P12.10
Set up a cylindrical coordinate system with $z$ along the wire. The $B$ field at point $P$ from any section of the wire will thus be in the $+\phi$ direction. Thus we don’t need to add up vector $B$ fields as we integrate, just the single scalar component.

Using Biot-Savart, we can write the contribution to the $B$ field from some incremental length $dl$ of the wire:

$$ dB_\phi = \frac{\mu_0 I}{4\pi} \frac{dl \sin \alpha}{r^2} $$

where $\alpha$ is the angle between $I$ and the unit vector $u_r$. This angle pops out of the cross product. Since both the angle and radius are changing as we integrate, we need to do some trig to get them in terms of a single variable. If you make a good drawing (required!) and appropriately label $\alpha$, you can use this to relate the increment $dl$ to the angle from the point $P$, by

$$ dl = r \, d\theta / \cos(\pi / 2 - \alpha) = r \, d\theta / \sin(\alpha) $$

Substitute this into the first expression to get

$$ dB_\phi = \frac{\mu_0 I}{4\pi} \frac{d\theta}{r} $$

and integrate from $\theta_1$ to $\theta_2$:

$$ B_\phi = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{d\theta}{r} \cos(\theta) d\theta $$

$$ = \frac{\mu_0 I}{4\pi} \left[ \int_{\theta_1}^{\theta_2} \cos(\theta) d\theta \right] $$

$$ = \frac{\mu_0 I}{4\pi} \left( \sin \theta_2 - \sin \theta_1 \right) $$

$$ = \frac{\mu_0 I}{4\pi a} \left( \sin \theta_2 - \sin \theta_1 \right) $$
P13.6 – The problem I meant to assign so gave as extra credit
First, we set up a cylindrical coordinate system with z along the axis. This system has
sufficient symmetry to use Ampere’s law, and there are magnetic materials involved so
we want to use \( H \) (which is created only by applied currents) not \( B \) (which is created by
all current).

\[
\oint H \, r d\phi = \begin{cases} \int_0^a dr \int_0^{2\pi} d\phi \frac{I}{\pi a^2} & r < a \\ \int_a^\infty dr \int_0^{2\pi} d\phi \frac{I}{\pi a^2} & r > a \end{cases}
\]

\[
H = \begin{cases} \frac{Ir}{2\pi a^2} & r < a \\ \frac{I}{2\pi r} & r > a \end{cases}
\]

Applying \( B = \mu \, H, \, M = (\mu_r - 1) \, H \) we have all of the field quantities in all of space. To
get the volume magnetization current \( J_m \) (which could be used to replace the material as
an equivalent source) we use Eq. 13.14. Since we’re in cylindrical coordinates, we look
up the expression for the curl because nobody remembers it:

\[
J_m = \nabla \times M
\]

\[
= \nabla \times [M_{\phi}(r)u_{\phi}]
\]

\[
= \frac{1}{r} \frac{\partial}{\partial r} [r M_{\phi}(r)] u_z
\]

\[
= (\mu_r - 1) \frac{I}{\pi a^2} u_z
\]

in the conductor and =0 elsewhere. The surface magnetization currents come from
equation 13.16 and are

\[
J_{m_s}(r = a) = -M_{\phi}(a)u_z
\]

\[
J_{m_s}(r = b) = +M_{\phi}(b)u_z
\]

\[
J_{m_s}(r = c) = -M_{\phi}(c)u_z
\]

This problem had virtually all of magnetostatics in it, so is a good one to understand
P13.16
A rather boring application of the boundary conditions, but here goes. Let material one be air and material 2 be the ferromagnetic material.

\[ B_1 = \sqrt{B_{\text{normal}}^2 + B_{\text{tangential}}^2} \]
\[ B_2 = \sqrt{B_{\text{normal}}^2 + B_{\text{tangential}}^2} \]
\[ = \sqrt{B_{\text{normal}}^2 + \left( \frac{\mu}{\mu_0} B_{\text{tangential}} \right)^2} \]
\[ >> B_1 \]

for \( \mu >> 1 \).

P14.6
First we note that the length of the solenoid is 10 times its diameter. In the engineering handbook, 10 = infinity, so we’ll treat the solenoid as infinitely long. The answer won’t be perfect, but it won’t be too far off. For an infinite solenoid, the flux is contained completely within the solenoid, so the flux through the outer loop is just the flux through the center of the solenoid. We know the flux through the outer loop and want the emf around it, which suggests Faraday’s law:

\[ B(t) = \mu_0 \frac{N_1}{L} i(t) \]
\[ \Phi(t) = \mu_0 \frac{N_1 \pi a^2}{L} i(t) \]
\[ e = -\frac{d\Phi(t)}{dt} \]
\[ = -\mu_0 \frac{N_1 N_2 \pi a^2}{L} \frac{d}{dt} i(t) \]
\[ = -0.116 \cos(314t) \text{ mV} \]

The amplitude of the induced electric field is simply the emf over the entire path length of the outer coil or \( e/(2 \pi N_2 b) \). Note that you get a minus sign if you assume the coils are wrapped in the same direction and a plus if you assume they are wrapped in the opposite directions.

P14.11
The liquid is flowing in the magnetic field so there is an induced E field \( \mathbf{E}_{\text{ind}} = \mathbf{v} \times \mathbf{B} \). Integrating this from the top of the fluid to the bottom yields the emf between the electrodes of \( V = \mathbf{v} \mathbf{B} \mathbf{d} \). Solving for the velocity, \( \mathbf{v} = V / (\mathbf{B} \mathbf{d}) \).