The Zero-Voltage Resonant Switch
"type b" or "ZVS"

Basic network: \( +v_0 \)

\[ \begin{array}{c}
V_T \\
I_L \uparrow \\
D_1 \\
D_2 \\
I_T \\
\end{array} \]

(half-wave example)

conducting devices: 

\[ \frac{I_T}{Q_2} \quad V_T \quad D_1/Q_1 \\
D_2 \\
\]

\[ \text{turn on} \]

\[ Q_1 \text{ during this time} \]

\[ \text{turn off} \]

\[ Q_1 \text{ here (converter control)} \]
The type b zero voltage switch is tolerant of MOSFET device capacitance $C_d$s, and also of stray and leakage inductances. However, the effective capacitance of diode $D_2$ leads to ringing and switching loss. The peak transistor current is $I_T$, as in the PWM switch (good), but the peak transistor blocking voltage is $V_T (1 + J_T)$, or $(1 + J_T)$ times that of the PWM switch (bad — may be much larger than in other switches).
interval 1 all semiconductor devices off capacitor charging interval

\[ 0 \leq t < \alpha \]

\[ \begin{array}{c}
V_T \\
C \\
L \\
+ V_C \\
- i_L \\
I_T
\end{array} \]

\[ L \text{ and } V_T \text{ are irrelevant because they are in series with current source } I_T. \]

Capacitor \( C \) is charged by current \( I_T \):

\[ C \frac{dV_C}{dt} = I_T, \quad V_C(0) = 0 \]

normalize:

\[ \frac{1}{\omega_0} w_c = J_T, \quad w_c(0) = 0 \]

solution:

\[ w_c = J_T \omega_0 t \]

interval ends when \( V_C = V_T \), or \( w_c = 1 \)

\[ a + \omega_0 t = \alpha \]

hence,

\[ 1 = \alpha J_T \]

\[ \alpha = \frac{1}{J_T} \]
\[
\text{interval 2} \quad \text{D}_2 \text{ conducts} \quad \alpha \leq t \leq \alpha + \beta \\
\text{tank ringing interval}
\]

\[
\begin{align*}
C \frac{d}{dt} v_c &= i_L \\
L \frac{d}{dt} i_L &= v_T - v_c \\
\text{with } &i_L(\alpha) = \dot{I}_T, \\
v_c(\alpha) &= V_T
\end{align*}
\]

normalized form:
\[
\frac{1}{\omega_0} \dot{m}_c = \dot{J}_L, \quad m_c(\alpha) = 1 \\
\frac{1}{\omega_0} \ddot{J}_L = 1 - m_c, \quad J_L(\alpha) = J_T
\]

solution in phase plane is circular arc centered at \( m_c = 1, \dot{J}_L = 0 \)
of radius \( J_T \)

geometry:
\[
\beta = \pi + \sin^{-1} \frac{1}{J_T} \\
J_L = \sqrt{J_T^2 - 1}
\]

note \( J_T \geq 1 \) is required for zero voltage switching to occur.
interval 3

$\alpha + \beta \leq \omega_0 t \leq \alpha + \beta + \delta$

$D_2$ conducts

$D_1/Q_1$ conducts

inductor charging interval

$$\begin{align*}
V_c &= 0 \\
L \cdot i_L &= V_T \\
i_L(\alpha + \beta) &= -I_{L1}
\end{align*}$$

normalize: $\frac{1}{\omega_0} \int L = 1 \Rightarrow \int L(\alpha + \beta) = -I_{L1}$

solution: $\int L(\omega_0 t) = -I_{L1} + \omega_0 t - (\alpha + \beta)$

interval ends at $\omega_0 t = \alpha + \beta + \delta$, when $j_L = J_T$

so $J_T = -I_{L1} + \delta$

$$\delta = J_T + I_{L1} = J_T + \sqrt{J_T^2 - 1}$$

interval 4

$\alpha + \beta + \delta \leq \omega_0 t \leq \alpha + \beta + \delta + \eta$

$Q_1$ conducts

$$\begin{align*}
i_L &= I_T \\
V_c &= 0 \\
I_T
\end{align*}$$
The resonant switch is a two-port device, driven by the independent inputs \( V_T \) and \( I_T \). The dependent outputs are \( i_S \) and \( v_S \). Let us compute the average values of \( i_S \) and \( v_S \), and use the result to model the entire converter.

Since the tank inductor current \( i_L \) does not change in value over one switching period (\( i_L(T) = i_L(T+T_S) = I_T \)), no net volt-seconds are applied to the inductor and the average inductor voltage is zero. Hence,

\[
v_S = V_T - V_C - V_L
\]

\[
\langle v_S \rangle = \langle V_T \rangle - \langle V_C \rangle - \langle V_L \rangle
\]

\[
\langle v_S \rangle = V_T - \langle V_C \rangle
\]
in normalized form, $<w_s> = 1 - <w_c>$

So we need to compute the average tank capacitor voltage.

\[ <v_c> = \frac{1}{T_s} \int_0^{T_s} v_c(t) \, dt = \frac{\lambda_1 + \lambda_2}{T_s} \]

$\lambda_1 = \frac{1}{2} (\text{base}) \cdot (\text{height}) = \frac{1}{2} \left( \frac{a}{\omega_0} \right) (V_T) \quad (\text{area of triangle})$

$\lambda_2$ is found by inductor flux-linkage arguments:

during interval (2), the tank capacitor, tank inductor, and source $V_T$ are connected in series. Hence,

$V_T = v_c + v_L \quad \Rightarrow \quad v_c = V_T - v_L$

The integral of $v_c$ over this interval is

\[ \lambda_2 = \int_{a/\omega_0}^{\omega_0} v_c(t) \, dt = \int_{a/\omega_0}^{\omega_0} V_T \, dt - \int_{a/\omega_0}^{\omega_0} v_L(t) \, dt \]
\[ \gamma = \nu T \frac{E}{\omega_0} \]

\( \gamma \) is the total change in flux linkages
in the tank inductor during interval 2:
\[ \gamma = L \Delta i_L = L \left[ i_L(\omega_0) - i_L(\omega_0) \right] = L \left[ -I_L - I_T \right] \]

\{ This could also be written \}
\[ \gamma = \int_a^b v_c dt \int_a^b L di_L = L (i_L(b) - i_L(a)) \}

So we have
\[ \gamma = \nu T \frac{E}{\omega_0} = L \left[ -I_L - I_T \right] \]

\[ \langle v_c \rangle = \frac{\gamma_1 + \gamma_2}{T_S} = \frac{1}{\omega_0 T_S} \left[ \frac{1}{2} \nu V_T + \beta V_T + \omega_0 L (I_L + I_T) \right] \]

normalize;
\[ \langle w_c \rangle = \frac{F}{2\pi} \left[ \frac{1}{2} \nu + \beta + J_L1 + J_T \right] \]

substitute for \( \alpha, \beta, J_L1 \):
\[ \langle w_c \rangle = \frac{F}{2\pi} \left[ \frac{1}{2} \frac{1}{J_T} + T + \sin^{-1} \left( \frac{1}{J_T} \right) + J_T + \sqrt{J_T^2 - 1} \right] \]

= FP(J_T)

note that this is the same as the result for the
half-wave zero current (type \( a \)) switch, except
\( J_T \) is replaced by \( \frac{1}{J_T} \).
Average output voltage:

\[ \langle m_s \rangle = 1 - \langle m_c \rangle \]

\[ \langle v_s \rangle = \mu V_T \quad \text{with} \quad \mu = 1 - FP(I_T) \]

\[ P = \frac{1}{2\pi} \left[ \frac{1}{2} \frac{1}{J_T} + \pi + \sin^{-1} \frac{1}{J_T} + J_T + \sqrt{J_T^2 - 1} \right] \]

Type b: \( P(I_T) = \) Type a: \( P\left(\frac{1}{J_T}\right) \)

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Average input current:

For ideal switch, \( P_{in} = P_{out} \)

\[ V_T \langle i_s \rangle = \langle v_s \rangle I_T \]

but \( \langle v_s \rangle = \mu V_T \)

so \( \langle i_s \rangle = \mu I_T \)

\[
\begin{bmatrix}
\langle v_s \rangle \\
\langle i_s \rangle \\
V_s \\
I_T
\end{bmatrix} = \mu
\begin{bmatrix}
V_T \\
I_T
\end{bmatrix}
\]

De transformer

\( \mu \) controlled by \( F_j \) but also depends on \( V_T \) and \( I_T \)
A similar result occurs for the full wave case:

\[ P_x \geq 1 \]

\[ \Rightarrow \mu = 1 - F \quad \text{independent of load current} \]

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Summary of switches studied so far:

<table>
<thead>
<tr>
<th>Switch</th>
<th>( \mu )</th>
<th>( P(J_T) )</th>
<th>load current range</th>
<th>voltage conversion range</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM</td>
<td>D</td>
<td>–</td>
<td>nearly infinite</td>
<td>0 ( \leq \mu \leq 1 )</td>
</tr>
<tr>
<td>type a 1/2 wave</td>
<td>FP(x)</td>
<td>( R_1(J_T) )</td>
<td>0 ( \leq J_T \leq 1 )</td>
<td>0 ( \leq \mu \leq 1 )</td>
</tr>
<tr>
<td>type a full wave</td>
<td>FP x F</td>
<td>( R_1(J_T) \approx 1 )</td>
<td>0 ( \leq J_T \leq 1 )</td>
<td>0 ( \leq \mu \leq 1 )</td>
</tr>
<tr>
<td>type b 1/2 wave</td>
<td>1-F FP(J_T)</td>
<td>( \frac{1}{2} \left( \frac{1}{J_T} \right) )</td>
<td>1 ( \leq J_T \leq \infty )</td>
<td>0 ( \leq \mu \leq 1 )</td>
</tr>
<tr>
<td>type b full wave</td>
<td>1-F FP x 1-F</td>
<td>( \left( \frac{1}{J_T} \right) \approx 1 )</td>
<td>1 ( \leq J_T \leq \infty )</td>
<td>0 ( \leq \mu \leq 1 )</td>
</tr>
</tbody>
</table>

with \( R_\frac{1}{2} (x) = \frac{1}{2\pi} \left[ \frac{1}{2} x + \pi + \sin^{-1} x + \frac{1}{x} \left( 1 + \sqrt{1 - x^2} \right) \right] \)

\( R_1 (x) = \frac{1}{2\pi} \left[ \frac{1}{2} x + 2\pi - \sin^{-1} x + \frac{1}{x} \left( 1 - \sqrt{1 - x^2} \right) \right] \)
Output characteristics, full wave type b switch:

\[ J \]

\[ F = \frac{f_s}{f_o} \]
\[ F = \frac{1}{2} \]
\[ F = \frac{1}{4} \]

\[ M = \frac{V}{V_g} = \mu \]
\[ J = J_T = \frac{IR_o}{V_g} \]

\[ V = (1 - \frac{f_s}{f_o}) V_g \quad \text{with} \quad I \geq \frac{V_g}{R_o} \]

With resistive load \( R \): \( I = \frac{V}{R} \geq \frac{V_g}{R_o} \)
\[ \Rightarrow R \leq \left(1 - \frac{f_s}{f_o}\right) R_o \]