17

Some Examples and Applications of Time-Invariant and Slowly Time-Varying Magnetic Fields

17.1 Introduction

Magnetic fields are present in many practical applications, as well as in the natural world. For example, we are continuously situated in the relatively strong time-invariant (or extremely slowly variant) magnetic field of the earth; this field may, for example, affect the way your computer monitor works depending on whether you happen to turn it on in the Northern or Southern Hemisphere. We also often find ourselves in magnetic and electric fields existing around high-voltage and high-current power lines, and it is interesting to calculate the order of magnitude of voltages induced in our body. Most electrical home appliances contain devices that use mag-
magnetic forces and moments of magnetic forces, and our computers, tape recorders, and video recorders use magnetic storage devices. The aim of this chapter is to review some of the more important and interesting applications of magnetic (along with electric) fields.

17.2 The Magnetic Field of the Earth

The earth behaves like a large permanent magnet whose magnetic field is similar to the field of a giant current loop with an axis declined 11 degrees with respect to the earth’s axis of rotation (Fig. 17.1a). (Geologists believe that the magnetic field is created by the difference in the speed of rotation of the earth’s liquid core and its solid mantle.) The planet’s geographic North Pole is approximately the south magnetic pole (this is why the north pole of the magnetic needle of a compass always points to the north). The magnitude of the earth’s magnetic flux density is about 50 μT at our latitude and about 20% stronger at the poles.

About 90% of the magnetic field measured at the earth’s surface is due to the field originating inside the planet. The rest is due to the currents produced by charged particles coming from the sun, and to the magnetism of the rocks in the crust. The region in which the earth’s magnetic field can be detected is called the magnetosphere. It is not symmetrical, but rather has the shape of a teardrop. This is due to the charged particles streaming from the sun that are deflected by the earth’s magnetic field; the earth forms a “shadow” for charges, which has the effect of elongating the magnetosphere.

Because the magnetic field everywhere on the surface of the earth is partly due to magnetization of rocks at that point, magnetometers can be used in geology for de-

![Diagram](image-url)

**Figure 17.1** (a) The earth produces a dc magnetic field roughly equivalent to the magnetic field of a very large current loop. The plane of the loop is declined with respect to the earth’s axis of rotation. (b) The region in which the earth’s magnetic field can be detected is called the magnetosphere and is asymmetrical due to the charges emitted from the sun.
tecting different types of ores. Measuring magnetization of rocks also gives us insight into the earth’s magnetic history. Rocks become magnetized when they are formed, or when they remelt and recool at some later time. When rocks are heated they lose their magnetization and are remagnetized by the earth’s magnetic field as they cool. Therefore, they carry a permanent record of what the earth’s magnetic field was like at the time of the rock formation. Measurements of rock magnetization show that the earth’s magnetic poles have wandered. Some rocks that formed over short time intervals show fossil magnetic polarities 180 degrees apart, which cannot be explained by a 180-degree rotation of a continent (the time of reversal was too short for this to be possible). The conclusion is that the earth’s magnetic field switched polarity, similarly to the field of a loop in which the current changes direction. These field reversals occurred many times during our planet’s geological history, and about 10 times in the last 4 million years. Rock magnetization indicates that the polarity does not flip instantly: it first slowly decreases and then increases in the opposite direction.

Questions and problems: Q17.1 to Q17.3

17.3 Applications Related to Motion of Charged Particles in Electric and Magnetic Fields

Charged particles in both electric and magnetic fields always move. In many instances one of the fields is of much less influence than the other. For example, we have seen that both an electric and a magnetic field act on moving charges that form an electric current in a conductor, but that the influence of the magnetic field is negligible. There are examples of the other kind, where electric forces exist but are negligible. In some cases, the effects of electric and magnetic fields on a moving particle are of the same order of magnitude and must both be taken into account.

The motion of charged particles in electric and magnetic fields may be in a vacuum (or very rarefied gas), in gases, and in solid or liquid conductors. This brief section is aimed at explaining the principles of motion of charged particles in electric and magnetic fields and at presenting examples of how some engineering applications take advantage of this motion.

We know that the force on a charge \( Q \) moving in an electric and a magnetic field with a velocity \( \mathbf{v} \) is the Lorentz force, Eq. (12.13), which is repeated here for convenience:

\[
\mathbf{F} = Q\mathbf{E} + Q\mathbf{v} \times \mathbf{B}. \quad (17.1)
\]

If the electric field can be neglected, we omit the first term of the Lorentz force. If the magnetic field can be neglected, we omit the second term.

If a charge moves in a vacuum, then this force in any instant must be equal in magnitude and opposite in direction to the inertial force. If the mass of the charge is \( m \), the equation of motion therefore has the form

\[
m \frac{\mathbf{d}\mathbf{v}}{dt} = Q\mathbf{E} + Q\mathbf{v} \times \mathbf{B}. \quad (17.2)
\]
In this equation, \( \mathbf{E} \) and \( \mathbf{B} \) in general are functions of space coordinates and of time. Except in rare cases, it is impossible to solve such a general equation for the velocity of the charge analytically, but it can always be solved numerically.

If a charge moves in a material (a gas, a liquid, or a solid), collisions influence the (macroscopic) charge motion to a great extent. For example, we have seen that in solid and liquid conductors the motion of free charges is always along the lines of vector \( \mathbf{E} \). An equation like (17.2) is not valid for average (drift) velocity.

We now discuss a few examples of the motion of charged particles in an electric and a magnetic field.

**Example 17.1—Motion of a charged particle in a uniform electric field.** In Example 11.1 we analyzed the simplest case of motion of a charged particle in a uniform electric field. Let us now consider a more general case, when a charge \( Q \) \( (Q > 0) \) moves in a uniform field with arbitrary initial velocity \( \mathbf{v}_0 = v_{0x} + v_{0y} \), as in Fig. 17.2a.

The equation of motion (17.2) becomes \( m(d\mathbf{v}/dt) = q\mathbf{E} \). Integrating the scalar \( x \) and \( y \) components of this equation twice, with respect to the position of the charge as a function of time, we obtain

\[
x(t) = \frac{QE}{2m}t^2 + v_{0x}t + x_0 \quad \text{and} \quad y(t) = v_{0y}t + y_0,
\]

where \( x_0 \) and \( y_0 \) are the initial \( x \) and \( y \) coordinates of the charge. Consequently, the charge will move along a parabola. This is the same as when we throw a stone at an angle (other than 90 degrees) with respect to the earth’s surface.

**Example 17.2—Deflection of an electron stream by a charged capacitor.** Imagine that we shoot an electron between the plates of a charged parallel-plate capacitor, perpendicularly to the electric field. We now know that the electron trajectory will curve toward the positive electrode. So if we put a screen behind the capacitor, with no voltage on the electrodes the

![Diagram showing charged particle in uniform electric field and deflection by charged capacitor](image)

**Figure 17.2** (a) Charged particle in a uniform electric field, revisited; (b) deflection of a charged particle by a charged capacitor
electron hits point $A$ in Fig. 17.2b. When a voltage is applied, the electron is deflected and hits point $B$ on the screen in Fig. 17.2b. Some cathode-ray tubes use this principle for deflecting the electron beam, although the practical deflections are rather small.

**Example 17.3—Motion of a charged particle in a uniform magnetic field.** Consider a charged particle $Q$ ($Q > 0$) moving in a magnetic field of flux density $B$ with a velocity $v$ normal to the lines of vector $B$, as in Fig. 17.3.

Since the magnetic force on the charge is $F_m = Qv \times B$, it is always perpendicular to the direction of motion. This means that a magnetic field cannot change the magnitude of the velocity (i.e., it cannot speed up or slow down charged bodies); it can only change the direction of the charged particle motion. In other words, magnetic forces cannot change the kinetic energy of moving charges.

In the case considered in Fig. 17.3, there is a magnetic force on the charged particle directed as indicated, tending to curve the particle trajectory. Since $v$ is normal to $B$, the force magnitude is simply $QvB$. It is opposed by the centrifugal force, $mv^2/R$, where $R$ is the radius of curvature of the trajectory. Therefore, we have

$$QvB = \frac{mv^2}{R},$$

so that the radius of curvature is constant, $R = (mv)/(QB)$. Thus the particle moves in a circle. It makes a full circle in

$$t = T = \frac{2\pi R}{v} = \frac{2\pi m}{QB},$$

which means that the frequency of rotation of the particle is equal to $f = 1/T = (QB)/(2\pi m)$. Note that $f$ does not depend on $v$. Consequently, all particles that have the same charge and mass make the same number of revolutions per second. This frequency is called the cyclootron frequency.

**Example 17.4—The cyclotron.** The cyclotron is a device used for accelerating charged particles. It is sketched in Fig. 17.4. The main part of the cyclotron is a flat metal cylinder, cut along its middle. The two halves of the cylinder are connected to the terminals of an oscillator.
A charged particle from source $O$ finds itself in an electric field that exists between the halves of the cylinder, and it accelerates toward the other half of the cylinder. While outside of the space between the two cylinder halves, the charge finds itself only in a magnetic field, and it circles around with a radius of curvature found as in the preceding example. We saw that the time the charge takes to go around a semicircle does not depend on its velocity. That means that the charge will always take the same amount of time to again reach the gap between the two cylinders. If the electric field variation in this region is adjusted in such a way that the charge is always accelerated, the charge will circle around larger and larger circles, with increasingly larger velocity, until it finally shoots out of the cyclotron. The velocity of the charge when it gets out of the cyclotron is $v = (QBa)/m$. This equation is valid only for velocities not close to the speed of light. If this is not the case, the relativistic effects increase the mass, i.e., the mass is not constant.

As a numerical example, for $B = 1 \text{T}$, $Q = e$, $a = 0.5 \text{m}$, $m = 1.672 \cdot 10^{-27} \text{kg}$ (a proton), we get $v = 47.9 \cdot 10^6 \text{m/s}$.

**Example 17.5—Cathode-ray tube.** Cathode-ray tubes (CRTs), used in some TVs and computer monitors, have controlled electron beams that show traces on a screen. One system for deflecting electron streams in CRTs is sketched in Fig. 17.2b. Basically, there are two mutually orthogonal parallel-plate capacitors, which can deflect the stream in two orthogonal directions. In this way the electron stream can hit any point of the screen, and precisely where it hits is controlled by appropriate voltages between the electrodes of the two capacitors.

We have already mentioned that the electric field can deflect electron streams only by relatively small distances. When a large deflection is required, as in television receivers, a magnetic field is used, as sketched in Fig. 17.5. The design of the magnetic deflecting system (a coil of a specific geometry and with many turns of wire) is rather complicated and is usually done experimentally. Part of the experimental adjustment is due to the effect of the earth's magnetic field on charged particles at any point on the planet.

If we think of the earth as of an equivalent current loop, as described in section 17.2, the horizontal component of the magnetic flux density vector is oriented along the north-south direction, and the vertical component is oriented downward in the Northern Hemisphere and
upward in the Southern Hemisphere. Therefore, CRTs that use magnetic field deflection have to be tuned to take this external field into account. It is likely that your computer monitor (if a CRT) will not work exactly the same way if you turn it sideways (it might slightly change colors or shift the beam by a couple of millimeters), or if you use it in the other hemisphere of the globe.

Example 17.6—The Hall effect. In 1879, Edwin Hall thought of a clever way of determining the sign of free charges in conductors. A ribbon made of the conductor we are interested in has a width \( d \) and is in a uniform magnetic field of flux density \( \mathbf{B} \) perpendicular to the ribbon (Fig. 17.6). A current of density \( \mathbf{J} \) flows through the ribbon. The free charges can in principle be positive, as in Fig. 17.6a, or negative, as in Fig. 17.6b. The charges that form the current are moving in a magnetic field, and therefore a magnetic force \( \mathbf{F} = Q \mathbf{v} \times \mathbf{B} \) is acting on them. Due to this force, positive charges accumulate on one side of the ribbon, and negative ones on the other side. These accumulated charges produce an electric field \( E_H \). This electric field, in turn, acts on the free charges with a force that is in the opposite direction to the magnetic force. The charges will stop accumulating when the electric force is equal in magnitude
to the magnetic force acting on each of the charges. So in the steady state,

\[ QvB = QE_H, \quad \text{or} \quad E_H = vB. \]

Between the left and right edge of the ribbon, we can measure a voltage equal to

\[ |V_{12}| = E_H d = vBd. \]

In the case shown in Fig. 17.6a this voltage is negative, and in Fig. 17.6b it is positive. So the sign of the voltage tells us the sign of free charge carriers, and a voltmeter can be used to determine this sign.

Since \( J = NQv \), where \( N \) is the number of free charges per unit volume, we can write

\[ |V_{12}| = \frac{Jd}{NQ} B. \]

Thus if we determine the coefficient \( Jd/NQ \) for either ribbon sketched in Fig. 17.6 (which is usually done experimentally), by measuring \( V_{12} \) we can measure \( B \). This ribbon has four terminals—two for the connection to a source producing current in the ribbon, and two for the measurement of voltage across it. Such a ribbon is called a Hall element.

For single valence metals, e.g., copper, if we assume that there is one free electron per atom, the charge concentration is given by

\[ N = \frac{N_A \rho_m}{M}, \]

where \( N_A \) is Avogadro’s number \( (6.02 \cdot 10^{23} \text{ atoms/mole}) \), \( \rho_m \) is the mass density of the metal, and \( M \) is the atomic mass.

Questions and problems: Q17.4 to Q17.7, P17.1 to P17.5

17.4 Magnetic Storage

Magnetic materials have been used for storing data since the very first computers. The first computer memories consisted of small toroidal ferromagnetic cores arranged in two-dimensional arrays, in which digital information was stored in the form of magnetization. These memories are bulky and slow, as can be concluded from their description in Example 17.7. Today’s memories are essentially electrostatic: capacitances inside transistors are used for storing bits of information in the form of charges.

The hard disk in every computer is also a magnetic memory. We can write to the disk by magnetizing a small piece of the disk surface, and we can read from the disk by inducing a voltage in a small loop that is moving in close proximity to the magnetized disk surface element. As technology has improved, the amount of information that can be stored on a standard-size hard disk has grown rapidly. Between 1995 and the standard capacity of hard disks on new personal computers shot from a few hundred megabytes to more than 2 gigabytes. The development is in the
direction not only of increasing disk capacity but also of increasing speed (or reducing the access time). As we will see in Example 17.8, these two requirements compete with each other, and the engineering solution, as is usually the case, needs to be a compromise.

**Example 17.7—History: magnetic core memories.** Magnetic core memories were used in computers around 1970 but are now completely obsolete. The principle of their operation, however, is clever.

A magnetic core memory uses the hysteresis properties of ferromagnetics. One "bit" of the memory is a small ferromagnetic torus, shown in Fig. 17.7a. Two wires, in circuits 1 and 2, pass through the torus. Circuit 1 is used for writing and reading, and circuit 2 is used only for reading. To write, a positive ("1") or negative ("0") current pulse is passed through circuit 1 in the figure. When a positive current pulse is sent through the circuit, the core is magnetized to the point labeled \( B_r \) on the hysteresis curve in Fig. 17.7b. When a negative current pulse is passed through the circuit, the core is magnetized to the point labeled \(-B_r\). So the point \( B_r \) corresponds to a "1," and \(-B_r\) to a "0."

![Diagram of magnetic core memory](image)

**Figure 17.7** (a) One bit of a magnetic core memory. (b) Hysteresis loop of the core. (c) The induced emf pulses produced in circuit 2 while reading out the binary value written in the core.
How is the reading performed? A negative current pulse is passed through circuit 1. If the core is at a "1," the negative current pulse will bring the operating point to $-B_s$ (the negative tip of the loop), and after the pulse is over the point will move to $-B_r$ on the hysteresis loop. If the core is at a "0," the negative pulse will make the point go to the negative tip of the loop, and then return to $-B_r$.

While this is done, an emf is induced in circuit 2, resulting in one of the two possible readings shown in Fig. 17.7c. These two "pulses" correspond to a "1" and a "0." The speed at which this is done is about $\Delta t = 0.5 - 5 \mu s$. The dimensions of the torus are small: the outer diameter is 0.55 to 2 mm, the inner diameter is 0.3 to 1.3 mm, and the thickness is 0.12 to 0.56 mm.

Elements of an entire memory are arranged in matrices, as shown in Fig. 17.8. Two wires pass through each torus, as in Fig. 17.7a. The current passing through each row or column is only half the current needed to saturate the torus, so both the row and column of a specific core need to be addressed.

**Example 17.8—Computer hard disks.** The hard disk in every computer has information written to and read from it. We will describe how both processes work in modern hard disks and discuss some of the engineering parameters important for hard disk design. The hard disk itself is coated with a thin coating of ferromagnetic material such as Fe$_2$O$_3$. The disk is organized in sectors and tracks, as shown in Fig. 17.9.

The device that writes data to the disk and reads data from it is called a magnetic head. Magnetic heads are made in many different shapes, but all operate according to the same principle. We will describe the read-write process for a simplified head, shown in Fig. 17.10. It is a magnetic circuit with a gap. The gap is in close proximity to the tracks, so there is some leakage flux between the head and the ferromagnetic track.
In the "write" process, a current flows through the winding of the magnetic head, thus creating a fringing magnetic field in the gap. The gap is as small as 5 μm. As the head moves along the track (usually the track rotates), the fringing field magnetizes a small part of the track, creating a south and a north pole in the direction of rotation. These small magnets are about 5 μm long by 25 μm wide. A critical design parameter is the height of the head above the track: the head cannot hit the track and get smashed, but it also needs to be as close as possible to maximize the leakage flux that magnetizes the track. Typically, the surface of the track is flat.
to within several micrometers, and the head follows the surface profile at a distance of about 1 micrometer or less above it. This is possible because the head aerodynamically flies above the disk surface, as shown in Fig. 17.11. The current in the head windings should be strong enough to saturate the ferromagnetic track. If the track is saturated, the voltage signal during readout is maximized.

In the "read" process, there is no current in the windings of the magnetic head. The residual magnetization of the magnetic head should be as small as possible, so that the head is demagnetized when the current is turned off in readout. Now the flux from the magnetized track induces a voltage between the winding open ends while the head is moving with respect to the track (according to Faraday's law). Since the largest changes in \( B \) occur when the magnetic field changes direction, i.e., between two tiny magnets along the track, the output voltage has a waveform consisting of positive and negative pulses, as shown in Fig. 17.12b. The volt-

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**Figure 17.11** The magnetic head aerodynamically flies over the disk surface at a distance of only about 1 micrometer above it, following the surface profile.

**Figure 17.12** (a) Hysteresis curve of the track material. \( B_r \) is the remanent magnetic flux density and should be large for good readout. (b) A typical voltage signal read from the disk.
age is proportional to the remanent magnetic flux density, $B_r$, of the ferromagnetic hysteresis curve in Fig. 17.12a.

The capacity of data storage is given by the information density per unit area of storage surface. The storage density per unit surface area is the product of the storage density per unit track length, times the track density per unit distance normal to the direction of relative motion. An increase in track density reduces the sharpness in magnetic field discontinuity, thus reducing the readout voltage. Note that magnetic disks are inherently binary storage devices and that the frequency of the voltage pulses in readout is doubled compared to the number of actual segments along the track.

**Questions and problems:** Q17.8 and Q17.9, P17.6 and P17.7

### 17.5 Transformers

A transformer is a magnetic circuit with (usually) two windings, "primary" and "secondary," on a common ferromagnetic core (Fig. 17.13a). When an ac voltage is applied to the primary coil, the magnetic flux through the core is the same at the secondary and induces a voltage at the open ends of the secondary winding. Ampère's law for this circuit can be written as

$$N_1i_1 - N_2i_2 = Hl,$$

where $N_1$ and $N_2$ are the numbers of the primary and secondary turns, $i_1$ and $i_2$ are the currents in the primary and secondary coils when a generator is connected to the primary and a load to the secondary, $H$ is the magnetic field in the core, and $l$ is the effective length of the core. Since $H = B/\mu$ and, for an ideal core, $\mu \to \infty$, both $B$ and $H$ in the ideal core are zero (otherwise the magnetic energy in the core would be infinite). Therefore for an ideal transformer we have

$$\frac{i_1}{i_2} = \frac{N_2}{N_1}. \quad (17.3)$$

![Figure 17.13 (a) A transformer, and (b) the equivalent circuit of an ideal transformer](image-url)
This is the relationship between the primary and secondary currents in an ideal transformer. For good ferromagnetic cores, the permeability is high enough that this is a good approximation.

From the definition of magnetic flux, we know that the flux through the core is proportional to the number of turns in the primary. From Faraday’s law, we know that the induced emf in the secondary is proportional to the number of times the magnetic flux in the core passes through the surface of the secondary winding, that is, to $N_2$. Therefore, we can write the following for the voltages across the primary and secondary windings:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}.\hspace{1cm}(17.4)$$

From our discussion of mutual inductance in Chapter 15, we see that the equivalent circuit of an ideal transformer is just a mutual inductance, as shown in Fig. 17.13b.

Assume that the secondary winding of an ideal transformer is connected to a resistor of resistance $R_2$. What is the resistance seen from the primary terminals? From Eqs. (17.3) and (17.4) we obtain

$$R_1 = \frac{v_1}{i_1} = \frac{v_2N_1/N_2}{i_2N_2/N_1} = R_2 \left( \frac{N_1}{N_2} \right)^2.\hspace{1cm}(17.5)$$

From this discussion, we see that the transformer’s name is appropriate: it transforms the values of the voltage, current, and resistance between the primary and secondary windings. The transformation ratio is dictated by the ratio of the number of turns. In an ideal transformer there are no losses, so all of the power delivered to the primary can be delivered to a load connected to the secondary.

In a realistic transformer there are several loss mechanisms: resistance in the wire of the windings, and eddy current losses and hysteresis losses in the ferromagnetic core. To minimize resistive losses in sometimes very long wires used for a large number of turns, a good metal such as copper is chosen. Eddy current losses are minimized by laminating the core, as discussed in Example 14.6, and hysteresis losses were discussed in Example 16.3. These losses, as well as the inductance of the windings, result in a realistic equivalent circuit for a transformer shown in Fig. 17.14.

![Figure 17.14](image-url)
high-frequency transformers, or in cases where transients are important, the capacitance between the winding turns also needs to be taken into account.

Questions and problems: Q17.10 and Q17.11, P17.8

17.6 Synchronous and Asynchronous (Induction) Electric Motors

Electric motors serve to continuously transform electric energy into mechanical energy. There are several types of electric motors, and we will briefly describe two types that use the concept of rotating magnetic fields.

Imagine we have a U-shaped magnet that rotates with an angular velocity $\omega$, as in Fig. 17.15. The magnetic field will rotate with the magnet; thus it is known as the rotating magnetic field. (We will see that a rotating magnetic field can be obtained with appropriate sinusoidal currents in stationary coils.) Let a small magnet, e.g., a compass needle, be situated in this field, with the axis of rotation the same as that of the U-shaped magnet. A magnetic torque will act on the small magnet. If the small magnet is stationary, and $\omega$ is large, there will be a torque on the small magnet that tends to rotate it in one and then in the other direction, so that it will only oscillate. However, if the small magnet is brought to rotate with the angular velocity $\omega$, the rotating magnetic field will act on it by a continuous torque in one direction, and the small magnet will rotate in synchronism with the magnetic field, even if it has to overcome a small friction (or a load). If the rotating magnetic field is obtained with currents in stationary coils, the same will happen, and we will have a simple synchronous motor. The name comes from the fact that the motor can rotate only in synchronism with the rotation of the magnetic field.

If instead of the small magnet we have in the rotating magnetic field a short-circuited wire loop, as in Fig. 17.16, a current will be induced in the loop because the magnetic flux through the loop is varying in time. According to Lentz’s law, the actual direction of the induced current will be as indicated in the figure. It is seen that there will be a torque on the loop, tending to rotate it with the field. If the rotating field is produced by currents in stationary coils, we obtain a simple induction motor.

![Figure 17.15 A small magnet in the rotating magnetic field of a rotating magnet](image-url)
Because there will be a time-average torque on the loop for any angular velocity of the loop rotation, whether or not it rotates in synchronism with the field, it is also known as the asynchronous (i.e., not synchronous) motor. Its rotating part, or rotor, is usually made in the form of a number of short-circuited loops at an angle, similar to a cage (the squirrel-cage rotor). The short-circuited loops are fixed in grooves in a ferromagnetic rotor core.

For large amounts of power, the rotating magnetic field is obtained directly from a three-phase current system. Let us examine a simpler way of obtaining a rotating magnetic field using two currents of equal amplitude that are 90 degrees out of phase (Fig. 17.17), a method used for low-power synchronous and asynchronous motors.

If the currents in the two coils in Fig. 17.17 are of equal magnitude and shifted in phase by 90 degrees, so are the magnetic flux densities they produce. Therefore (see Fig. 17.17),

\[ B_x(t) = B_m \cos \omega t, \quad \text{and} \quad B_y(t) = B_m \sin \omega t. \]

The total magnetic flux density has a magnitude of

\[ B_{\text{total}}(t) = \sqrt{B_x^2(t) + B_y^2(t)} = B_m, \]

which means that it has a constant magnitude. The vector \( \mathbf{B} \) is rotating, however, since

\[ \tan \alpha(t) = \frac{B_y(t)}{B_x(t)} = \tan \omega t, \]

so that

\[ \alpha(t) = \omega t, \]
which means that, indeed, the vector $\mathbf{B}$ rotates with a constant angular velocity $\omega$. We have thus obtained a rotating magnetic field with sinusoidal currents in two stationary coils.

Three-phase motors and generators were invented by Nikola Tesla, a Serbian immigrant who came to America with 4 cents in his pocket. In 1891, he filed about 50 patents related to different kinds of ac generators and motors, but he had to fight Thomas Edison’s promotion of dc power. Eventually George Westinghouse, who supported Tesla’s inventions, won the battle and the first ac power plant was built on Niagara Falls. In 1891, the same year Tesla filed the patents that provoked strong reactions and resistance in the scientific community, mines in Telluride, Colorado, had already installed polyphase motors and generators based on his patents.

**Questions and problems:** Q17.12 and Q17.13, P17.9

### 17.7 Rough Calculation of the Effect of Power Lines on the Human Body

We often hear that the electric and magnetic fields “radiated” from power lines may be harming our bodies. We are now ready to do some rough calculations of the voltages induced in the body. We will do the calculations on the example of a human head (which is the most important and most sensitive part of our body). We will assume that our head is a sphere with a radius of 10 cm, and made mostly of salty water. In this example, let the power lines be as close as 20 m to a human, and let them carry an
unbalanced current of 100 A, as in Fig. 17.18. (The total current in a balanced power line is zero.)

We need to consider two effects: the induced voltage from the magnetic field, since the current in the line is sinusoidally varying in time at 60 Hz, and the voltage due to the electric field of the wire.

First, the magnetic flux density 20 m from a wire carrying 100 A is equal to $B = \mu_0 I / (2\pi r) = 1 \mu T$. For comparison, the earth's average dc magnetic field is 50 $\mu T$ on the earth's surface. (This dc field induces currents in our bodies only if we move, and we are probably adapted to this small effect.) The induced electric field around our head (which is a conductor) can be calculated from Faraday's law:

$$\oint_{\text{head perimeter}} \mathbf{E}_{\text{ind}} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_{\text{head cross section}} \mathbf{B} \cdot d\mathbf{S}.$$  

The left-hand side of the equation is approximately $2\pi a E_{\text{ind}}$, and the right-hand side equals $-a^2 \pi \partial B / \partial t$. In complex notation we thus have

$$2\pi a E_{\text{ind}} = -j\omega B \pi a^2,$$

hence

$$|E|_{\text{ind}} = \omega B a / 2. \quad (17.6)$$

The rms value of the voltage due to this induced electric field across a single cell in our head (which is about 10 $\mu m$ wide) is $V_{\text{cell}} \approx 33$ pV at 60 Hz. This is very small; for comparison, the normal neural impulses that pass through the cells are spikes on the order of 100 mV in amplitude, and they last about a millisecond, with a frequency between 1 and 100 Hz.

Let us now consider the electric field effect. The approximate value for the electric field around power lines depends on the power line voltage rating and the distance of the point from the line, but a resonable value would be $E_0=1$ kV/m. Our cells are made of essentially salty water, which has a resistivity of about 1 $\Omega \cdot m$. To find the voltage across an individual cell we reason as follows.
The sphere approximating the head is conducting. It is situated in an approximately uniform electric field. Therefore, surface charges are induced on its surface as determined in Example 11.3, Eq. (11.9):

$$\sigma(\theta) = 3\varepsilon_0 E_0 \cos \theta,$$

where $\theta$ is the angle between the radius to the cell considered (a point on the sphere surface) and the direction of vector $E_0$. However, this charge is not time-constant, as in Example 11.3, but rather time-varying, since $E_0$ is time-varying. Consequently, there is a time-varying current inside the sphere, which can be determined approximately in the following manner.

The total charge on one hemisphere is given by

$$Q = \int_0^{\pi/2} \sigma(\theta) 2\pi a \sin \theta \, d\theta = \int_0^{\pi/2} 3\varepsilon_0 E \cos \theta \, 2\pi a \sin \theta \, d\theta = 3\pi \varepsilon_0 a^2 E_0 a.$$ (17.7)

For $E_0 = 1 \text{kV/m}$, this amounts to $Q = 835 \text{pC}$.

If the charge is time-varying, there is a time-varying current in the sphere obtained as $i(t) = dQ(t)/dt$. For a sinusoidal field of frequency $f = 60 \text{Hz}$, the rms value of the current is

$$I = \omega Q = 2\pi f Q = 0.315 \mu A.$$ (17.14)

The current density inside the sphere, in the equatorial plane of the sphere and normal to $E_0$, is hence

$$J = \frac{I}{a^2 \pi} \approx 10 \mu A/\text{m}^2,$$

so that the electric field inside the sphere is not zero, but has an rms value $E = \rho J = 10 \mu V/\text{m}$. So the voltage across a cell equals about $10 \mu V/\text{m} \times 10 \mu m = 100 \text{pV}$. This is somewhat larger than the voltage due to the time-varying magnetic field, but it is probably still negligible with respect to 100-mV voltage spikes due to normal neural impulses.

In conclusion, voltages induced in our body when we are close to power lines are much smaller than the normal electric impulses flowing through our nerve cells. Nevertheless, it is hard to say with absolute certainty that these orders-of-magnitude lower voltages do not have any effect on us, because biological systems are often at a very unstable equilibrium.

Questions and problems: Q17.14

**Questions**

**Q17.1.** Where is the earth’s south magnetic pole?

**Q17.2.** What is the order of magnitude of the earth’s magnetic flux density?

**Q17.3.** Approximately how fast would you need to spin around your axis in the magnetic field of the earth to induce 1 mV around the contour of your body?
Q17.4. Turn your computer monitor sideways or upside down while it is on (preferably with some brightly colored pattern on it). Do you notice changes in the screen? If yes, what and why?

Q17.5. What do you expect to happen if a magnet is placed close to a monitor? If you have a small magnet, perform the experiment (note that the effect might remain after you remove the magnet, but it is not permanent). Explain.

Q17.6. Explain how the Hall effect can be used to measure the magnetic flux density.

Q17.7. Explain how the Hall effect can be used to determine whether a semiconductor is p- or n-doped.

Q17.8. What magnetic material properties are chosen for the tracks and heads in a hard disk?

Q17.9. Sketch and explain the time-domain waveform of the induced emf (or current) in the magnetic head coil in “read” mode as it passes over a piece of information recorded on a computer disk as “110.” (Assume that a “1” is a small magnet along the track with a N-S orientation from left to right, and a “0” is in the opposite direction.)

Q17.10. Write Ampère’s law for an ideal transformer, and derive the voltage, current, and impedance (resistance) transformation ratio. The number of turns in the primary and secondary are $N_1$ and $N_2$.

Q17.11. What are the loss mechanisms in a real transformer, and how does each of the contributors to loss depend on frequency?

Q17.12. Explain how a synchronous motor works.

Q17.13. How is an asynchronous motor different from the synchronous type?

Q17.14. Describe the two mechanisms by which ac currents can affect our body. Use formulas in your description.

**PROBLEMS**

P17.1. What is the minimum magnitude of a magnetic flux density vector that will produce the same magnetic force on an electron moving at 100 m/s that a 10-kV/cm electric field produces?

P17.2. Calculate the velocity of an electron in a 10-kV CRT. The electric field is used to accelerate the electrons, and the magnetic field to deflect them.

P17.3. How large is the magnetic flux density vector needed for a 20-cm deflection in the CRT in problem P17.2, if the length of the tube is 25 cm?

P17.4. A thin conductive ribbon is placed perpendicularly to the field lines of a uniform B field. When the current is flowing in the direction shown in Fig. 17.6a, there is a measured negative voltage $V_{12}$ between the two edges of the ribbon. Are the free charges in the conductive ribbon positive or negative?

P17.5. What is the voltage $V_{12}$ equal to in problem P17.4 if $B = 0.8$ T, the ribbon thickness $t = 0.5$ mm, $I = 0.8$ A, and the concentration of free carriers in the ribbon is $N = 8 \cdot 10^{28}$ m$^{-3}$?

P17.6. The magnetic head in Figure 17.10 is in write mode. Calculate the magnitude of the current $i$ in the winding that would be needed to produce a $B_0 = 1$ μT field in the gap. There are $N = 5$ turns on the core, the core can approximately be considered as linear, of relative permeability of $\mu_r = 1000$, the gap is $L_0 = 20$ μm wide, the cross-sectional
area of the core is $S = 10^{-9} m^2$, and the mean radius of the core is $r = 0.1 \text{ mm}$. Assume that the fringing field in the gap makes the gap cross-sectional area effectively 10% larger than that of the core.

**P17.7.** The head and the tracks in magnetic hard disks are made of different magnetic materials because they perform different functions. Sketch and explain the preferred hysteresis curves for the two materials, indicating the differences. Which has higher loss in the ac regime?

**P17.8.** A CRT needs 10 kV to produce an electric field for electron acceleration. Design a wall-plug transformer to convert from 110 V in the U.S. and Canada or 220 V in Europe and Asia. Assume you have a core made of a magnetic material that has a very high permeability.

**P17.9.** Assume that in Fig. 17.17 you have three instead of two coils. The axes of the coils are now at 60 degrees, not 90 degrees, with respect to each other. What is the relative phasing of three sinusoidal currents in the coils that will give a rotating magnetic field, as described in section 17.6 for the case of two currents? Plot the current waveforms as a function of time.