4

The Electric Scalar Potential

4.1 Introduction

You may recall from your physics courses that the gravitational field at any point can be described in two ways. One is by the force acting on a small mass located at that point. This is analogous to the electric force. The other is by specifying how large the energy at the point of that small mass is, per unit mass. This is known as the gravitational potential. The electric potential is analogous to the gravitational potential. It tells us how large the energy of a small charge at a point of the electric field is, per unit charge. Note that force is a vector, whereas energy is a scalar. Therefore the description of the field in terms of the potential is mathematically simpler than in terms of the field vector.

4.2 Definition of the Electric Scalar Potential

To understand the concept of the electric scalar potential, consider a test charge, $\Delta Q$, at a point $A$ in an electrostatic field. The electric force acting on $\Delta Q$ will tend to move the test charge. Let the force move the charge along a line $a$ to a point $B$, as sketched in Fig. 4.1. How much work was done by the electric force in this case?
We know from physics that if a force $\mathbf{F}$ moves a body along a small vector distance $dl$, the work done by the force is

$$dA = F \, dl \cos(\text{angle between vectors } \mathbf{F} \text{ and } dl) \quad \text{joules (J)}, \quad (4.1)$$

where $F$ and $dl$ are the magnitudes of the two vectors.

This type of product of two vectors occurs frequently in physics and engineering. (If necessary, before proceeding further please read Section A1.2 of Appendix 1.) It is known as the scalar product, or dot product. For any two vectors $\mathbf{X}$ and $\mathbf{Y}$, the dot product is defined as

$$\mathbf{X} \cdot \mathbf{Y} = XY \cos(\text{angle between vectors } \mathbf{X} \text{ and } \mathbf{Y}). \quad (4.2)$$

Hence, instead of Eq. (4.1) we can use the shorthand

$$dA = \mathbf{F} \cdot dl. \quad (4.3)$$

Work is a scalar quantity. Therefore, to obtain the work done by the electric force in moving the test charge from point $A$ to point $B$, we simply add all elemental works of the form as in Eq. (4.3), from $A$ to $B$:

$$A_{\text{from } A \text{ to } B} = \int_{A}^{B} \mathbf{F} \cdot dl \quad \text{(J).} \quad (4.4)$$

This type of integral (which is nothing but a sum of many very small terms) is known as a line integral.

The electric force on $\Delta Q$ is $\Delta Q \mathbf{E}$, and $\Delta Q$ is a constant that can be taken out of the integral sign. With this in mind, if we divide Eq. (4.4) by $\Delta Q$, we get

$$\frac{A_{\text{from } A \text{ to } B}}{\Delta Q} = \int_{A}^{B} \mathbf{E} \cdot dl \quad \text{(J/C = V).} \quad (4.5)$$

Note that the right-hand side of this equation does not depend on $\Delta Q$. It represents the work that would be done by the electric field in moving the test charge from point $A$ to point $B$, per unit charge.

Imagine now that the field moves the test charge from $A$ to $B$ but along a different path, for example path $b$ in Fig. 4.1. How much work is done by the electric
forces in that case? It is easy to understand that the answer must be the same as for path $a$. Assume for a moment that the work that the electric field does when moving the charge along path $b$ is larger than the work done along path $a$. We could then let the field move the test charge along $b$ first. At point $B$, it would have a certain velocity, which means a certain kinetic energy. This energy would be greater than the work that needs to be done to return the test charge to point $A$ along path $a$. So we would come back to $A$ with extra energy, in spite of the system being again the same as in the beginning. Evidently, this is contrary to the law of conservation of energy. Therefore the work done by the field or against the field in moving the test charge from one point of the field to another does not depend on the particular path between the two points.

Since this is so, we can adopt the point $B$ to be a fixed point and call it the reference point, $R$. We can next describe the field at all other points by specifying how large the expression in Eq. (4.5) is at these points. With the adoption of the fixed reference point $R = B$, we in fact have a scalar function of coordinates describing the field. It is known as the electric scalar potential, $V_A$, at a point $A$ of the electric field:

$$V_A = \int_A^R \mathbf{E} \cdot d\mathbf{l} \quad \text{volts (V).} \quad (4.6)$$

(Definition of the electric scalar potential)

The unit of potential is the volt (abbreviated V), hence the unit V/m for $E$.

We have convinced ourselves that the line integral of $E$ in an electrostatic field between two points does not depend on the path of integration. An important conclusion follows from this property: the line integral of the electric field strength along any closed contour $C$ is zero:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0. \quad (4.7)$$

(Law of conservation of energy in the electrostatic field)

Note that the contour $C$ can be completely, but also only partly, in the field. The integral on the left side of this equation is known as a contour integral. Note also that Eq. (4.7) represents the mathematical expression of the law of conservation of energy for the electrostatic field. It is, therefore, a fundamental property of the electrostatic field.

Questions and problems: Q4.1 to Q4.8

4.3 Electric Scalar Potential of a Given Charge Distribution

Let us determine the potential at a point $A$, which is a distance $r$ away from a positive point charge $Q$. Assume that the reference point is a distance $r_R$ away from the
Figure 4.2 A field point, $A$, and the reference point, $R$, in the field of a point charge

charge, Fig. 4.2. We know that we can go from $A$ to $R$ along any path, so we adopt the simplest route: we first go from point $A$ along a radius to the point $B$ where it intersects with the circle of radius $r_R$. Along this path segment, vectors $\mathbf{E}$ and $d\mathbf{l}$ are parallel. Therefore the product $\mathbf{E} \cdot d\mathbf{l}$ is simply $E \, dl$ (cosine of zero is unity). We then continue to the point $R$ along the arc of the circle, where the product $\mathbf{E} \cdot d\mathbf{l}$ is zero (cosine of $\pi/2$ is zero). We thus have

$$V_A = \int_A^B \mathbf{E} \cdot d\mathbf{l} + \int_B^R \mathbf{E} \cdot d\mathbf{l} = \frac{Q}{4\pi \varepsilon_0} \int_r^{r_R} \frac{dr}{r^2}.$$  \hspace{1cm} (4.8)

The integral is a standard one, and the result is

$$V_A = \frac{Q}{4\pi \varepsilon_0} \left( \frac{1}{r} - \frac{1}{r_R} \right) \quad (V).$$  \hspace{1cm} (4.9)

This is the formula for the potential of a point charge at a distance $r$ from the charge, and with respect to the reference point a distance $r_R$ from it.

So far, we have not discussed where the reference point should be. This can be 
any point. It is convenient to adopt it so that the expression for the potential is the
simplest. In the case of Eq. (4.9), this is obtained if we assume that $r_R$ is very large,
theoretically infinite, i.e., if the reference point is at infinity. In that case the potential
at point $A$ of the field of a point charge $Q$ becomes

$$V_A = \frac{Q}{4\pi \varepsilon_0 r} \quad \text{(reference point at infinity)} \quad (V).$$  \hspace{1cm} (4.10)

(Potential at a distance $r$ from a point charge)

The reference point at infinity is the most convenient and is used most often. We shall see, however, that this point cannot be used if there are charges at infinity (e.g., for an infinitely long line charge).
How does the choice of the reference point influence the scalar potential function? For example, what happens if instead of \( R \) we adopt \( R_1 \) to be the new reference point? It is left to the reader to prove that in that case the potential at all points will be increased by the same amount:

\[
\Delta V = \int_{R}^{R_1} \mathbf{E} \cdot d\mathbf{l}.
\]  

(4.11)

Once we know the potential of a point charge, it is quite simple to determine the potential of a given distribution of volume, surface, or line charges. Referring to Fig. 4.3, the potential of a volume charge distribution is given by

\[
V_p = \frac{1}{4\pi \varepsilon_0} \int_V \frac{\rho \, dv}{r} \quad \text{(reference point at infinity)}
\]  

(4.12a)

(Potential of volume distribution of charges)

that of a surface charge distribution is obtained as

\[ dQ = \sigma \, dS \]

\[ dQ = Q' \, dl \]

Figure 4.3 (a) A charged cloud, (b) a charged surface, and (c) a charged line, with a point \( P \) at which the electric scalar potential is calculated
\[ V_p = \frac{1}{4\pi \varepsilon_0} \int_S \frac{\sigma \, dS}{r} \quad \text{(reference point at infinity)} \quad \text{(V)}, \quad (4.12b) \]

(Potential of surface distribution of charges)

and the potential of a line charge distribution is, by analogy,

\[ V_p = \frac{1}{4\pi \varepsilon_0} \int_L \frac{Q' \, dl}{r} \quad \text{(reference point at infinity)} \quad \text{(V)}. \quad (4.12c) \]

(Potential of line distribution of charges)

**Example 4.1—Potential on the axis of a charged ring.** Let us find the potential on the axis of a thin ring of radius \( R \), uniformly charged along its length with a line charge density \( Q' \), Fig. 4.4. The element \( dl \) of the ring has a charge \( dQ = (Q/(2\pi R)) \, dl \). The potential due to this charge is the same as that of a point charge, except that \( Q \) needs to be replaced by \( dQ \). The potential at a point \( P \) on the ring axis (Fig. 4.4) is therefore obtained as

\[ V_p = \frac{1}{4\pi \varepsilon_0} \int_C \frac{Q}{2\pi R} \frac{dl}{r} = \frac{Q}{8\pi^2 \varepsilon_0 R} \int_{\text{ring}} dl. \]

Since the integral of \( dl \) around the ring equals its circumference, \( 2\pi R \), we finally obtain

\[ V_p = \frac{Q}{4\pi \varepsilon r} = \frac{Q}{4\pi \varepsilon_0 \sqrt{R^2 + x^2}}. \]

As already mentioned, we rarely know what the distribution of charges is. Therefore these formulas, similarly to those for the electric field strength of a given distribution of charges, do not have wide practical applicability. However, as in the case of the electric field strength, Eq. (4.12b), for example, can be used to calculate the charge distribution over a conducting body numerically.

**Questions and problems:** Q4.9 to Q4.13, P4.1 to P4.9

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**Figure 4.4** A thin ring uniformly charged along its length with a line charge density \( Q' \)
4.4 Potential Difference and Voltage

An important concept in circuit theory is the potential difference or voltage between two points in an electrostatic field. We shall see that voltage is a wider concept than just potential difference. *Only in the electrostatic field are the two concepts equivalent.*

We denote the voltage with the same letter $V$ as the potential, either with two subscripts or with no subscripts at all (such as in the case of the potential difference between two terminals of a voltage source). The two subscripts tell us between which two points the potential difference is considered—for example, $V_{12}$ is the voltage between points 1 and 2. In the case of the potential at a point, of course, there is only one subscript, for example $V_1$, although in electrostatics we could also write it as $V_{1R}$, where $R$ denotes the reference point.

According to the definition of the potential in Eq. (4.6), the potential difference between points $A$ and $B$ is given by

$$V_{AB} = V_A - V_B = \int_A^R E \cdot dl - \int_B^R E \cdot dl \quad (V). \quad (4.13)$$

If in the second integral the upper and lower limits of integration are interchanged, the line element, $dl$, changes sign. Hence we can rewrite Eq. (4.13) as

$$V_{AB} = \int_A^R E \cdot dl + \int_R^B E \cdot dl. \quad (4.14)$$

So we have to integrate the dot product $E \cdot dl$ from $A$ to $R$, and then from $R$ to $B$, i.e., from $A$ to $B$ over $R$. We know, however, that the path between $A$ and $B$ does not affect the result. Therefore we can calculate this integral along any path, not necessarily traversing point $R$. Thus we finally have

$$V_{AB} = \int_A^B E \cdot dl \quad (V). \quad (4.15)$$

*(Potential difference between points $A$ and $B)*

Consequently, the position of the reference point does not influence the voltage between two points in an electrostatic field. This, of course, was to be expected—we know that a change in the position of the reference point changes the potential at all points by the same amount, $\Delta V$ in Eq. (4.11).

If we compare Eqs. (4.15) and (4.5), we see that the potential difference between two points can be given the following physical interpretation: it equals the work that would be done by the electric forces in moving a test charge from the first to the second point, per unit test charge.

What is the range of voltages encountered in practice? The smallest (time-varying) voltage we can measure is on the order of $1 \text{ pV} = 10^{-12} \text{ V}$. The voltage of batteries for watches and calculators is about $1.5 \text{ V}$. The voltage in the plugs in our
homes is, for example, 110 V in the United States and Canada, and 220 V in Europe. The largest voltage used in power transmission by high-voltage transmission lines is on the order of 1 MV = 10^6 V.

Questions and problems:  Q4.14 to Q4.17, P4.10 to P4.14

4.5 Evaluation of Electric Field Strength from Potential

Here is the final basic question we may ask about the electric scalar potential \( V \): we know how to determine \( V \) if we know \( \mathbf{E} \) along any path from \( A \) to \( R \), but can we determine \( \mathbf{E} \) if we know \( V \)? This is quite simple to do.

Consider two close points, \( A \) and \( B \), in an electrostatic field (Fig. 4.5). Let the potential at \( A \) be \( V_A \), and at \( B \) be \( V_B = V_A + dV \). Assume that the vector line element from \( A \) to \( B \) is \( dl \), and let it be along the \( x \) coordinate axis so that \( dl = dx \). The potential difference between \( A \) and \( B \) is then simply \( \mathbf{E} \cdot dl = E dx \cos \alpha = E_x dx \) [the integral in Eq. (4.15) consists of a single small term]. So we have

\[
V_A - V_B = V_A - (V_A + dV) = -dV = E_x dx. \tag{4.16}
\]

In other words, the component \( E_x \) of vector \( \mathbf{E} \) in the \( x \) direction is obtained by

\[
E_x = -\frac{dV}{dx} \quad \text{(V/m)}. \tag{4.17}
\]

This is a very simple result. Assume that at a point in the electrostatic field we know \( V \) as a function of coordinate \( x \) along an \( x \) axis in any direction at that point. We can then determine the projection \( E_x \) of the vector \( \mathbf{E} \) on the \( x \) axis at that point simply as the negative derivative of \( V(x) \). The reference direction for the projection is the \( x \) axis.

Example 4.2—Electric field of a point charge found from the potential. Consider a point charge \( Q \). Let the \( x \) axis be any radial line beginning at the charge. The potential is then

![Figure 4.5 Determination of vector \( \mathbf{E} \) from known \( V \) at two close points](image)
given by Eq. (4.10), except that we have to replace \( r \) by \( x \). According to Eq. (4.17), we have

\[
E_x = -\frac{d}{dx} \left( \frac{Q}{4\pi \varepsilon_0 x} \right) = \frac{Q}{4\pi \varepsilon_0 x^2},
\]

as we know it should be from Coulomb’s law.

Since we now know how to determine the projection of the vector \( E \) in any direction at a point, we can easily determine the complete vector \( E \) at that point. We simply define three coordinate axes at the point, calculate the projections of the vector \( E \) on all the three axes, and sum the three components as vectors. For example, let the three axes be the \( x \), \( y \), and \( z \) axes of a rectangular coordinate system. Then the vector \( E \) at any point is given by

\[
E = -\left( \frac{\partial V}{\partial x} u_x + \frac{\partial V}{\partial y} u_y + \frac{\partial V}{\partial z} u_z \right) \quad \text{(V/m)},
\]

where \( u_x, u_y \) and \( u_z \) are unit vectors of the three coordinate axes. Partial derivatives must be used instead of ordinary derivatives because the potential \( V = V(x, y, z) \) is a function of all three coordinates. To determine a projection of \( E \) on any one of the three coordinate axes, we have to differentiate \( V(x, y, z) \) with respect to that coordinate only, considering the other two as constants. This is exactly the definition of the partial derivative of a function of several variables.

We know from mathematics that the expression in the parentheses on the right-hand side of Eq. (4.19) is called the gradient of the scalar function \( V \). (If necessary, please read Section A1.4.1 of Appendix 1 before proceeding further.) It is sometimes written as \( \nabla V \), but much more frequently we use the so-called nabla operator or del operator. The del operator in the rectangular coordinate system is defined as

\[
\nabla = \left( \frac{\partial}{\partial x} u_x + \frac{\partial}{\partial y} u_y + \frac{\partial}{\partial z} u_z \right) \quad \text{(1/m),}
\]

(Definition of nabla or del operator)

with the assumption that the expression \( \nabla V \) is a shorthand for the expression in parentheses on the right-hand side in Eq. (4.19). So we can write

\[
E = -\text{grad}V = -\nabla V \quad \text{(V/m)},
\]

(Evaluation of the electric field strength from potential)

where, in the rectangular coordinate system,
\[ \nabla V = \frac{\partial V}{\partial x} \mathbf{u}_x + \frac{\partial V}{\partial y} \mathbf{u}_y + \frac{\partial V}{\partial z} \mathbf{u}_z \quad (V), \quad \frac{\mathbf{V}}{m} \]  

(Gradient of a scalar function \( V \) in rectangular coordinates)

**Example 4.3—Vector \( \mathbf{E} \) on the axis of a charged ring.** As an example of the determination of \( \mathbf{E} \) from the scalar potential, consider again the ring in Fig. 4.4. \( \mathbf{E} \) is obtained as \(-\nabla V\). The scalar potential along the ring axis is given at the end of Example 4.1. Note that it is a function of the coordinate \( x \) only. Therefore at a point \( x \) on the ring axis

\[
\mathbf{E} = -\nabla \left( \frac{Q}{4\pi \varepsilon_0 \sqrt{R^2 + x^2}} \right) \mathbf{u}_x = \frac{Qx}{4\pi \varepsilon_0 (R^2 + x^2)^{3/2}} \mathbf{u}_x.
\]

**Questions and problems:** Q4.18 to Q4.23, P4.15 to P4.18

### 4.6 Equipotential Surfaces

A surface in an electrostatic field having the same potential at all points is called an **equipotential surface.** This is an important concept. For example, we will see that in electrostatics, the surface of any conductor is always equipotential. It can also aid in visualizing the electric field, usually in combination with electric field lines.

Since all points of an equipotential surface are at the same potential, the potential difference between two close points \( A \) and \( B \) on the surface is zero. Let \( d\mathbf{l} \) be the position vector of point \( B \) with respect to point \( A \). Because \( d\mathbf{l} \) is very small, the potential difference in Eq. (4.15) becomes simply \( dV = \mathbf{E} \cdot d\mathbf{l} \). Since this potential difference \( dV \) is zero (we assumed \( A \) and \( B \) to be on the same equipotential surface), the electric field strength vector at any equipotential surface is normal to that surface.

**Example 4.4—Equipotential surfaces in the field of a point charge.** As an example, we know that the expression for the potential of a point charge is \( V(r) = Q/(4\pi \varepsilon_0 r) \). Therefore the equation of the equipotential surface at a potential \( V_0 \) is obtained from

\[
V(r) = \frac{Q}{4\pi \varepsilon_0 r} = V_0,
\]

from which we obtain

\[
r = \frac{Q}{4\pi \varepsilon_0 V_0}.
\]

For different \( V_0 \), equipotential surfaces are spheres centered at the charge, and vector \( \mathbf{E} \) is normal to these spheres.

If plotted, equipotential surfaces usually have the same potential difference from one surface to the next. Let this potential difference be \( \Delta V \). For \( V_0 = 0 \) in the
preceding equation we would then have $r_0 = \infty$, for $V_0 = 1 \times \Delta V$ the radius of the equipotential surface is $r_1 = Q/(4\pi \varepsilon_0 V_0)$, and so on. With this convention, therefore, equipotential surfaces for a point charge are as in Fig. 4.6.

Questions and problems:  Q4.24

4.7 Chapter Summary

1. The electric scalar potential is a scalar quantity that can be used instead of vector $E$ for the description of the electrostatic field. It is defined as the line integral of $E$ from any point of the field to an arbitrary reference point.

2. The electric scalar potential is not unique (it depends on the choice of the reference point), but for two reference points the potential at all points differs only by a constant. If there are no charges at infinity, the reference point is always adopted at infinity, but if the distribution of charges extends (theoretically) to infinity, this is not possible.

3. If we know the electric scalar potential as a function of coordinates, it is easy to obtain the component of $E$ in any direction, and hence to obtain the complete vector $E$. For this, we need the mathematical concept of the gradient of a scalar function. In the rectangular coordinate system, the gradient of $V$ is obtained by the $\nabla$ operator acting on $V$, and $E = -\nabla V$.

4. Being a scalar quantity, the electric scalar potential is more convenient than the vector $E$ for the analysis of electrostatic fields.

5. An equipotential surface is defined as a geometrical surface with all points at the same potential. Lines of the electric field strength vector are normal to equipotential surfaces.

Questions

Q4.1. Consider a uniform electric field of electric field strength $E$, and two planes normal to vector $E$, that are a distance $d$ apart. What is the work done by the field in moving a
Q4.2. Is it possible to have an electrostatic field with circular closed field lines, with the vector E in the same direction along the entire lines? Explain.

Q4.3. Is it possible to have an electrostatic field with parallel lines, but of different magnitude of vector E in the direction normal to the lines? Explain.

Q4.4. If the potential of the earth were taken to be 100,000 V (instead of the usual 0 V), would it be dangerous to walk around? What influence would this have on the potential at various points, and on the difference of the potential at two points?

Q4.5. If we know \( E(x, y, z) \), is the electric scalar potential \( V(x, y, z) \) determined uniquely? Explain.

Q4.6. Equation (4.7) is satisfied by the electric field of a point charge. Does the expression for the electric field of a point charge follow from Eq. (4.7)?

Q4.7. Why does Eq. (4.7) represent the law of conservation of energy in the electrostatic field?

Q4.8. What is the potential of the reference point?

Q4.9. As we approach a point charge \( Q \) (\( Q > 0 \)), the potential tends to infinity. Explain.

Q4.10. How much energy do you transfer to the electric field of a point charge when you move the reference point from a point at a distance \( r_R \) from the charge to a point at infinity?

Q4.11. Why do we usually adopt the reference point at infinity?

Q4.12. Is the potential of a positively charged body always positive, and that of a negatively charged body always negative? Give examples that illustrate your conclusions.

Q4.13. Why are the expressions for the potential in Eqs. (4.12a-c) valid for a reference point at infinity?

Q4.14. Does it make sense to speak about voltage between a point in the field and the reference point? If it does, what is this voltage?

Q4.15. A charge \( \Delta Q \) is moved from a point where the potential is \( V_1 \) to a point where the potential is \( V_2 \). What is the work done by the electric forces? What is the work done by the forces acting against the electric forces?

Q4.16. A charge \( \Delta Q \) (\( \Delta Q < 0 \)) is moved from a point at potential \( V_1 \) to a point at potential \( V_2 \). What is the work done by the electric forces?

Q4.17. Is \( V_{AB} = -V_{BA} \)? Explain.

Q4.18. Why is the vector \( E \) at a point directed toward the adjacent equipotential surface of lower potential?

Q4.19. Why do we have \( E = -\text{grad} \ V \), and not \( E = +\text{grad} \ V \)?

Q4.20. A cloud of positive and negative ions is situated in an electrostatic field. Which ions will tend to move toward the points of higher potential, and which toward the points of lower potential?

Q4.21. Suppose that \( V = 0 \) at a point. Does it mean that \( E = 0 \) at that point? Explain.

Q4.22. Assume we know \( E \) at a point. Is this sufficient to determine the potential \( V \) at that point? Conversely, if we know \( V \) at that point, can we determine \( E \)?

Q4.23. The potential in a region of space is constant. What is the magnitude and direction of the electric field strength vector in the region?

Q4.24. Prove that \( E \) is normal to equipotential surfaces.
PROBLEMS

P4.1. Two point charges, \( Q_1 = -3 \cdot 10^{-9} \text{C} \) and \( Q_2 = 1.5 \cdot 10^{-9} \text{C} \), are \( r = 5 \text{cm} \) apart. Find the potential at the point that lies on the line joining the two charges and halfway between them. Find the zero-potential point(s) lying on the straight line that joins the two charges.

P4.2. Two small bodies, with charges \( Q \) (\( Q > 0 \)) and \( -Q \), are a distance \( d \) apart. Determine the potential at all points with respect to the reference point at infinity. Is there a zero-potential equipotential surface? How much work do the electric forces do if the distance is increased to \( 2d \)?

P4.3. A ring of radius \( a \) is charged with a total charge \( Q \). Determine the potential along its axis normal to the ring plane with reference to the ring center.

P4.4. A soap bubble of radius \( R \) and very small wall thickness \( a \) is at a potential \( V \) with respect to the reference point at infinity. Determine the potential of a spherical drop obtained when the bubble explodes, assuming all the soap in the bubble is contained in the drop.

P4.5. A volume of a liquid conductor is sprayed into \( N \) equal spherical drops. Then, by some appropriate method, each drop is given a potential \( V \) with respect to the reference point at infinity. Finally, all these small drops are combined into a large spherical drop. Determine the potential of the large drop.

P4.6. Two small conducting spheres of radii \( a \) and \( b \) are connected by a very thin, flexible conductor of length \( d \). The total charge of the system is \( Q \). Assuming that \( d \) is much larger than \( a \) and \( b \), determine the force \( F \) that acts on the wire so as to extend it. Charges may be considered to be located on the two spheres only, and to be distributed uniformly over their surfaces. (Hint: when connected by the conducting wire, the spheres will be at the same potential—see Chapter 6.)

P4.7. Two small conducting balls of radii \( a \) and \( b \) are charged with charges \( Q_a \) and \( Q_b \), and are at a distance \( d \) (\( d \gg a, b \)) apart. Suppose that the balls are connected with a thin conducting wire. What will the direction of flow of positive charges through the wire be? Discuss the question for various values of \( Q_a, Q_b, a, \) and \( b \). (Hint: when connected by the conducting wire, the balls will be at the same potential—see Chapter 6.)

*P4.8. The source of an electrostatic field is a volume charge distribution of finite charge density \( \rho \), distributed in a finite region of space. Prove that the electric scalar potential has a finite value at all points, including the points inside the charge distribution.

*P4.9. Prove that the electric scalar potential due to a surface charge distribution of density \( \sigma \) over a surface \( S \) is finite at all points, including the points of \( S \).

P4.10. The reference point for the potential is changed from point \( R \) to point \( R' \). Prove that the potential of all points in an electric field changes by the voltage between \( R \) and \( R' \).

P4.11. Four small bodies with equal charges \( Q = 0.5 \cdot 10^{-9} \text{C} \) are located at the vertices of a square with sides \( a = 2 \text{cm} \). Determine the potential at the center of the square, and the voltage between the square center and a midpoint of a square side. What is the work of electric forces if one of the charges is moved to a very distant point?

P4.12. An insulating disk of radius \( a = 5 \text{cm} \) is charged by friction uniformly over its surface with a total charge of \( Q = -10^{-8} \text{C} \). Find the expression for the potential of the points which lie on the axis of the disk perpendicular to its surface. Plot your result. What are the numerical values for the potential at the center of the disk, and at a distance
\[ z = a \] from the center, measured along the axis? What is the voltage between these two points equal to?

**P4.13.** The volume charge density inside a spherical surface of radius \( a \) is such that the electric field vector inside the sphere is pointing toward the center of the sphere, and varies with radial position as \( E(r) = E_0 r/a \) (\( E_0 \) is a constant). Find the voltage between the center and the surface of the sphere.

**P4.14.** Two large parallel equipotential plates at potentials \( V_1 = -10 \, \text{V} \) and \( V_2 = 55 \, \text{V} \) are a distance \( d = 2 \, \text{cm} \) apart. Determine the electric field strength between the plates.

**P4.15.** Determine the potential along the line joining two small bodies carrying equal charges \( Q \). Plot your result. Starting from that expression, prove that the electric field strength at the midpoint between the bodies is zero.

**P4.16.** Two small bodies with charges \( Q_1 = 10^{-10} \, \text{C} \) and \( Q_2 = -Q_1 \) are a distance \( d = 9 \, \text{cm} \) apart. Determine the potential along the line joining the two charges, and from that expression determine the electric field strength along the line. Plot your results.

**P4.17.** From the expression for potential found in problem P4.3, find the electric field strength vector along the ring axis. (See problem P3.20.)

**P4.18.** From the general expression for the potential along the axis of the disk from problem P4.12, determine the electric field strength along the disk axis. (See problem P3.21.)