P24.6. In a radio link at \( f = 900 \text{ MHz} \), with two half-wave dipoles a distance \( r = 100 \text{ m} \) apart, one dipole (e.g., dipole 1) is replaced by a more directive antenna (for a cellular phone, this would be the base-station antenna). Calculate the ratio of the powers received by the second dipole in the two cases, for a directivity of (1) 6 dB, (2) 12 dB and (3) 20 dB of antenna 1. —
\[ \begin{align*}
(1) & \ 2.45, \ 6.1, \ 67, \ \ (2) \ 2.34, \ 8.5, \ 89, \ \ (3) \ 1.85, \ 9.3, \ 52.
\end{align*} \]

P24.7. Assume that in a communications link two matched bowtie antennas, \( A \) and \( B \), are \( r \) apart in each other's far fields. The antenna directivities and effective areas in the line-of-sight direction are \( D_A, A_A \) and \( D_B, A_B \), respectively. First antenna \( A \) transmits a power \( P_A \), while antenna \( B \) receives a power \( P_B \). Then antenna \( A \) transmits a power \( P_{BA} \) while antenna \( A \) receives a power \( P_{AB} \). Using the reciprocity condition, which says that \( P_{AB}/P_A = P_{BA}/P_B \) (think about what this means), show that the ratio of the directivity to the effective area is a constant for any antenna. — Hint: use Eqs. (24.9) and (24.10) to obtain the Friis formula with the directivity of the transmitting antenna and the effective area of the receiving antenna; combine both such formulas, one for antenna \( A \) as the transmitting antenna and the other for antenna \( B \) as the transmitting antenna, with the reciprocity conditions.

P24.8. Derive the Friis formula in terms of effective area only. In a microwave relay system for TV each antenna is a reflector with an effective area of \( 1 \text{ m}^2 \), independent of frequency. The antennas are 10 km apart. If the required received power is \( P_r = 1 \text{ mW} \), what is the minimum transmitted power \( P_t \) required for transmission at 1 GHz, 3 GHz, and 10 GHz? — Hint: use Eqs. (24.9) and (24.10). \( \begin{align*}
(1) & \ 9 \text{ mW}, \ 1 \text{ mW}, \text{ and } 0.09 \text{ mW}, \ \ (2) \ 90 \text{ mW}, \ 1 \text{ mW}, \ \text{ and } 0.09 \text{ mW}, \ \ (3) \ 9 \text{ W}, \ 1 \text{ W}, \ \text{ and } 0.09 \text{ W}.
\end{align*} \]

P24.9. Derive the Friis formula in terms of directivities only. — Hint: use Eqs. (24.9) and (24.10).

25. Some Practical Aspects of Electromagnetic Waves

- Applications of electromagnetic waves are numerous, ranging from cooking food to controlling a happening spacecraft and receiving information from it. In these applications, a number of practical problems need to be solved. For example, when one sends or receives information using electromagnetic (radio) waves, the maximum distance at which this can be done is limited.
by the amount of power available at the sending end, and the loss of the wave energy by the time it gets to the receiving end, assuming a constant receiver sensitivity. The path loss varies with the medium through which the wave is propagating, as well as the frequency (wavelength) of the wave. For example, for a coaxial cable the attenuation constant is given by

\[ \alpha = \frac{R'}{2Z_0} = \frac{R'}{2} \sqrt{\frac{C'}{L'}} \]  

(25.1)

Note that, due to skin effect, \( R' \) increases with frequency.

- For long two-conductor transmission lines, the attenuation can be reduced drastically by introducing additional series inductances along the line. This is seen from the expression for the attenuation constant,

\[ \alpha \approx \frac{1}{2} \left( R' \sqrt{\frac{C}{L}} + G' \sqrt{\frac{L}{C}} \right) \]  

(25.2)

noting that, in practice, the first term is much greater (several orders of magnitude) than the second, due to relatively large value of \( R' \).

- Attenuation is also present in waveguides, although they have relatively large surfacer for current flow. For example, the attenuation constant for the dominant mode (TE\(_{10}\)) in a rectangular waveguide is given by

\[ \alpha = R_n \sqrt{\frac{\sigma a}{b} + \frac{2\pi^2 f^2}{c}} \]  

(TE\(_{10}\)).

(25.3)

Since \( R_n \) is related directly to skin depth, the attenuation constant depends on the metal conductivity and frequency. For example, a 10 GHz waveguide has an attenuation constant of about \( \alpha = 0.0883 \text{Np/m} = 0.767 \text{dB/m} \).

- Optical fibers are used as waveguides for electromagnetic waves in the visible an infrared part of the frequency spectrum, with wavelengths between roughly 300 nm and 10 \( \mu \)m. Fibers are so-called dielectric waveguides. The simplest dielectric waveguide is a flat dielectric slab, where the total reflection at the slab faces is used to guide the wave.

- In all of the above guided wave cases, the ratio of the received and transmitted power is of the form \( P_{\text{rec}}/P_{\text{trans}} = e^{-2\alpha r} \).

- In a line-of-sight radio link (where a radio wave travels between two antennas directly, with no reflections), the power loss is given by the Friis transmission formula. If the two antennas are equal, \( n^2 \) large, and we assume they are well designed so that the effective areas are roughly equal to their geometric areas, the Friis formula becomes

\[ \frac{P_{\text{rec}}}{P_{\text{trans}}} = \frac{n^2 \lambda^2}{r^2} \]  

(25.4)
CHAPTER 25: SOME PRACTICAL ASPECTS OF ELECTROMAGNETIC WAVES

This means that the larger the antennas are (measured in wavelengths), the lower the loss of power between the transmitter and receiver.

- AM broadcasting systems rely on surface wave transmission between two points on the earth's surface. Short wave radio systems use bounce off of the ionosphere and the earth's surface. VHF and VHF radio waves used for communications by airplanes, as well as microwave in radio relay links, use the line-of-sight propagation, in which case the range is limited by the curvature of the earth. Therefore, almost all radio relay stations are put up on high peaks. If the height of the two antennas are $h_{trans}$ and $h_{rec}$, and the radius of the earth is $R$, the approximate range of the line-of-sight link obtained in practice is

$$r = \sqrt{\frac{2R}{3}h_{trans}} + \sqrt{\frac{2R}{3}h_{rec}}.$$  \hspace{1cm} (25.5)

In this formula, no attenuation due to the atmosphere is taken into account.

- The upper layer of the atmosphere, approximately between 30 km and 500 km above the earth's surface, is a highly rarified ionized gas, known as the ionosphere. It has a very pronounced influence on the propagation of electromagnetic waves in a wide frequency range, from about 10 kHz up to about 30 MHz, but also at higher frequencies than these. The presence of the ionosphere is equivalent to a reflection in permittivity. The equivalent (or effective) permittivity of an ionized gas is defined as

$$\varepsilon' = \varepsilon_0 \left(1 - \frac{NQ^2}{\omega^2 \epsilon_0 m} \right) = \varepsilon_0 \left(1 - \frac{\omega_i^2}{\omega^2} \right),$$  \hspace{1cm} (25.6)

where

$$\omega_i = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{NQ^2}{\epsilon_0 m}}.$$  \hspace{1cm} (25.7)

in the critical frequency of the ionized gas. Thus, the propagation coefficient and the phase velocity of the wave are

$$\beta = \omega \sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{\varepsilon_0} \sqrt{\left(1 - \frac{\omega_i^2}{\omega^2} \right)}, \quad c_p = \frac{\omega}{\beta} = \frac{\varepsilon_0}{\sqrt{1 - \omega_i^2/\omega^2}}.$$  \hspace{1cm} (25.8)

It is seen that waves of angular frequencies $\omega < \omega_i$ cannot propagate in the ionized gas. Ionized gases also introduce a wave attenuation, due to the collisions of ions accelerated by the electric field of the wave with neutral molecules.

- Let a plane wave be emitted from the earth's surface towards the ionosphere at an arbitrary angle $\theta_0$ with respect to the vertical. The wave is either reflected back, or passes through the ionosphere, depending on the wave frequency, $f$, the highest critical frequency in the ionosphere, $f_{cr max}$, and the angle $\theta_0$. If the wave is reflected, it bounces back at a height corresponding to a critical frequency given by the equation
Waves of higher frequencies emitted at the same angle will pass through the ionosphere.

- From this summary, it is seen that the optimal frequency range for different applications is very different. For example, the communications via satellites require frequencies not influenced by the ionosphere, while short-wave communications use the ionosphere as a mirror. At lower frequencies and when the two ends can be physically connected, coaxial cables are often used (e.g., cable TV). TV signals can also be received from satellites, in which case frequencies on the order of 10 GHz are used. For cellular telephony, mostly frequencies around 900 MHz and 2 GHz are used, to avoid manmade and atmospheric noise. For radio communication with submarines, frequencies as low as about 10 kHz are used, since these waves have relatively large skin depth (penetration) in sea water.

- Radars are essentially a radio link, where the transmitter and receiver are located at the same place. The transmitter sends a wave of power $P_T$, which partly reflects off of the target. The power density at the target is $P_T D/(4\pi r^2)$, where $r$ is the distance to the target, and $D$ is the radar antenna directivity. The target scatters the wave proportionally to a quantity called the radar scattering cross section, usually denoted by $\sigma(\theta, \phi)$, which is essentially an effective area of the target acting as a receiving antenna. When it reflects the wave, the target acts as a transmitting antenna with a directivity of $4\pi r^2/L^2$. The reflected wave is received at the transmitting point and conclusions are then made about the target. The applications of radars are very diverse, ranging from many military purposes, to weather radars in meteorology, anti-collision radars for cars, etc. The radar equation tells us how much power is received back for a given transmitted power. If the same antenna is used for transmission and reception, it reads

$$P_{rec} = P_T \frac{D^2(\theta, \phi)\sigma^2(\theta, \phi)}{16\pi^2r^4}.$$  

In a Doppler radar, the transmitted signal is frequency modulated, which enables the speed of the target to be measured.

- In computers and other digital systems, radiation and electromagnetic coupling becomes progressively more pronounced as the clocks in these systems become faster. This can result in a high crosstalk between different segments of the system.

- In normal ovens, food is heated mainly by infrared radiation from the heaters. These waves have very small skin depth, so that the food is heated from the surface inwards by thermal conduction. Using microwave frequencies instead, the skin depth is greatly increased, so that direct electromagnetic heating takes place in a large part of the heated food, reducing greatly the cooking time. In microwave ovens, the standard frequency is 2.45 GHz.

**QUESTIONS**

Q25.1. Explain what the physical origin of loss in coaxial waveguides is. — *Hint: recall the skin effect.*

Q25.2. Explain what the physical origin of loss in metallic waveguides is, and why the loss can be smaller than in coaxial cables. — *Hint: recall the skin effect and compare the areas available for current flow in the two cases.*
Q25.3. Explain what the physical origin of loss in optical fiber is, and why the loss can be smaller than in metallic structures. — *Hint*: recall the polarization losses.

Q25.4. Explain what the physical origin of loss in a line-of-sight antenna link is. — *Hint*: think of the loss due to the spherical character of the wave, of the influence of the atmosphere, of possible raindrops along the way and of possible multiple waves reaching the receiver.

Q25.5. What is the range in a line-of-sight link limited by? — (a) Frequency. (b) Height of transmitter. (c) Height of transmitter and receiver. and the radius of the earth.

5 Q25.6. Explain in your own words why there is attenuation in an ionized medium with neutral gas molecules. — *Hint*: think what happens when the electric field accelerates a charged particle, and the particle collides with a neutral atom.

**Answer.** The electric field of the wave accelerates charged particles, and thus transfers to them some energy. If there are no collisions with neutral particles, this energy is returned to the wave in a latter time interval. If, however, a charged particle collides with a neutral gas molecule, the acquired energy of the particle is partly transferred to the molecule. There is no mechanism for an uncharged particle to interact with the wave and to return this energy back to it. So the energy of the wave is reduced.

Q25.7. A wave of frequency higher than the highest critical frequency for the ionosphere needs to be used for communication between two points of the earth. Is this possible? Explain. — (a) No. (b) Yes, if the angle of incidence on the ionosphere is large enough. (c) Yes, if the signal is frequency modulated.

5 Q25.8. A wave of extremely low frequency (e.g., below 100 Hz) coming from outer space penetrates through the ionosphere and reaches the earth's surface. Explain. — (a) The ions in the ionosphere are not affected by such low frequencies. (b) The process of wave reflection takes a certain depth. When the wavelength is long, this depth can be the entire ionosphere, and the wave reaches the earth's surface. (c) Low frequencies carry more energy, and therefore pass the ionosphere.

**Answer.** The free-space wavelength of such a wave is on the order of few thousand kilometers, i.e., much greater than the ionosphere thickness. So the ionosphere does not have a sufficient shielding effect for such waves.

Q25.9. Imagine a line-of-sight link in a hallway with conducting walls on top and bottom, and absorbing walls on the sides. How many waves can contribute to the received signal? How would you construct antenna images that approximate the influence of the walls? — *Hint*: assume no reflections off the absorbing walls, and approximate the top and bottom walls by perfectly conducting planes.

5 Q25.10. Derive the radar equation, Eq. (25.16).

**Answer.** As the target, the received power is $P_r = \frac{P_t \cdot \sigma}{(4\pi)^2} \cdot \frac{D_A^2}{4\pi}$. This power is reflected in an amount proportional to the “directivity” of the target, $D_A = \frac{\sigma}{(4\pi)}$, and is received by the radar in proportion to its antenna effective area, $A = \frac{D_A^2}{(4\pi)}$. So, we have

$$P_r = P_t \cdot \frac{1}{4\pi^2} \cdot \frac{\sigma}{4\pi} \cdot \frac{D_A^2}{4\pi} = P_t \cdot \frac{\sigma}{16\pi^2} \cdot \frac{D_A^2}{4\pi}$$
Q25.11. Consider a Doppler radar at 10 GHz. The received signal from one car is in the audio range, and can be between 300 Hz and 4 kHz. What is the range of speed this radar can detect?

Answer. For velocities much smaller than the speed of light, $c$, the Doppler shift is given by $f_D/f = \nu/c$, so $\nu = cf_D/f$. For $f_D = 300$ Hz, $\nu = 9$ m/s (corresponding to a car moving at about 30 km/h), and for $f_D = 4$ kHz, $\nu = 129$ m/s (corresponding to a slow plane, flying at about 220 km/h).

Q25.12. Consider an FM ranging radar in which the frequency varies linearly from $f_1 = 10$ GHz to $f_2 = 10$ GHz $f_1$ in $T = 10\mu s$. How would you choose $f_2$ in order to be able to detect targets 1 km away, if the radar bandwidth is 500 MHz?

Answer. The range is given by $r = (\pi f_2 f_1) / (2f $f_2 - f_1$)$. For $f_2 = 10$ GHz and $r = 1$ km, we get that $f = (2f_2 - 10)/3$. If we choose $f_2$ so that the full bandwidth is used, i.e., $f_2 = 10$ GHz + 500 MHz = 10.5 GHz, we get $f = 10.33$ GHz for a 1-km target range, which is within the radar bandwidth.

PROBLEMS

P25.1. Calculate how much power is received in England if 1 MW is sent from Boston along a transatlantic 50Ω cable at 10kHz. You can assume that the main loss in the cable is due to conductor loss and that $R = 0.005 \Omega/m$. — (a) 1024 kW. (b) 4.39 kW. (c) 0.53 kW.

P25.2. What value of Pupin coils would you choose and how would you place them to reduce the loss in the cable with $C = 167$ pF/m, $G = 0.3$ pS/m, $L' = 0.2 \mu H/m$, and $R' = 0.0055 \Omega/m$. — Hint: use coils at every kilometer to artificially increase the inductance per unit length about 85 times.

P25.3. Calculate the skin depth and attenuation coefficient of a rectangular waveguide with dimensions $a = 23$ mm and $b = 10$ mm, at 10 GHz, if the waveguide is made of (1) copper, (2) aluminum, (3) silver, or (4) gold. What do you think are the engineering problems associated with each metal? Can you think of any combined solution? — We give possible answers for gold: (a) $\delta = 7.86 \cdot 10^{-7}$ m, $\alpha = 0.0149 \, \text{Np/m}$. (b) $\delta = 4.53 \cdot 10^{-7}$ m, $\alpha = 0.0231 \, \text{Np/m}$. (c) $\sigma = 9.7 \cdot 10^{-7}$ m, $\alpha = 0.0189 \, \text{Np/m}$.

P25.4. Calculate the skin depth of gold in the optical domain, at wavelengths of 500 nm, 830 nm, 1.33 µm, and 1.55 µm. How thin would one need to make a sheet of gold to see through it? — The sheet should be thinner than about (a) 100 nm. (b) 20 nm. (c) 5 nm.

P25.5. Compare the loss in the inner conductor and outer conductor of a coaxial cable at 1 MHz. Assume the conductors are made of copper, that the cable is filled with a dielectric of permittivity $\varepsilon_r = 3$, and that the dimensions are such that the inner conductor radius $a = 0.45$ mm, and inner radius of the outer conductor $b = 0.9$ mm. — (a) 0.05. (b) 0.27. (c) 0.3.

P25.6. Plot the power attenuation in dB versus distance from 1 m to 1,000 km on a logarithmic scale for: coaxial cable at 100Hz with $a = 0.5 \, \text{dB/m}$, waveguide with $a = 0.1 \, \text{dB/m}$, 1.55-µm single-mode optical fiber with $a = 0.1 \, \text{dB/km}$, and a free space link at 10 GHz with a horn antenna with 20-dB directivity and a 1-m diameter dish antenna. — Hint: recall the attenuation constants in the four cases.

P25.7. Calculate the dimensions for a rectangular waveguide with a dominant $TE_{02}$ mode at cable TV frequencies between 100 and 600 MHz. — Let $a/b = 2$. (a) $a = 30$ cm. (b) $a = 50$ cm. (c) $a = 100$ cm.
CHAPTER 25: SOME PRACTICAL ASPECTS OF ELECTROMAGNETIC WAVES

25.8. A UHF radio system for communication between airplanes uses antennas with a directivity of 2. What is the maximum line-of-sight range between two airplanes at an altitude of 10 km? If the required received power is 10 μW, what is the minimum transmitted power $P_t$ required for successful transmission at 100 MHz, 300 MHz, and 1 GHz?

Solution. The range is $r = 2 \sqrt{\frac{P_t D_c D_r}{4 \pi I_b}} = 824.4 \text{ km}$. For this range, the minimal required transmitted power is $P_t = \left(\frac{4 \pi I_b}{r^2}\right)^2 P_c = 268 \text{ (watts)}$. where $I_b$ is in milliwatts. At 100 MHz, 300 MHz and 1 GHz, the respective wavelengths are 3 m, 1 m and 0.3 m, and the transmitted powers are 30 W, 268 W and 34 W.

25.9. Calculate the effective area of a dish antenna for TV that requires a 1-degree beamwidth in both $\theta$ and $\phi$ planes, assuming one of the standard cable frequencies (for example, 225 MHz). Is this a practical antenna? (Note: you can use an approximate formula for the maximal directivity given the beamwidths, $\alpha_1$ and $\alpha_2$, in the two planes, $D \approx 35.000/(\alpha_1 \alpha_2)$, where the beamwidths are given in degrees.)

- (a) 356 m$^2$.
- (b) 132.8 m$^2$.
- (c) 5527 m$^2$.

25.10. If a satellite is 1000 km above the earth's surface, and has a 0.1 degree beamwidth in both planes, calculate the corresponding directivity using the approximate formula in the previous problem. Find the size of the footprint on the earth's surface, and the effective area of the antenna at a satellite frequency of 4 GHz. — The radius of the footprint is (a) 37 m, (b) 168 m, (c) 956 m. The antenna effective area is (a) 1432 m$^2$, (b) 5271 m$^2$, (c) 761 m$^2$.

25.11. Derive the radar equation (25.10) for a radar that uses two antennas, one for transmitting and another for receiving. — Hint: what you need to know in this case instead of the directly squared of the antenna?

25.12. Assuming a 10-GHz police radar uses an antenna with a directivity of 20 dB (standard horn), and your car has a scattering cross section of 100 λ$^2$, plot the received power as a function of target distance, for a transmitted power of 1 W. If the receiver sensitivity is 10 mW, how close to the radar would you need to drive down to avoid getting a speeding ticket? — The radar starts detecting the speed of a car at approximately (a) 35 m, (b) 489 m, (c) 1213 m.

25.13. How large is the dynamic range of the radar from problem 25.12? (The dynamic range is the ratio of the largest to smallest signal power detected, expressed in decibels.)

- (a) 50 dB.
- (b) 90 dB.
- (c) 64 dB.