ECEN 3400, HW assignment 7
With solutions.

Material: Chapter 19
Assigned: Saturday, Nov 5
Recital: Monday, Nov 7
Due: Monday, Nov 14, beginning of class (4 pm)

Problems:

P19.1: Simple, but worth emphasizing. Add the following two questions:

- Does the addition of the concept of displacement current alter the law of current continuity? Why or why not?
- Does the addition of the concept of displacement current alter Ampere’s law? Why or why not? Your answer should go beyond “it adds a term”. Why is it necessary to add this term?

\[
\begin{align*}
 i_{\text{Displacement}} &= \frac{\partial D}{\partial t} \left[ \frac{A}{m^2} \right] S[m^2] \\
 &= \frac{\partial \sigma}{\partial t} S \\
 &= \frac{\partial (Q/S)}{\partial t} S \\
 &= \frac{\partial Q}{\partial t} \\
 &= i
\end{align*}
\]

This does not modify the law of current continuity. A closed surface surrounding one of the capacitor plates would have a current \( i \) across the surface and a charge accumulation inside equal to \( Q \), the charge on the plate. The time rate of change of this enclosed charge is equal the current \( i \) across the boundary. Displacement current, in this sense, is not a “real” current made of moving charges.

This does modify Ampere’s law. We must add this term to the equation in order to maintain the concept that the current is the same through any open surface stretched across a closed contour.

Equivalence of integral and differential forms of Maxwell’s equations
A common numerical technique to solve Maxwell’s equations is called the Finite Difference method. All derivatives are replaced by a simple “finite difference” approximation:
The figure below shows a small portion of such a simulation space in which 5 field components are known at 5 discrete locations. X points out of the page and the figure is in the YZ plane. Write an expression for the time derivative of $H_x$ at the center of the grid using first the integral form of Faraday’s law (19.5) and then the differential form (19.16) and show that they are the same.

Start with the integral form, divide through by the area of the region and re-arrange terms:

\[
\oint_C \mathbf{E}_{\text{ind}} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}
\]

\[
E_y(z - \Delta z/2)\Delta y + E_z(y + \Delta y/2)\Delta z - E_y(z + \Delta z/2)\Delta y - E_z(y - \Delta y/2)\Delta z = -\frac{d\mu H_x}{dt}(\Delta y\Delta z)
\]

\[
\mu \frac{dH_x}{dt} = \frac{E_y(z + \Delta z/2) - E_y(z - \Delta z/2)\Delta y}{\Delta z} - \frac{E_z(y + \Delta y/2) - E_z(y - \Delta y/2)\Delta y}{\Delta y}
\]

The last term is the x component of the differential form.

P19.12 This is a simple plug and chug problem, but contains a very important idea. Add the following:

- Use expressions from chapter 18 to simplify the expression down to a comparison of $d$ and one quantity.
- This is the condition under which circuit theory is derived as an approximation to Maxwell’s equations. Explain why this condition permits you to ignore transmission-line effects.
From above, we see that we are comparing the circuit dimension with the wavelength of the associated radiating wave. This is the fundamental approximation on which all of circuit theory is founded. In this limit, you can not observe the wave behavior of voltages and currents on a transmission line, leading to the simplification that voltages and currents are constant along wires. Similarly, propagation delays can be ignored since propagation at $c$ across a distance $d$ will be much smaller than the period of the oscillation $1/f$.

\[ d < \frac{0.1}{\omega \sqrt{\mu \varepsilon}} = \frac{0.1}{c} \frac{c}{2\pi f} = \frac{0.1}{2\pi} \lambda \]

\[
\begin{aligned}
&\left\{ 
80 \text{ km} & f = 60\text{Hz} \\
0.48 \text{ m} & f = 10\text{MHz} \\
0.48 \text{ mm} & f = 10\text{GHz}
\right. 
\end{aligned}
\]