \varepsilon \frac{d\varepsilon}{dx} \]  

Integrating this expression from 0 to \( x \), where we assume the electric field to be zero\(^{18}\) at \( x = 0 \) one obtains:

\[ \frac{J x}{\varepsilon \mu} = \frac{\varepsilon^2}{2} \text{ or } \varepsilon(x) = \sqrt{\frac{2 x J}{\varepsilon \mu}} \]  

integrating once again from \( x = 0 \) to \( x = d \) with \( V(0) = V \) and \( V(d) = 0 \), one finds:

\[ V = \int_{0}^{d} \varepsilon \, dx = \sqrt{\frac{2 J d^{3/2}}{\varepsilon \mu^{3/2}}} \]  

from which one obtains the expression for the space-charge-limited current:

\[ \text{---------------------} \]

\(^{18}\)This implies an infinite carrier density. The analysis can be modified to allow for a finite carrier density. However the carrier pile-up do to the current restriction typically provides a very high carrier density at \( x=0 \).
3.3.4 Ballistic Transport in insulators

Ballistic transport is carrier transport without scattering or any other mechanism which would cause a loss of energy. Combining energy conservation, current continuity and Gauss's law one finds the following current-voltage relation:

\[ J = \frac{4 \varepsilon \mu V^2}{9 d^3} \sqrt{\frac{2 q}{m^*}} \frac{V^{3/2}}{d^2} \]  \[3.3.12\]

where \( d \) is the thickness of the insulator and \( m^* \) is the effective mass of the carriers.