An LLC inverter - Example

Input impedance of tank:

\[ Z_{io} = \frac{1}{sC} + sL_s \quad \quad Z_{i\infty} = \frac{1}{sC} + s(L_s + L_p) \]

\[ f_0 = \frac{1}{2\pi \sqrt{L_s C}} \]

\[ f_{oo} = \frac{1}{2\pi \sqrt{(L_s + L_p) C}} \]
For $f_s < f_{oo}$: $\mathcal{Z}_{io}$ and $\mathcal{Z}_{ioo}$ are both capacitive
(i.e., $\angle \mathcal{Z}_{io} = -90^\circ$, $\angle \mathcal{Z}_{ioo} = -90^\circ$)

\[ \Rightarrow \text{2CS for all } R \]

For $f_s > f_o$: $\mathcal{Z}_{io}$ and $\mathcal{Z}_{ioo}$ are both inductive
(i.e., $\angle \mathcal{Z}_{io} = +90^\circ$ and $\angle \mathcal{Z}_{ioo} = +90^\circ$)

\[ \Rightarrow \text{2VS for all } R \]

For $f_{oo} < f_s < f_o$: $\mathcal{Z}_{io}$ is capacitive $\Rightarrow$ 2CS for small $R$ ($R > R_{crit}$)
$\mathcal{Z}_{ioo}$ is inductive $\Rightarrow$ 2VS for large $R$ ($R > R_{crit}$)

\[ \text{with } R_{crit} = \frac{\mathcal{Z}_{ioo}}{\mathcal{Z}_{io}} \sqrt{-\frac{s \mathcal{Z}_{ioo}}{s \mathcal{Z}_{io}}} \]

We want to choose $f_s > f_m$ so that switch and tank currents are small at light load.

\[ \text{Tank transfer function } H(s) \]

\[ H(s) = \frac{R s L_p}{R s L_p + s L_s + \frac{1}{s C}} \]

\[ = \frac{s^3 L_p C}{1 + s L_p + s^2 C (L_p + L_s) + s^3 L_s L_p C} \]

\[ \text{Output impedance } Z_{oo}(s) \]

\[ Z_{oo}(s) = \frac{S L_p (s L_s + \frac{1}{s C})}{1 + s^2 (L_s + L_p) C} \]

\[ R_{ooo} = \omega_{oo} L_p = \frac{L_p}{C} \sqrt{\frac{1}{L_p L_s}} \]

\[ R_{oo} = \omega_0 L_p L_s \sqrt{\frac{1}{L_p + L_s}} \]
open-circuit transfer function \( H_\infty(s) = \frac{H(s)}{R \to \infty} \)

\[
H_\infty(s) = \frac{sL_p}{sL_p + sL_s + \frac{s}{sC}} = \frac{s^2L_pC}{1 + s^2(L_pL_s)C}
\]

Output characteristic

\( \text{load current (mA)} \) vs. \( \text{load voltage (V)} \)

- we want to operate with \( f_s > f_{oo} \) to obtain GVS
- can't control converter when \( f_s \gg f_{oo} \) because \( ||u_{in}|| \to \frac{L_p}{L_pL_s} \)
- so choose \( f_{oo} < f_s < f_o \)

\[ V_{oc} = \frac{||u_{in}|| ||u_{sw}||}{4} \text{ as above} \]

\[ J_{sc} = \frac{V_{oc}}{||i_{2o}||} \text{, with } ||i_{2o}|| \text{ as on previous page} \]

\[
\frac{V^2}{V_{oc}} + \frac{I^2}{J_{sc}} = 1 . \text{ If resistive load: } V = IR \text{ then can solve for } V:\n\]

\[ V = \frac{V_{oc}}{\sqrt{1 + \frac{||i_{2o}||^2}{R^2}}} \]
At \( f = f_m \), \( \| z_{io} \| = \| z_{i\infty} \| \)

Note that, at \( f = f_m \),
\[
\angle z_{io} = -90^\circ \\
\angle z_{i\infty} = +90^\circ
\]

So at \( f = f_m \), \( z_{io}(j\omega) = -z_{i\infty}(j\omega) \)

\[
\frac{1}{j\omega C} + j\omega L_s = -\left( \frac{1}{j\omega C} + j\omega L_s + j\omega L_p \right)
\]

\( \Rightarrow 2j\omega L_s + j\omega L_p = -2 \frac{1}{j\omega C} \)

\( a \omega^2 L_s C + \omega^2 L_p C = 2 \)

\( \omega^2 = \frac{2}{(2L_s + L_p) C} \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{(L_s + \frac{1}{2}L_p) C}} \)

\( \Rightarrow f_m = \frac{1}{2\pi \sqrt{(L_s + \frac{1}{2}L_p) C}} \)
A Design Example

Specifications
Converter should drive a 20W, 100V rms load at 100kHz

\( R = \frac{V^2}{P} = \frac{100^2}{20} = 500\ \Omega \)

Obtain ZVS for \( R \geq 500\ \Omega \)

Choose \( L_s, L_p, C, n \)

Output characteristic

To obtain ZVS for \( R \geq 500\ \Omega \), we need \( R_{crit} \leq 500\ \Omega \) referred to secondary

\[
R_{crit} = \frac{n^2 L_{2i0} \sqrt{-\frac{2V_{in}}{B_i0}}}{L_{2i0}}
\]

\[
\Rightarrow \quad \frac{L_{2i0}}{L_{2i0}} = \left( \frac{R_{crit}}{n^2 L_{2i0}} \right)^2
\]

To obtain some margin, choose \( R_{crit} = 400\ \Omega \).
To obtain good efficiency at light load, we want

\[ |Z_{2\text{on}}| >> |Z_{2\text{off}}| \]

\[ \Rightarrow \text{operate with } f_s > f_m \]

Let's choose \(|Z_{2\text{on}}|\) approximately 5 times greater than \(|Z_{2\text{off}}|\). This will make the transistor current vary by a factor of five as \(R\) varies from 500 \(\Omega\) to \(\infty\).

So

\[ \frac{|Z_{2\text{on}}|}{|Z_{2\text{off}}|} = 5 = \left( \frac{R_{crit}}{n^2|Z_{2\text{off}}|} \right)^2 \]

\[ \Rightarrow |Z_{2\text{on}}| = \frac{400 \Omega}{n^2 \sqrt{5}} = \frac{179 \Omega}{n^2} \]

We can now evaluate the elliptical output characteristic to find \(V_{oc}\):

\[ \frac{V^2}{V_{oc}^2} + \frac{I^2}{I_{sc}^2} = 1 \]

with

\[ V = 100 \text{ V rms} \]

\[ I = 0.2 \text{ A rms} \]

\[ I_{sc} = \frac{V_{oc}}{n^2 |Z_{2\text{on}}|} = \frac{V_{oc}}{179 \Omega} \]

So

\[ \frac{(100)^2}{V_{oc}^2} + \frac{(0.2)^2 (179)^2}{V_{oc}^2} = 1 \]

\[ \Rightarrow V_{oc} = \sqrt{(100)^2 + (0.2)^2 (179)^2} = 106 \text{ V rms} \]

but

\[ V_{oc, \text{rms}} = n \left( \frac{V_s}{\sqrt{2}} \right) |H_{\text{on}}| = n \left( \frac{4}{\pi} \frac{V_s}{\sqrt{2}} \right) |H_{\text{on}}| \]

So we need

\[ n |H_{\text{on}}| = \frac{(106)}{\left( \frac{4}{\pi}\frac{12}{\sqrt{2}} \right)} = 9.8 \]
A design that comes close to these desired values:

\[ L_s \quad C \]
\[ 2.5 \mu H \quad 0.4 \mu F \]

\[ \frac{M}{1 \cdot 7.5} \]
\[ L_p \quad 15 \mu H \]

This leads to

- \( V_{oc} = 109 \text{V rms} \)
- \( I_{sc} = 0.16 \text{A rms} \)
- \( V = 102 \text{V rms} \) at 100kHz, 500Ω
- \( R_{crit} = 310 \Omega \) so 2VS for \( R > 310 \Omega \)

\[ 2i \]

open-circuit tank current \( = \frac{V_s}{Z_{io}} \)
\[ = \frac{4}{d} \frac{12V}{70} = 2.2 \text{A peak} \]

short-circuit tank current \( = \frac{V_s}{Z_{io}} \)
\[ = \frac{4}{d} \frac{12V}{240} = 0.3 \text{A peak} \]