Chapter 15 Transformer Design

Some more advanced design issues, not considered in previous chapter:
- Inclusion of core loss
- Selection of operating flux density to optimize total loss
- Multiple winding design: as in the coupled-inductor case, allocate the available window area among several windings.
- A transformer design procedure
- How switching frequency affects transformer size

15.1 Transformer Design: Basic Constraints

Core loss

\[ P_c = K_{fe} (\Delta B)^2 A \]

Typical value of \( K_{fe} \) for ferrite materials: 2.6 or 2.7

\( \Delta B \) is the peak value of the ac component of \( B(t) \), i.e., the peak ac flux density.

So increasing \( \Delta B \) causes core loss to increase rapidly.

This is the first constraint.
Flux density
Constraint #2

Flux density \( B(t) \) is related to the applied winding voltage according to Faraday’s Law. Denote the volt-seconds applied to the primary winding during the positive portion of \( v_1(t) \) as \( \lambda_1 \).

\[
\lambda_1 = \int_{t_1}^{t_2} v_1(t) \, dt
\]

This causes the flux to change from its negative peak to its positive peak. From Faraday’s law, the peak value of the ac component of flux density is

\[
\Delta B = \frac{\lambda_1}{2A_n}
\]

To attain a given flux density, the primary turns should be chosen according to

\[
n_1 = \frac{\lambda_1}{2MB}
\]

Copper loss
Constraint #3

- Allocate window area between windings in optimum manner, as described in previous section.
- Total copper loss is then equal to

\[
P_{cu} = \rho \left( \frac{MLT}{W_A} \right) n_1^2 I_{tot}^2
\]

with

\[
I_{tot} = \sum_{j=1}^{k} n_j I_j
\]

Eliminate \( n_1 \), using result of previous slide:

\[
P_{cu} = \left( \frac{\rho \lambda_1^2 I_{1m}^2}{4K_n} \right) \left( \frac{(MLT)}{W_A} \right) \left( \frac{1}{\Delta B} \right)^3
\]

Note that copper loss decreases rapidly as \( \Delta B \) is increased.

Total power loss

4. \( P_{tot} = P_{cu} + P_{fe} \)

There is a value of \( \Delta B \) that minimizes the total power loss

\[
P_{sw} = P_c + P_n
\]

\[
P_{fe} = K_f (\Delta B) \beta_A I_{1m}
\]

\[
P_{sw} = \left( \frac{\rho \lambda_1^2 I_{1m}^2}{4K_n} \right) \left( \frac{(MLT)}{W_A} \right) \left( \frac{1}{\Delta B} \right)^2
\]
5. Find optimum flux density $\Delta B$

Given that

$$P_{tot} = P_{fe} + P_{cu}$$

Then, at the $B$ that minimizes $P_{tot}$, we can write

$$\frac{dP_{tot}}{dB} = \frac{dP_{fe}}{dB} + \frac{dP_{cu}}{dB} = 0$$

Note: optimum does not necessarily occur where $P_{fe} = P_{cu}$. Rather, it occurs where

$$\frac{dP_{fe}}{dB} = - \frac{dP_{cu}}{dB}$$

**Take derivatives of core and copper loss**

$$P_{fe} = K_{fe} (\Delta B)^{3/2} A_{lm}$$

$$P_{cu} = \rho_\lambda I_{tot}^2 \left( \frac{MLT}{W_{c} A_{c}} \right)$$

$$\frac{dP_{fe}}{dB} = \frac{3}{2} \rho_\lambda I_{tot}^2 \left( \frac{MLT}{W_{c} A_{c}} \right) (\Delta B)^{1/2}$$

$$\frac{dP_{cu}}{dB} = -2 \rho_\lambda I_{tot}^2 \left( \frac{MLT}{W_{c} A_{c}} \right) (\Delta B)^{-3/2}$$

Now, substitute into

$$\frac{dP_{fe}}{dB} = - \frac{dP_{cu}}{dB}$$

and solve for $\Delta B$:

$$\Delta B = \left( \frac{\rho_\lambda I_{tot}^2}{W_{c} A_{c} (MLT)} \right)^{2/3}$$

Optimum $\Delta B$ for a given core and application

**Total loss**

Substitute optimum $\Delta B$ into expressions for $P_{fe}$ and $P_{cu}$. The total loss is:

$$P_{tot} = \left( A_{lm} K_{fe} \right) \left( \frac{\rho_\lambda I_{tot}^2}{4W_{c} A_{c}} \right) \left[ \left( \frac{MLT}{W_{c} A_{c}} \right)^{2/3} \right]$$

Rearrange as follows:

$$\left( \frac{\rho_\lambda I_{tot}^2}{4W_{c} A_{c}} \right) \left( \frac{MLT}{W_{c} A_{c}} \right)^{2/3} = \frac{\rho_\lambda I_{tot}^2}{4W_{c} A_{c}} \left( P_{fe} \right)^{1/3}$$

Left side: terms depend on core geometry

Right side: terms depend on specifications of the application
The core geometrical constant $K_{fe}$

Define $K_{fe} = \frac{W_A A_c}{(\beta/2)^{2} - (\beta/2)^{2}}$

Design procedure: select a core that satisfies

$$K_{fe} \geq \frac{\rho \lambda_{1}^{2} I_{t}^{2} K_{fe}^{2}}{4 \beta K_u P_{tot}^{10/\beta} + 2}$$

Appendix D lists the values of $K_{fe}$ for common ferrite cores

$K_{fe}$ is similar to the $K_{g}$ geometrical constant used in Chapter 14:

- $K_{g}$ is used when $B_{max}$ is specified
- $K_{fe}$ is used when $B$ is to be chosen to minimize total loss

15.2 Step-by-step transformer design procedure

The following quantities are specified, using the units noted:

- Wire effective resistivity $\rho$ (Go=cm)
- Total rms winding current, ref to pri $I_{t}$ (A)
- Desired turns ratios $n_{1}, n_{2}, n_{3}, \ldots$
- Applied pri volt-sec $V_{s}$ (V-sec)
- Winding fill factor $K_{u}$
- Core loss exponent $\beta$
- Core loss coefficient $K_{fe}$ (W/cm$^{3}$T$^{\beta}$)

Other quantities and their dimensions:

- Core cross-sectional area $A_c$ (cm$^2$)
- Core window area $W_A$ (cm$^2$)
- Mean length per turn $MLT$ (cm)
- Magnetic path length $l_e$ (cm)
- Wire areas $A_w, \ldots$ (cm$^2$)
- Peak ac flux density $B_{max}$ (T)

Procedure

1. Determine core size

$$K_{fe} \geq \frac{\rho \lambda_{1}^{2} I_{t}^{2} K_{fe}^{2}}{4 \beta K_u P_{tot}^{10/\beta} + 2}$$

Select a core from Appendix D that satisfies this inequality.

It may be possible to reduce the core size by choosing a core material that has lower loss, i.e., lower $K_{fe}$. 


2. Evaluate peak ac flux density

\[ \Delta B = \left[ 10^\frac{2\beta J_2^2}{2\mu_0 N_1} (MLT) W_{ac} L_1 \right]^{\frac{1}{1+\beta}} \]

At this point, one should check whether the saturation flux density is exceeded. If the core operates with a flux dc bias \( B_{dc} \), then \( \Delta B + B_{dc} \) should be less than the saturation flux density \( B_{sat} \).

If the core will saturate, then there are two choices:

- Specify \( \Delta B \) using the \( k_p \) method of Chapter 14, or
- Choose a core material having greater core loss, then repeat steps 1 and 2.

3. and 4. Evaluate turns

Primary turns:

\[ n_1 = n_1 = \frac{N_1}{N_1} \times 10^4 \]

Choose secondary turns according to desired turns ratios:

\[ n_2 = n_1 \times \frac{n_2}{n_1} \]
\[ n_3 = n_1 \times \frac{n_3}{n_1} \]

5. and 6. Choose wire sizes

Choose wire sizes according to:

\[ A_{	ext{tot}} = \frac{\alpha_K W}{n_1} \]

Fraction of window area assigned to each winding:

\[ \alpha_{1} = \frac{n_1}{N_1} \]
\[ \alpha_{2} = \frac{n_2}{N_2} \]
\[ \vdots \]
\[ \alpha_{k} = \frac{n_k}{N_k} \]
Check: computed transformer model

Predicted magnetizing inductance, referred to primary:
\[ L_{M} = \frac{\mu_1}{\pi} A_{cm} \]

Peak magnetizing current:
\[ i_{M, pk} = \frac{\lambda_1}{2L_{M}} \]

Predicted winding resistances:
\[ R_1 = \rho_n \frac{A_{w1}}{n_1} LMT \]
\[ R_2 = \rho_n \frac{A_{w2}}{n_2} LMT \]

Predicted magnetizing inductance, referred to primary:
\[ i_{M} = \frac{\lambda_1}{2L_{M}} \]

Peak magnetizing current:
\[ i_{M, pk} = \frac{\lambda_1}{2L_{M}} \]

Predicted winding resistances:
\[ R_1 = \rho_n \frac{A_{w1}}{n_1} LMT \]
\[ R_2 = \rho_n \frac{A_{w2}}{n_2} LMT \]

15.4.1 Example 1: Single-output isolated Cuk converter

Use a ferrite pot core, with Magnetics Inc. P material. Loss parameters at 200 kHz are
\[ K_{fe} = 24.7 \]
\[ K_{v} = 2.6 \]

Waveforms

Applied primary volt-seconds:
\[ \lambda_i = D \frac{V_i}{n} = 0.5 (5 \text{ vac}) (25 \text{ V}) = 62.5 \text{ V·sec} \]

Applied primary rms current:
\[ I_1 = \sqrt{\frac{1}{D} \left( \frac{V_i}{n} \right)^2} = 4 \text{ A} \]

Applied secondary rms current:
\[ I_2 = n I_1 = 20 \text{ A} \]

Total rms winding current:
\[ I_{tot} = I_1 + I_2 = 8 \text{ A} \]
Choose core size

\[
K_{fe} = \left(1.724 \times 10^{-6}\right) \left(62.5 \times 10^{-6}\right)^2 \left(8\right)^2 \left(24.7\right)^2 \left(10^3\right) / (0.5) (0.25) 4.6 / 2.6 = 0.00295
\]

Pot core data of Appendix D lists 2213 pot core with

\[K_{fe} = 0.0049\]

Next smaller pot core is not large enough.

Evaluate peak ac flux density

\[
\Delta B = \left(1.724 \times 10^{-6}\right) \left(62.5 \times 10^{-6}\right)^2 \left(0.0858\right) (0.635) 3 \left(3.15\right) / (2.6) (24.7) 1 \approx 0.0858 \text{ Tesla}
\]

This is much less than the saturation flux density of approximately 0.35 T. Values of \(\Delta B\) in the vicinity of 0.1 T are typical for ferrite designs that operate at frequencies in the vicinity of 100 kHz.

Evaluate turns

\[
n_1 = \left(62.5 \times 10^{-6}\right) / 0.0858 / (0.635) 3 \left(3.15\right) / (2.6) (24.7) 1 = 5.74 \text{ turns}
\]

\[n_2 = n_1 / (2.6) = 1.15 \text{ turns}\]

In practice, we might select

\[n_1 = 5 \text{ and } n_2 = 1\]

This would lead to a slightly higher flux density and slightly higher loss.
Determine wire sizes

Fraction of window area allocated to each winding:

\[ \alpha_1 = \frac{4A}{8A} = 0.5 \]
\[ \alpha_2 = \frac{20A}{8A} = 0.5 \]

(Since, in this example, the ratio of winding rms currents is equal to the turns ratio, equal areas are allocated to each winding)

Wire areas:

\[ A_{w1} = \left(0.5\times0.5\times0.297\times5\times10^{-3}\right) \text{ cm}^2 = 14.8 \times 10^{-3} \text{ cm}^2 \text{ AWG #16} \]
\[ A_{w2} = \left(0.5\times0.5\times0.297\times1\times10^{-3}\right) \text{ cm}^2 = 74.2 \times 10^{-3} \text{ cm}^2 \text{ AWG #9} \]

Wire sizes: discussion

Primary
5 turns #16 AWG
Secondary
1 turn #9 AWG

- Very large conductors!
- One turn of #9 AWG is not a practical solution

Some alternatives
- Use foil windings
- Use Litz wire or parallel strands of wire

Effect of switching frequency on transformer size for this P-material Cuk converter example

- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases
15.3.2 Example 2
Multiple-Output Full-Bridge Buck Converter

- Switching frequency: 150 kHz
- Transformer frequency: 75 kHz
- Turns ratio: 110:5:15
- Optimize transformer at $D = 0.75$

\[ n_2 + v_1(t) - D + n_3 Q_2 D + n_3 Q_3 D + n_3 Q_4 i_1(t) + 5 V - D + n_5 i_2(t) + 15 V - D + n_7 i_3(t) \]

Other transformer design details
- Use Magnetics, Inc. ferrite $P$ material. Loss parameters at 75 kHz:
  - $K_{Fe} = 7.6 \text{ W/T cm}^3$
  - $\beta = 2.6$
- Use E-E core shape
- Assume fill factor of $K_u = 0.25$ (reduced fill factor accounts for added insulation required in multiple-output offline application)
- Allow transformer total power loss of $P_{tot} = 4 \text{ W}$ (approximately 0.5% of total output power)
- Use copper wire, with
  - $\rho = 1.724 \times 10^{-8} \Omega \cdot \text{cm}$

Applied transformer waveforms
Applied primary volt-seconds

\[ \lambda_1 = DT_s V_p = (0.75 \times 6.67 \mu\text{sec}) \times (160 \text{ V}) = 800 \text{ V-\mu sec} \]

Applied primary rms current

\[ I_1 = \left( \frac{n_2}{n_1} I_{5V} + \frac{n_3}{n_1} I_{15V} \right) \sqrt{D} = 5.7 \text{ A} \]

Applied rms current, secondary windings

\[ I_2 = \frac{1}{2} I_{3V} \sqrt{T + D} = 66.1 \text{ A} \]
\[ I_3 = \frac{1}{2} I_{3V} \sqrt{T + D} = 9.9 \text{ A} \]
RMS currents, summed over all windings and referred to primary:

\[ I_{tot} = \sum_{\text{windings}} n_j \frac{I_j}{n_1} = I_1 + 2 \frac{n_2}{n_1} I_2 + 2 \frac{n_3}{n_1} I_3 \]

\[ = (5.7 \, \text{A}) + \frac{5}{\sqrt{11}} (66.1 \, \text{A}) + \frac{15}{\sqrt{11}} (9.9 \, \text{A}) \]

\[ = 14.4 \, \text{A} \]

Select core size:

\[ K_{geo} \geq \left( 1.724 \times 10^{-6} \right) \left( 800 \times 10^{-6} \right)^2 \left( 14.4 \right)^2 \left( 7.6 \right)^2 / 2.6 \]

\[ = 0.00937 \]

From Appendix D

Evaluate ac flux density \( \Delta B \):

\[ B_{max} = 10^3 \frac{\beta_k J_k}{2\pi} \left( \frac{MLT}{W_A I_{true}} \right) \frac{1}{|B|^2} \]

Plug in values:

\[ \Delta B = 10^3 \left( 1.724 \times 10^{-6} \right) \left( 800 \times 10^{-6} \right)^2 \left( 14.4 \right)^2 \left( 7.6 \right)^2 / 2.6 \]

\[ = 0.23 \, \text{Tesla} \]

This is less than the saturation flux density of approximately 0.35 T.
Evaluate turns

Choose \( n_1 \) according to Eq. (15.21):

\[
\frac{n_1}{n_2} = \frac{B_2}{B_1} = \frac{10^{4}}{10^{6}} \frac{800 \times 10^{-6}}{20 \times 10^{-6}} = 13.7 \text{ turns}
\]

Choose secondary turns according to desired turns ratio:

\[
\frac{n_2}{n_1} = \frac{5}{110} \text{ turns}
\]

\[
\frac{n_3}{n_1} = \frac{15}{110} \text{ turns}
\]

Rounding the number of turns to obtain desired turns ratio of 110:5:15, we might round the actual turns to 22:1:3

Increased \( n_1 \) would lead to:
- Less core loss
- More copper loss
- Increased total loss

Loss calculation with rounded turns

With \( n_1 = 22 \), the flux density will be reduced to:

\[
\Delta B = \frac{800 \times 10^{-6}}{22^2/(1.27)^2} = 0.143 \text{ Tesla}
\]

The resulting losses will be:

\[
P_{fe} = (7.6)(0.143)^2(1.27)(7.7) = 0.47 \text{ W}
\]

\[
P_{cu} = (1.724 \times 10^{-6})(800 \times 10^{-6})^{2}(14.4)^2 \left( \frac{8.5}{4(0.25)} \right) \left( \frac{1}{(1.1)(1.27)^2} \right) \left( \frac{1}{(0.143)^2} \right) = 5.4 \text{ W}
\]

\[
P_{tot} = P_{fe} + P_{cu} = 5.9 \text{ W}
\]

Which exceeds design goal of 4 W by 50%. So use next larger core size: EE50.

Calculations with EE50

Repeat previous calculations for EE50 core size. Results:

\[
\Delta B = 0.14 \text{ T}, \ n_1 = 12, \ P_{tot} = 2.3 \text{ W}
\]

Again round \( n_1 \) to 22. Then:

\[
\Delta B = 0.08 \text{ T}, \ n_1 = 12, \ P_{tot} = 2.3 \text{ W}
\]

Which is close enough to 4 W.
Wire sizes for EE50 design

<table>
<thead>
<tr>
<th>Window allocations</th>
<th>Wire gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_1 = \frac{I_1}{I_{tot}} = 0.396 )</td>
<td>( A_{w1} = \frac{n_1 K_u W A}{\alpha_1} = (0.396)(0.25)(1.78) = 8.0 \times 10^{-3} \text{ cm}^2 )</td>
</tr>
<tr>
<td>( n_2 = \frac{I_2}{I_{tot}} = 0.209 )</td>
<td>( A_{w2} = \frac{n_2 K_u W A}{\alpha_2} = (0.209)(0.25)(1.78)(1) = 93.0 \times 10^{-3} \text{ cm}^2 )</td>
</tr>
<tr>
<td>( n_3 = \frac{I_3}{I_{tot}} = 0.094 )</td>
<td>( A_{w3} = \frac{n_3 K_u W A}{\alpha_3} = (0.094)(0.25)(1.78)(3) = 13.9 \times 10^{-3} \text{ cm}^2 )</td>
</tr>
</tbody>
</table>

 Might actually use foil or Litz wire for secondary windings

Discussion: Transformer design

- Process is iterative because of round-off of physical number of turns and, to a lesser extent, other quantities
- Effect of proximity loss
  - Not included in design process yet
  - Requires additional iterations
- Can modify procedure as follows:
  - After a design has been calculated, determine number of layers in each winding and then compute proximity loss
  - Also effective resistivity of wire to compensate: define
    \[ \rho_{eff} = \frac{P_{cu}}{P_{dc}} \]
    where \( P_{cu} \) is the total copper loss (including proximity effects) and \( P_{dc} \) is the copper loss predicted by the dc resistance.
  - Apply transformer design procedure using this effective wire resistivity, and compute proximity loss in the resulting design.
  - Further iterations may be necessary if the specifications are not met.

15.4 AC Inductor Design

Design a single-winding inductor, having an air gap, accounting for core loss (note that the previous design procedure of this chapter did not employ an air gap, and inductance was not a specification)
### Outline of Key Equations

<table>
<thead>
<tr>
<th>Obtain specified inductance:</th>
<th>Total loss is minimized when</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = \mu_0 \frac{\mu_n A_n^2}{l} ]</td>
<td>[ \Delta B = \left[ \frac{\mu_0 I}{2 A_n} \left( \frac{M_{LT}}{W_A. L_A} \right) \left( \frac{1}{K_f} \right) \right] ]</td>
</tr>
<tr>
<td>Relationship between applied volt-seconds and peak ac flux density:</td>
<td>Must select core that satisfies</td>
</tr>
<tr>
<td>[ \Delta B = \frac{\lambda}{2 l^2} ]</td>
<td>[ K_m = \frac{\rho \lambda^4 \mu_0 n^{(0)}}{2 K_f [P_{tot}]/(\mu_f + \mu_k)} ]</td>
</tr>
<tr>
<td>Copper loss (using dc resistance):</td>
<td>See Section 15.4.2 for step-by-step design equations</td>
</tr>
<tr>
<td>[ P_{cu} = \frac{\rho A_c (M_{LT})}{K_f W_A} I^2 ]</td>
<td></td>
</tr>
</tbody>
</table>

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Chapter 15: Transformer design