12.3.3 Discussion

The “more accurate” controller model accounts for the differences between $i_L$ and $i_c$ that arise by two mechanisms:

- Inductor current ripple
- Artificial ramp

$F_g$ and $F_v$ blocks model small-signal effects of inductor current ripple: how the difference between $i_L$ and $i_c$ varies with applied voltages.

For operation deep in CCM, ripple is small—$F_g$ and $F_v$ blocks can then be ignored.
Effect of artificial ramp

- Artificial ramp also causes inductor current to differ from $i_c$
- $F_m$ block models effect of artificial ramp. $F_m$ varies inversely with $M_a$, and becomes infinite with zero $M_a$
- With zero $M_a$, the input to the $F_m$ block tends to zero. The controller block diagram then predicts that

$$\frac{\hat{d}}{F_m} = 0 = \hat{i}_c - \hat{i}_L - F_g \hat{v}_g - F_v \hat{v}$$

With negligible inductor current ripple, $F_g$ and $F_v$ blocks go to zero. The model then predicts

$$0 = \hat{i}_c - \hat{i}_L$$

which coincides with the ideal model of Section 12.2
Ideal limiting case

General expression for control-to-output transfer function:

\[ G_{vc}(s) = \frac{\hat{v}(s)}{\hat{i}_c(s)} \bigg|_{\hat{v}_g(s) = 0} = \frac{F_m G_{vd}}{1 + F_m(G_{id} + F_v G_{vd})} \]

In the limit when \( F_m \to 0 \), \( F_g \to 0 \), \( F_v \to 0 \), then the control-to-output transfer function reduces to the following:

\[ \lim_{F_m \to \infty} G_{vc}(s) = \frac{G_{vd}}{G_{id}} \]

and the line-to-output transfer function reduces to:

\[ \lim_{F_m \to \infty} G_{vg-cpm}(s) = \frac{G_{vg} G_{id} - G_{ig} G_{vd}}{G_{id}} \]

These expressions coincide with the transfer functions predicted by the ideal model of Section 12.2.
Large artificial ramp
Duty-cycle control

In the extreme case of a very large artificial ramp, the controller degenerates to duty-cycle control. For large $M_a$ (small $F_m$) and for $F_g \to 0$ and $F_v \to 0$, the control-to-output transfer function reduces to

$$\lim_{\text{small } F_m} G_{vc}(s) = F_m G_{vd}(s)$$

(current-mode controller becomes pulse-width modulator having gain $F_m$, and duty-cycle controller gain $G_{vd}(s)$ is obtained)

For large $M_a$ (small $F_m$) and for $F_g \to 0$ and $F_v \to 0$, the line-to-output transfer function reduces to

$$\lim_{F_m \to \infty} G_{vg-cpm}(s) = G_{vg}$$

(the duty-cycle controller gain $G_{vg}(s)$ is obtained)
### 12.3.4 Evaluation of transfer functions

**Buck converter example**

<table>
<thead>
<tr>
<th>Control-to-output transfer function $G_{vc}(s)$</th>
<th>Ideal current mode control</th>
<th>General result</th>
<th>Duty cycle control</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\hat{v}}{i_c} = \frac{R}{1 + sRC}$</td>
<td>$G_{vd}(s) = \frac{F_m G_{vd}}{1 + F_m (G_{id} + F_v G_{vd})}$</td>
<td>$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s \frac{L}{R} + s^2 LC}$</td>
<td></td>
</tr>
</tbody>
</table>

| Line-to-output transfer function $G_{vg-cpm}(s)$ | | | |
| $\frac{\hat{v}_v}{\hat{v}_g} = 0$ | $G_{vg}(s) = D \frac{1}{1 + s \frac{L}{R} + s^2 LC}$ |

- Evaluate general result for the buck converter
- Need to evaluate $G_{vd}$, $G_{vg}$, $G_{id}$, $G_{ig}$
Buck converter model

Transfer functions $G_{vd}$, $G_{vg}$, $G_{id}$, $G_{ig}$

Analyze model to find:

\[ G_{vd}(s) = \frac{V}{D} \frac{1}{\text{den}(s)} \]
\[ G_{vg}(s) = D \frac{1}{\text{den}(s)} \]
\[ G_{id}(s) = \frac{V}{DR} \frac{(1 + sRC)}{\text{den}(s)} \]
\[ G_{ig}(s) = \frac{D}{R} \frac{(1 + sRC)}{\text{den}(s)} \]

All transfer functions of a given circuit have the same poles:

\[ \text{den}(s) = 1 + s \frac{L}{R} + s^2 LC \]
Control-to-output transfer function $G_{vc}(s)$

Substitute into expression for $G_{vc}(s)$:

$$G_{vc}(s) = \frac{F_m G_{vd}}{1 + F_m[G_{id} + F_v G_{vd}]} = \frac{F_m \left( \frac{V}{D} \frac{1}{\text{den}(s)} \right)}{1 + F_m \left[ \frac{V}{DR} \frac{1 + sRC}{\text{den}(s)} \right] + F_v \left( \frac{V}{D} \frac{1}{\text{den}(s)} \right)}$$

Algebra:

$$G_{vc}(s) = \frac{F_m \frac{V}{D}}{\text{den}(s) + \frac{F_m V}{DR} \left( 1 + sRC \right) + F_m F_v \frac{V}{D}}$$

$$G_{vc}(s) = \frac{G_{c0}}{1 + \frac{s}{Q_c \omega_c} + \left( \frac{s}{\omega_c} \right)^2}$$
Summary of results for buck converter

Simple model

\[ \frac{\hat{v}}{\hat{i}_c} = \frac{R}{1 + sRC} \]
\[ \frac{\hat{i}}{\hat{v}_g} = 0 \]

Duty cycle controlled gains

\[ G_{vd}(s) = \frac{V}{D} \frac{1}{\text{den}(s)} \]
\[ G_{id}(s) = \frac{V}{DR} \frac{1 + sRC}{\text{den}(s)} \]
\[ G_{vg}(s) = D \frac{1}{\text{den}(s)} \]
\[ G_{ig}(s) = \frac{DR}{R} \frac{1 + sRC}{\text{den}(s)} \]
\[ \text{den}(s) = 1 + s \frac{L}{R} + s^2LC \]

More accurate model

\[ \frac{\hat{v}}{\hat{i}_c} = G_{vc}(s) = G_{c0} \frac{1}{1 + \frac{s}{Q_c \omega_c} + \left( \frac{s}{\omega_c} \right)^2} \]
\[ \omega_c = \frac{1}{\sqrt{LC}} \sqrt{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}} \]

\[ \frac{\hat{i}}{\hat{v}_g} = G_{vg-cpm}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_c \omega_c} + \left( \frac{s}{\omega_c} \right)^2} \]

\[ G_{c0} = \frac{V}{D} \left( 1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D} \right) \left( 1 + \frac{RC F_m V}{DL} \right) \]
\[ Q_c = R \sqrt{\frac{C}{L}} \left( 1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D} \right) \left( 1 + \frac{RC F_m V}{DL} \right) \]
\[ G_{g0} = D \left( 1 - \frac{F_m F_g V}{D^2} \right) \left( 1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D} \right) \]

Fundamentals of Power Electronics 62  Chapter 12: Current Programmed Control
Effect of current programming on $Q$-factor of poles

$$Q_c = R \sqrt{\frac{C}{L}} \sqrt{1 + \frac{F_m V}{D R} + \frac{F_m F_v V}{D}} \left(1 + \frac{RCF_m V}{D L}\right)$$

- Expressed above as duty-cycle control result, multiplied by a factor that accounts for current programming
- Current programming tends to reduce the $Q$-factor
- For large $F_m$ (small $M_a$), $Q_c$ varies as $F_m^{-1/2}$
- If the artificial ramp is not too large, then poles become real and well separated
The low-Q approximation:
Low-frequency pole

Apply the low-Q approximation (Section 8.1.7) to factor the poles, for the case of large $F_m$:

Low-frequency pole becomes

$$Q_c \omega_c = \frac{R}{L} \left( \frac{1 + \frac{F_m V}{D R} + \frac{F_m F_v V}{D}}{1 + \frac{RCF_m V}{D L}} \right)$$

For large $F_m$ and small $F_v$, this can be further approximated as:

$$Q_c \omega_c \approx \frac{1}{RC}$$

which coincides with the prediction of the ideal CPM model.
The low-\(Q\) approximation:
High-frequency pole

The low-\(Q\) approximation predicts that the high-frequency pole is

\[
\frac{\omega_c}{Q_c} = \frac{1}{RC} \left( 1 + \frac{RCF_m V}{DL} \right)
\]

For large \(F_m\), this can be further approximated as:

\[
\frac{\omega_c}{Q_c} \approx \frac{F_m V}{DL} = f_s \frac{M_2}{D M_a}
\]

The high-frequency pole is typically predicted to lie near to, or greater than, the switching frequency \(f_s\).

It should be pointed out that the converter switching and modulator sampling processes lead to discrete-time phenomena that affect the high-frequency behavior of the converter, and are not predicted by continuous-time averaged analyses such as this one. So this model is valid only at frequencies sufficiently less than \(f_s/2\).
Line-to-output transfer function $G_{vg-cpm}(s)$

General expression:

$$G_{vg-cpm}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{i_c(s) = 0} = \frac{G_{vg} - F_m F_g G_{vd} + F_m \left( G_{vg} G_{id} - G_{ig} G_{vd} \right)}{1 + F_m \left( G_{id} + F_v G_{vd} \right)}$$

Note that, for an ideal CPM buck converter, this transfer function is equal to zero, and is given by

$$\lim_{F_m \to \infty} \lim_{F_g \to 0} \lim_{F_v \to 0} G_{vg-cpm}(s) = \frac{G_{vg} G_{id} - G_{ig} G_{vd}}{G_{id}}$$

Therefore, $(G_{vg} G_{id} - G_{vd} G_{ig}) = 0$ and the general expression reduces to

$$G_{vg-cpm}(s) = \frac{G_{vg} - F_m F_g G_{vd} + F_m \left( G_{vg} G_{id} - G_{ig} G_{vd} \right)}{1 + F_m \left( G_{id} + F_v G_{vd} \right)}$$
Line-to-output transfer function $G_{vg-cpm}(s)$ (continued)

Substitute expressions:

$$G_{vg-cpm}(s) = \frac{\frac{D}{\text{den}(s)} - F_m F_g \frac{V}{D} \frac{1}{\text{den}(s)}}{1 + F_m \left( \frac{V}{DR} \frac{1 + sRC}{\text{den}(s)} + F_v \frac{V}{D} \frac{1}{\text{den}(s)} \right)}$$

Simplify, then write in normalized form:

$$G_{vg-cpm}(s) = \frac{\left( D - F_m F_g V \right)}{\text{den}(s) + \frac{F_m V}{DR} \left( 1 + sRC \right) + F_m F_v V}$$

$$G_{vg-cpm}(s) = \frac{G_{g0}}{1 + \frac{s}{Q_c \omega_c} + \left( \frac{s}{\omega_c} \right)^2}$$

which has the same poles as $G_{vc}$, and the following dc gain:

$$G_{g0} = D \left( \frac{1 - \frac{F_m F_g V}{D^2}}{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}} \right) = D \left( \frac{1 - \frac{M_2}{2M_a}}{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}} \right)$$
DC gain of $G_{vg\text{-}cpm}(s)$

\[
G_{g0} = D \frac{1 - \frac{F_m F_g V}{D^2}}{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}} = D \frac{1 - \frac{M_2}{2M_a}}{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}}
\]

- For duty-cycle control ($F_m \to 0$), $G_{g0}$ is equal to $D$
- For ideal current programmed control ($F_m \to \infty$, $F_g \to 0$ and $F_v \to 0$), $G_{g0}$ is equal to 0
- For nonideal current programmed control, $G_{g0}$ and hence $G_{vg\text{-}cpm}(s)$ are in general nonzero
- For the special case $M_a = 0.5 M_2$, $G_{g0}$ is equal to 0 (for the current programmed buck converter). The effective feedforward path from $\hat{v}_g$ through $F_g$ then cancels the $\hat{v}_g$-induced variations that propagate through $G_{vg}$.
12.3.5 Results for basic converters

Buck converter

Simple model

\[
\frac{\hat{v}}{i_c} = \frac{R}{1 + sRC}
\]

\[
\frac{\hat{v}}{v_g} = 0
\]

Duty cycle controlled gains

\[
G_{vd}(s) = \frac{V}{D} \frac{1}{\text{den}(s)}
\]

\[
G_{id}(s) = \frac{V}{DR} \frac{1 + sRC}{\text{den}(s)}
\]

\[
G_{vg}(s) = D \frac{1}{\text{den}(s)}
\]

\[
G_{ig}(s) = \frac{D}{R} \frac{1 + sRC}{\text{den}(s)}
\]

\[
\text{den}(s) = 1 + s \frac{L}{R} + s^2LC
\]

More accurate model

\[
\frac{\hat{v}}{i_c} = G_{vc}(s) = G_{c0} \frac{1}{1 + \frac{s}{Q_c \omega_c} + \left(\frac{s}{\omega_c}\right)^2}
\]

\[
\omega_c = \frac{1}{\sqrt{LC}} \sqrt{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}}
\]

\[
\frac{\hat{v}}{v_g} = G_{vg-cpm}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_c \omega_c} + \left(\frac{s}{\omega_c}\right)^2}
\]

\[
G_{c0} = \frac{V}{D} \left(1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D} \right)
\]

\[
Q_c = R \sqrt{\frac{C}{L}} \sqrt{1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D}} \left(1 + \frac{RCF_m V}{DL} \right)
\]

\[
G_{g0} = D \left(1 - \frac{F_m F_g V}{D^2} \right) \left(1 + \frac{F_m V}{DR} + \frac{F_m F_v V}{D} \right)
\]

Results of this section are expressed as duty-cycle-controlled value, multiplied by factor that accounts for effects of current programming.
Boost converter

Simple model

\[
\frac{\dot{v}}{i_c} = DR \frac{1 - s \frac{L}{D^2R}}{1 + s \frac{RC}{2}}
\]

\[
\frac{\dot{v}}{v_g} = \frac{1}{2DR} \frac{1}{1 + s \frac{RC}{2}}
\]

Duty cycle controlled gains

\[
G_{vd}(s) = \frac{V}{D} \frac{1 - s \frac{L}{D^2R}}{\text{den}(s)}
\]

\[
G_{vd}(s) = \frac{2V}{D^2R} \frac{1 + s \frac{RC}{2}}{\text{den}(s)}
\]

\[
G_{id}(s) = \frac{1}{D} \frac{1}{\text{den}(s)}
\]

\[
G_{id}(s) = \frac{1}{D^2R} \frac{1 + sRC}{\text{den}(s)}
\]

\[
\text{den}(s) = 1 + s \frac{L}{D^2R} + s^2 \frac{LC}{D^2}
\]

More accurate model

\[
\frac{\dot{v}}{i_c} = G_{vc}(s) = G_{c0} \frac{1 - s \frac{L}{D^2R}}{1 + s \frac{Q_c \omega_c}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2}
\]

\[
\omega_c = \frac{D}{\sqrt{LC}} \sqrt{1 + \frac{2F_m V}{D^2R} + \frac{F_m F_g V}{D^2}}
\]

\[
\omega_{g0} = \frac{1 + \frac{s}{\omega_{g0}}}{1 + \frac{s}{\omega_{g0}} + \left(\frac{s}{\omega_{g0}}\right)^2}
\]

\[
G_{g0} = \frac{1}{D} \left(1 - \frac{F_m F_g V}{D^2R} + \frac{F_m V}{D^2R}\right)
\]

\[
G_{g0} = \frac{1}{D} \left(1 - \frac{2F_m V}{D^2R} + \frac{F_m F_g V}{D^2}\right)
\]

\[
\omega_{g0} = \frac{D^2R}{L} \frac{1 - \frac{F_m F_g V}{D^2R} + \frac{F_m V}{D^2R}}{\frac{F_m V}{D^2R}}
\]
Buck-boost converter

Simple model

\[
\frac{\dot{v}}{v_c} = -\frac{DR}{1+D} \left(1 - s \frac{DL}{D^2R} \right) \left(1 + s \frac{RC}{1+D} \right)
\]

\[
\frac{\dot{v}}{v_g} = -\frac{D^2}{1-D^2} \frac{1}{1 + s \frac{RC}{1+D} \frac{1}{D^2}}
\]

Duty cycle controlled gains

\[
G_{vd}(s) = -\frac{|V|}{DD} \left(1 - s \frac{DL}{D^2R} \right) \frac{1+D}{DD^2R} \frac{1}{den(s)}
\]

\[
G_{vg}(s) = -\frac{|V|}{DD^2R} \frac{1}{den(s)}
\]

\[
G_{id}(s) = -\frac{F_m V D}{DD^2R} \frac{1}{den(s)}
\]

\[
G_{ig}(s) = -\frac{|V|}{DD^2R} \frac{1}{V} \frac{1}{den(s)}
\]

\[
den(s) = 1 + s \frac{1}{D^2R} + s^2 \frac{LC}{D^2}
\]

More accurate model

\[
\dot{v} = G_{vc}(s) = G_{c0} \left( 1 - s \frac{DL}{D^2R} \right) \left( 1 + s \frac{Q_c \omega_c}{\omega_c^2} + \left( \frac{s}{\omega_c} \right)^2 \right)
\]

\[
\omega_c = \frac{D'}{\sqrt{LC}} \sqrt{1 + s \frac{F_m V \frac{1+D}{DD^2R} - F_m F_c |V|}{DD}}
\]

\[
\dot{v} = G_{vg-pm}(s) = G_{g0} \left( 1 + s \frac{1}{\omega_c} \right) \left( 1 + s \frac{1}{\omega_c} \right)^2
\]

\[
\omega_{g0} = \frac{D}{D'} \left( 1 + \frac{F_m V}{DD^2R} - \frac{F_m F_c |V|}{DD^2R} \right)
\]

\[
\omega_{g0} = \frac{DD^2R}{|V|} \left( 1 + \frac{F_m V}{DD^2R} - \frac{F_m F_c |V|}{DD^2R} \right)
\]