The One Dimensional Schrödinger Equation

\[ i\hbar \frac{d\psi(x, t)}{dt} = -\frac{\hbar^2}{2\mu} \frac{\partial^2 \psi(x, t)}{dx^2} + V(x)\psi(x, t) \]

The 1-D Paraxial Wave Equation

\[ ik_0 \frac{d\phi(x, z)}{dz} = -\frac{1}{2m} \frac{\partial^2 \phi(x, z)}{dx^2} - k^2(x)\phi(x, z)/2 \]
The 1-D Time Independent Schrödinger Equation

\[ \frac{\partial^2 \psi(x)}{dx^2} + \frac{2\mu}{\hbar^2} (E - V(x)) \psi(x) = 0 \]

The 1-D Paraxial Wave Eigen Equation

\[ \frac{\partial^2 \phi(x)}{dx^2} + \left( k^2(x) - 2i k_0 \beta \right) \phi(x) = 0 \]
The 1-D Schrödinger Equation, with \( V(x) = V_0 \) inside

\[
\frac{\partial^2 \psi(x)}{dx^2} + \frac{2\mu}{\hbar^2} (E - V_0) \psi(x) = 0
\]

and its solutions

\[
\psi(x) = A \cos kx + B \sin kx
\]

\[
k = \sqrt{\frac{2\mu (E - V_0)}{\hbar^2}}
\]
The 1-D Schrödinger Equation, with \( V(x) = V_0 \) outside

\[
\frac{\partial^2 \psi(x)}{dx^2} + \frac{2\mu}{\hbar^2} (V_0 - E) \psi(x) = 0
\]

and its solutions

\[
\psi(x) = A \exp(-\kappa x) + B \exp(\kappa x)
\]

\[
\kappa = \sqrt{\frac{2\mu (V_0 - E)}{\hbar^2}}
\]
The 1-D Schrödinger Equation, with \( V(x) = \mu \omega^2 x^2 / 2 \)

\[
\frac{\partial^2 \psi(x)}{dx^2} + \frac{2\mu}{\hbar^2} (E - V(x)) \psi(x) = 0
\]

and its solutions

\[
\psi_n(\xi) = A_n h_n(\xi) \exp(-\xi^2 / 2) \\
\xi = \sqrt{\frac{\mu \omega}{\hbar}} x
\]
The solution of the 1-D Schrödinger Equation is

\[ |\psi(t)\rangle = \sum_n \int dx \langle \psi_0|x\rangle \langle x|n\rangle |n\rangle \exp(-iE_n t/\hbar) \]

which is equivalent to

\[ |\psi(t)\rangle = \sum_n \langle \psi_0|n\rangle |n\rangle \exp(-iE_n t/\hbar) \]