The zero-voltage switching quasi-square wave resonant switch (ZVS-QSW)

Resonant switch network reduces to

![Resonant switch network diagram]

Features:

- Tank capacitor is effectively in parallel with all semiconductor elements. All devices operate with zero-voltage switching.
- The peak voltage applied to all semiconductor devices is identical to the peak voltages in the (ideal) parent dc-dc converter. This resonant switch does not increase the voltage stresses on the components.
- Peak currents are increased, and are similar to the peak currents in PWM converters that operate near the CCM/DCM boundary.
- Magnetics are small, and are similar to magnetics of DCM converters.
- Voltage waveforms resemble those of hard-switched PWM converters, except that the switching transitions are resonant and smooth.
- With single transistor: variable frequency operation, with restricted conversion ratio \( \frac{1}{3} \leq \alpha \leq 1 \). These disadvantages are not present in two-transistor synchronous rectifier versions.
Several implementations of ZVS-WSW converters:

**Buck**

![Buck circuit diagram](image)

\[ V_y \geq V \geq \frac{1}{2} V_g \]

**Flyback**

![Flyback circuit diagram](image)

\[ nV_y \leq V \]

**Boost**

![Boost circuit diagram](image)

\[ V \geq 2V_g \]

In each case, the resonant tank elements are \( L \) and \( C \), while \( C_F \) is a filter element having small ripple. The small tank inductor \( L \) effectively replaces the converter filter inductor; see following pages for explanation. In the flyback and SEPIC, this can lead to a very small transformer.
A voltage regulator module (VRM) using interleaved (phase-shifted) QSW-ZVS buck converters with synchronous rectifiers.

Constant frequency operation. Tanks are composed of small inductor L and MOSFET output capacitances Cds. Q1 and Q2 have approximately-complementary gate drives, but with small delays inserted to allow the resonant transitions to occur. The second converter (Q3 and Q4) operates 180° out of phase with the first, so that there is substantial cancellation of the input and output current ripples. The inductor current ripples are greater than their dc components, so that zero-voltage switching occurs. Control (such as current mode control) is needed to ensure that the paralleled converters share the load.

This approach finds significant commercial application in supplying low-voltage high-current microprocessors. Switching frequencies of several MHz are not unusual.
1. Analysis of the Quasi-Squarewave Resonant Switch

ZVS (type d) version

Implementation in a buck converter:

Tank capacitor $C$  \( \rightarrow \) small in value, large ripple

Tank inductor $L$  \( \rightarrow \) small in value, large ripple

Filter inductor $L_F$  \( \rightarrow \) large in value, small ripple

Filter capacitor $C_F$  \( \rightarrow \) large in value, small ripple

Since $L_F$ and $L$ are in parallel, they can be combined into a single inductor of value $L_F \parallel L \approx L$.

Averaged model of switch network: determine average terminal quantities $\langle v_1 \rangle$, $\langle i_1 \rangle$, $\langle v_2 \rangle$, $\langle i_2 \rangle$.

Note that $\langle v_1 \rangle = V_g$ and $\langle i_2 \rangle = I$

Let's find expressions for $\langle i_1 \rangle$ and $\langle v_2 \rangle$ and express as functions of $\langle v_1 \rangle$ and $\langle i_2 \rangle$. 
Two-port model of switch network:

control input: \( f_s \)

\[ \langle i_r \rangle \]

\[ \langle i_e \rangle \]

\[ \langle v_1 \rangle \]

\[ \langle v_2 \rangle \]

We can view two of these terminal quantities as given independent variables. The control input \( f_s \) is also an independent input. The remaining two terminal quantities are dependent variables; we must write equations that express the dependent variables (outputs) as functions of the independent variables (inputs).

The choice of which variable is dependent is somewhat arbitrary. We will choose \( \langle v_2 \rangle \) and \( \langle i_1 \rangle \) as the dependent variables.

In a PWM buck converter, we know that

\[ \langle v_2 \rangle = d \langle v_1 \rangle \]

\[ \langle i_1 \rangle = d \langle i_2 \rangle \]

\[ \text{Conventional PWM buck} \]

We will use the Averaged Switch Modeling approach to express the terminal equations of the QSW ZVS switch network using the switch conversion ratio \( \mu \):

\[ \langle v_2 \rangle = \mu \langle v_1 \rangle \quad \text{and} \quad \langle i_1 \rangle = \mu \langle i_2 \rangle \]

\( \mu \) can be viewed as a generalized or effective duty ratio. In general, \( \mu \) is a function of the control input \( f_s \) and the other independent inputs \( \langle v_1 \rangle \) and \( \langle i_2 \rangle \).
Tank waveforms

Note that
\[ \langle v_1 \rangle = v_g \]
\[ \langle v_2 \rangle = v = \mu \langle v_1 \rangle \]

Normalization

Define
\[ j_L = \frac{i_L}{I_{\text{base}}} \]
\[ m_v = \frac{v_c}{V_{\text{base}}} \]

Let
\[ V_{\text{base}} = \langle v_1 \rangle \]
\[ R_{\text{base}} = R_0 = \sqrt{\frac{L}{C}} \]
\[ I_{\text{base}} = \frac{\langle v_1 \rangle}{R_0} \]

In steady state
\[ \langle i_L \rangle = \langle i_2 \rangle = \text{dc load current} \]
\[ \langle v_c \rangle = \langle v_2 \rangle \]
Interval 1 begins when \( i_L(t) \) reaches zero
ends when \( Q_1 \) turns off
During this interval: \( Q_1 \) is on
\( D_1 \) and \( D_2 \) are reverse-biased

\[
\begin{align*}
\langle v_i \rangle & \quad \uparrow \quad i_L \\
C & \quad \longrightarrow \quad \frac{L}{i_L} \\
\downarrow & \quad \langle v_2 \rangle
\end{align*}
\]

\[
\frac{di_L}{dt} = \frac{v_g - v}{L} = \frac{\langle v_i \rangle - \langle v_2 \rangle}{L} = \langle v_i \rangle \frac{1 - \mu}{L}
\]

\( i_L(t) \) increases with constant slope.
At end of interval, \( \omega t = \alpha \)

\[
i_L \left( \frac{\alpha}{w_0} \right) = \left( \frac{v_g - v}{L} \right) \left( \frac{\alpha}{w_0} \right) = I_{L_1}
\]

\[
= \langle v_i \rangle \frac{1 - \mu}{L} \frac{\alpha}{w_0}
\]

normalize: \( (1 - \mu) \alpha = J_{L_1} \)
Interval 2: Begins when $Q_i$ turns off. $L$ and $C$ ring sinusoidally. Interval 2 ends when $D_2$ becomes forward-biased, when $v_c$ reaches zero. During interval 2, $Q_i$, $D_1$, and $D_2$ are off.

Circuit rings sinusoidally. Center is at DC solution $v_c = V_j$, $i_L = 0$, $v_c = V_j$, $i_L = 0$.

Initial values are $v_c = V_j \Rightarrow v_c = 1$

$c_c = 3L_1$

From initial values: $r_1^2 = (1 - 1) + 3L_1^2$

Final values: $r_1^2 = \mu^2 + 3L_2^2$

So $3L_2^2 = (1 - \mu)^2 - \mu^2 + 3L_1^2$

$3L_2^2 = 1 - 2\mu + 3L_1^2$

$J_{L_2} = \sqrt{1 - 2\mu + 3L_1^2}$

Interval length:

\[ \beta_1 = \tan^{-1} \left( \frac{1 - \mu}{J_{L_1}} \right) \]

\[ \beta_2 = \tan^{-1} \left( \frac{\mu}{J_{L_2}} \right) \]

$\beta = \beta_1 + \beta_2 = \tan^{-1} \left( \frac{1 - \mu}{J_{L_1}} \right) + \tan^{-1} \left( \frac{\mu}{J_{L_2}} \right)$
Interval 3

Begins when $I_2$ turns on, $v_c = 0$

Ends when $I_2$ turns off, $i_L = 0$

During this interval, $I_2$ is on, $Q_1$ & $Q_2$ are off

\[
L \frac{di_L}{dt} = -V = -\mu <v_1>
\]

\[
\frac{di_L}{dt} = -\frac{\mu <v_1>}{L}
\]

So

\[
\Delta i_L = \left( -\frac{\mu <v_1>}{L} \right) \left( \frac{t}{\omega_0} \right)
\]

Initial $i_L = I_{L2}$

Final $i_L = 0$

Change in $i_L = -I_{L2}$

Length of interval: $\omega_0 t = \delta$

$\Rightarrow t = \frac{\delta}{\omega_0}$

Normalize:

\[
-I_{L2} = -\mu \delta
\]

\[
J_{L2} = \mu \delta \quad \Rightarrow \quad \delta = \frac{J_{L2}}{\mu}
\]
Interval 4

Begins when \( Q_2 \) turns off at \( i_L = 0 \)
ends when \( Q_1 \) turns on, when \( v_c \) reaches \( V_s \)

During this interval, all semiconductors are off.

\[ V = \langle v_2 \rangle = \mu \langle v_i \rangle \]

Initial values \( i_L = 0 \)
\( v_c = 0 \)

Resonant circuit rings sinusoidally centered at
the dc solution \( v_c = V = \mu \langle v_i \rangle \)
\( i_L = 0 \)
\( w_c = \omega_c \) \( i_L = 0 \)

Note that \( r_2 = \mu \).
If \( \mu < \frac{1}{2} \), then \( w_c \) never reaches 1 \( \Rightarrow \) no 2VS
so we require \( \mu > \frac{1}{2} \).

\[ r_2^2 = J_{L3}^2 + (1 - \mu)^2 \]
so \( J_{L3}^2 = -(1 - \mu)^2 + r_2^2 = -(1 - \mu)^2 + \mu^2 = -1 + 2\mu \)

\[ J_{L3} = \sqrt{1 + 2\mu} \]

\[ \theta = \pi - \tan^{-1} \left( \frac{J_{L3}}{1 - \mu} \right) = \pi - \cos^{-1} \left( \frac{1 - \mu}{\mu} \right) \]
Interval \( 5 \)
- Begins when \( D_1 \) becomes forward biased
- \( Q_1 \) may be turned on at any time during interval \( 5 \) with zero-voltage switching
- \( D_2 \) is off

\[ \langle i_L \rangle = \langle v_i \rangle \]
\[ \frac{1}{L} \frac{d}{dt} i_L = \frac{\langle v_i \rangle - \mu \langle v_i \rangle}{\frac{5}{\omega_o}} \]

Initial current \( i_L = -I_L3 \)
Interval ends at \( i_L = 0 \)

\[ I_L3 = \frac{\langle v_i \rangle - \mu \langle v_i \rangle}{\frac{5}{\omega_o}} \]

 Normalize:
\[ J_L3 = (1-\mu) \frac{5}{\omega_o} \]

Average terminal current
\[ \langle i_1 \rangle = \mu \langle i_2 \rangle \]

\[ \langle i_1 \rangle = \frac{\text{area}}{T_3} = \frac{1}{2} (I_{L1} - I_{L3}) \frac{\alpha + \beta}{\omega_o} \]

Normalize:
\[ \langle j_1 \rangle = \frac{F}{4\pi} (J_{L1} - J_{L3}) (\alpha + \beta) \]

\[ F = \frac{f_s}{f_0} \]
\[ \omega_0 T_s = \alpha + \beta + \delta + \frac{\xi}{2} \quad \text{Length of switching period} \]

\[ \frac{2\pi}{F} = \frac{J_{L1}}{1 - \mu} + \tan^{-1}\left(\frac{1 - \mu}{J_{L1}}\right) + \tan^{-1}\left(\frac{\mu}{J_{L2}}\right) + \frac{J_{L2}}{\mu} + \pi - \cos^{-1}\left(\frac{1 - \mu}{\mu}\right) + \frac{J_{L3}}{1 - \mu} \]

with \[ J_{L1} = \sqrt{\frac{4\pi}{F} \mu (1 - \mu) J_2} + \sqrt{1 + 2\mu} \]

\[ J_{L2} = \sqrt{1 - 2\mu + J_{L1}^2} \]

\[ J_{L3} = \sqrt{1 + 2\mu} \]

Must iterate to solve the above equations numerically.

\[ \mu = \text{function of } J_2 \text{ and } F \]

If \( \alpha \) is given as the control input.
There are several ways to write the equation that relates $\mu$, $F$, and $J_2$. The approach on the previous page, of summing the integral angles and equating to $w_0 T_0$, is one valid way. Another is to directly write the equation for the dc component of $i_L$:

$$\langle i_L \rangle = \frac{1}{T_0} \int i_L \, dt = \frac{1}{T_0} \left[ \frac{I_{L_1} - I_{L_3}}{2} \frac{\alpha + \xi}{\omega_0} + \int i_L \, dt + \frac{1}{2} I_{L_2} \frac{\xi}{\omega_0} \left( 1 + \epsilon \right) + \int i_L \, dt \right]$$

during intervals $\circ$ and $\circ$,$\circ$, $i_L$ flows through capacitor $C$ (backwards)

$$-\int i_L \, dt = \text{change in charge on } C = C \cdot AV = C (-V_0)$$

Likewise, $\int i_L \, dt = C \cdot (+V_5)$, so $\int + \int = 0$

so $\langle i_L \rangle = \frac{1}{T_0} \left[ \frac{I_{L_1} - I_{L_3}}{2} \frac{\alpha + \xi}{\omega_0} + \frac{1}{2} I_{L_2} \frac{\xi}{\omega_0} \right]$

$$\langle i_L \rangle = \frac{F}{4\pi} \left[ (I_{L_1} - I_{L_3}) (\alpha + \xi) + I_{L_2} \xi \right]$$

$$J_2 = \frac{F}{4\pi} \left[ (J_{L_1} - J_{L_3}) (\alpha + \xi) + J_{L_2} \xi \right]$$

$$= \frac{F}{4\pi} \left[ (1 - \mu)(1 - \mu) \xi (\alpha + \xi) + \mu \xi^2 \right]$$

It remains to eliminate the angles $\alpha$, $\xi$, and $\xi$. 
Length of switching period

$$\omega_0 T_s = \alpha + \beta + \delta + \xi + \varepsilon = \frac{2\pi}{F}$$

Transistor/diode Q, D conduction angle

define $\Theta = \alpha + \varepsilon$

The transistor/diode duty cycle can be defined as

$$d = \frac{\Theta}{\omega_0 T_s} = \frac{\Theta F}{2\pi}$$

$\Theta$ could be viewed as a control input.

Average output voltage

$$\langle v_2 \rangle = \frac{1}{T_s} \int_0^{T_s} v_2 \, dt = \frac{1}{3} \left[ \frac{\Theta}{\omega_0} v_3 + \gamma_1 + \gamma_2 \right]$$

$V_2 = V_C$

\[ \begin{array}{c}
\text{area } \gamma_1 \\
\text{area } \gamma_2
\end{array} \]

\[ \begin{array}{c}
\alpha \\
\beta \\
\delta \\
\xi \\
\varepsilon
\end{array} \]

$\Omega \xrightarrow{\Delta \rightarrow} 1$ \xrightarrow{\delta \rightarrow} \xrightarrow{\xi \rightarrow'}$ 

$\Theta \xrightarrow{\Omega \rightarrow} 1$ \xrightarrow{\omega_0 T_s}$
\[ \lambda_1 = \int v_2 \, dt = \lambda_{L1} + V \frac{B}{\omega_0} \]

with \( \lambda_{L1} = L \cdot (\text{change in } i_L \text{ over interval (2)}) \)
\[ = L (I_{L2} - I_{L1}) \]

so \( \lambda_1 = L (I_{L2} - I_{L1}) + V \frac{B}{\omega_0} \)

\[ \lambda_2 = \int v_2 \, dt = \lambda_{L2} + V \frac{C}{\omega_0} \]

\[ \lambda_{L2} = L \cdot (\text{change in } i_L \text{ over interval (4)}) \]
\[ = -L I_{L3} \]

so \( \lambda_2 = V \frac{C}{\omega_0} - L I_{L3} \)

Hence,
\[ \langle v_2 \rangle = \frac{1}{T_S} \left( \frac{\Theta}{\omega_0} V + L I_{L2} - L I_{L1} + V \frac{B}{\omega_0} + V \frac{C}{\omega_0} - L I_{L3} \right) \]
Solution, and plotting the characteristics

Given $\mu$ and $\theta$, we can find $J_2$ without iteration (and also $F$), and then plot the output characteristics of the switch. The relevant equations are:

1. \[ J_{L3} = \sqrt{2\mu - 1} \] (from p. 7)

2. \[ J_{L1} = -J_{L3} + \theta(1 - \mu) \]
   which follows from the slope of the current during the transistor conduction interval:
   \[
   \begin{array}{c}
   \text{i}\_L \parallel \text{v}_{gs} \rightarrow \text{i}_{L1} \\
   \text{i}_{L3} \rightarrow \text{i}_{L1} \rightarrow \text{not} \\
   \theta \rightarrow \alpha \rightarrow \beta \rightarrow \gamma
   \end{array}
   \]
   So \[ I_{L1} = -J_{L3} + \frac{V_g - V}{L} \theta \]
   normalize to get (2)

3. \[ J_{L2} = \sqrt{1 - 2\mu + J_{L1}^2} \] (from p. 5)

4. \[ \beta = \tan^{-1}\left(\frac{1 - \mu}{J_{L1}}\right) + \tan^{-1}\left(\frac{\mu}{J_{L2}}\right) \] (from p. 5)

5. \[ \delta = \frac{J_{L2}}{\mu} \] (from p. 6)

6. \[ \xi = \pi - \tan^{-1}\left(\frac{J_{L3}}{1 - \mu}\right) \] (from p. 7)

7. \[ F = \frac{2\pi}{\theta + \beta + \delta + \xi} \text{ length of switching interval } \\
   \omega_0 T_s = \theta + \beta + \delta + \xi
   \]

8. \[ J_2 = \frac{F}{4\pi} \left[ (J_{L1} - J_{L3}) \theta + J_{L2} \delta \right] \] (from previous pages)
So given $\mu$ and $\Theta$, we can evaluate the above equations in order to find the normalized dc inductor current without iteration.

The following pages contain plots of the normalized characteristics of the $\text{ASW-215}$ switch network.

Also attached are Microsoft Visual Basic functions to evaluate the above equations. These functions can be called from an Excel spreadsheet.

- $J_{2}(\mu, \Theta)$ evaluates $J_{2}$ in closed form, given $\mu$ and $\Theta$.
- $F(\mu, \Theta)$ evaluates $F$ in closed form, given $\mu$ and $\Theta$.
- $J(\mu, F)$ iterates to find $J$, given $\mu$ and $F$.

An Excel spreadsheet that includes the above functions as macros, and that produces the plots on the following two pages, can be downloaded from the course web site. It is virus-free.
QSW-ZVS Characteristics

Basic single-transistor resonant switch
QSW-ZVS Characteristics

Basic single-transistor switch
Transistor \((Q_1/D_1)\)
conduction angle \(\theta\)
Option Explicit

Public Function J2(mu, theta)
' quasi-square-wave resonant switch (basic single transistor version)
' calculation of normalized current J, as function of Q1/D1 conduction angle theta and switch conversion ratio mu
    Dim JL1, JL2, JL3, beta, delta, ksi, F, Pi
    Pi = 3.14159265
    If mu < 0.5 Then
        J2 = ""
        Exit Function
    End If
    JL3 = Sqr(2 * mu - 1)
    JL1 = -JL3 + theta * (1 - mu)
    If (1 - 2 * mu + JL1 ^ 2) < 0 Then
        J2 = ""
        Exit Function
    End If
    JL2 = Sqr(1 - 2 * mu + JL1 ^ 2)
    beta = Atn((1 - mu) / JL1) + Atn(mu / JL2)
    delta = JL2 / mu
    ksi = Pi - Atn(JL3 / (1 - mu))
    F = 2 * Pi / (theta + beta + delta + ksi)
    J2 = F / (4 * Pi) * ((JL1 - JL3) * theta + JL2 * delta)
End Function

Public Function F(mu, theta)
' quasi-square-wave resonant switch (basic single transistor version)
' calculation of normalized switching frequency F, as function of Q1/D1 conduction angle theta and switch conversion ratio mu
    Dim JL1, JL2, JL3, beta, delta, ksi, Pi
    Pi = 3.14159265
    If mu < 0.5 Then
        F = ""
        Exit Function
    End If
    JL3 = Sqr(2 * mu - 1)
    JL1 = -JL3 + theta * (1 - mu)
    If (1 - 2 * mu + JL1 ^ 2) < 0 Then
        F = ""
        Exit Function
    End If
    JL2 = Sqr(1 - 2 * mu + JL1 ^ 2)
    beta = Atn((1 - mu) / JL1) + Atn(mu / JL2)
    delta = JL2 / mu
ksi = Pi - Atn(JL3 / (1 - mu))
F = 2 * Pi / (theta + beta + delta + ksi)

End Function

Public Function J(mu, F)
' quasi-square-wave resonant switch (basic single transistor version)
' iteration to find normalized current J, as function of normalized switching frequency F and switch conversion ratio mu

Dim JL1, JL2, JL3, beta, delta, ksi, Pi, F1, F2, F3, theta1, theta2, theta3, thetamin, m, conv, eps, dF
Pi = 3.14159265
convg = 1e-06   ' set to 1e-05 for faster performance
eps = 0.01
If mu < 0.5 Then
  J = ""
  Exit Function
End If

JL3 = Sqr(2 * mu - 1)

thetamin = 2 * Sqr(2 * mu - 1) / (1 - mu) ' minimum theta
theta2 = 2 * Pi * mu / F ' starting guess
If theta2 < thetamin Then
  J = ""
  Exit Function
End If

theta3 = theta2 * 1.1  ' set up initial values
F2 = Fit(JL3, theta2, mu)
F3 = Fit(JL3, theta3, mu)
Do                                      ' iteration loop
theta1 = theta2
theta2 = theta3
F1 = F2
F2 = F3
m = (F2 - F1) / (theta2 - theta1)
theta3 = theta2 + eps * (F - F2) / m
F3 = Fit(JL3, theta3, mu)
dF = Abs(F - F3)
Loop Until dF < convg

JL1 = -JL3 + theta3 * (1 - mu)       ' calculate solution after convergence
JL2 = Sqr(1 - 2 * mu + JL1 ^ 2)

beta = Atn(((1 - mu) / JL1) + Atn(mu / JL2))
delta = JL2 / mu
ksi = Pi - Atn(JL3 / (1 - mu))
J = F3 / (4 * Pi) * ((JL1 - JL3) * theta3 + JL2 * delta)

End Function
Public Function Fit(JL3, theta, mu)
    ' computation of F, used in iterative routine of function J(mu, F) above
    Dim JL1, JL2, beta, delta, ksi, Pi
    Pi = 3.14159265
    JL1 = -JL3 + theta * (1 - mu)
    JL2 = Sqr(1 - 2 * mu + JL1 ^ 2)
    beta = Atn((1 - mu) / JL1) + Atn(mu / JL2)
    delta = JL2 / mu
    ksi = Pi - Atn(JL3 / (1 - mu))
    Fit = 2 * Pi / (theta + beta + delta + ksi)
End Function