What’s a lens?

Paraxial approximation

Let us assume, for a moment, that a lens is a thin phase function that connects all of the rays from an *object* to an *image*.

\[-nt + n't' = n\sqrt{t^2 + r^2} + n'\sqrt{t'^2 + r^2} + S_{\text{lens}}(r)\]

**Fermat’s principle**

\[\approx -nt - \frac{r}{2t} n + n't' + \frac{r}{2t'} n' + S_{\text{lens}}(r)\]

**Binomial (paraxial) approximation**

\[S_{\text{lens}}(r) = -\frac{r^2}{2} \left( -\frac{n}{t} + \frac{n'}{t'} \right) = -\frac{r^2}{2f}\]

**OPL of a paraxial thin lens**

\[\frac{1}{f} \equiv \phi = -\frac{n}{t} + \frac{n'}{t'}\]

**Gaussian thin lens equation**

- **f** Focal length of lens [m]
- **ϕ=1/f** Power of lens [diopters]

Note that Mouroulis uses *K* for power.
Power of a lens

Physical meaning

\[ \frac{1}{t'} = \frac{1}{t} + \phi \]

In air

The power of a lens is the algebraic increment in curvature added to the incident wavefront.

Note that this means that two thin lenses in contact are equivalent to a single lens with the sum of the powers.
Graphical ray tracing
Solving Maxwell’s Eq. with a ruler

1. A ray through the center of the lens is undeviated
2. An incident ray parallel to the optic axis goes through the back focal point
3. An incident ray through the front focal point emerges parallel to the optic axis. *and occasionally useful*
4. Two rays that are parallel in front of the lens intersect at the back focal plane.
5. *Corollary*: two rays that intersect at the front focal plane emerge parallel.

\[
M \equiv \frac{y'}{y} = \frac{t'}{t} < 0
\]
Graphical tracing

Negative lenses

1. A ray through the center of the lens is undeviated
2. An incident ray parallel to the optic axis appears to emerge from the front focal point
3. An incident ray directed towards the back focal point emerges parallel to the optic axis. *and occasionally useful*
4. Two rays that are parallel in front of the lens intersect at the back focal plane.
5. *Corollary:* two rays that intersect at the front focal plane emerge parallel.

\[ M \equiv \frac{y'}{y} = \frac{t'}{t} > 0 \]
Real and virtual
Images and objects

If you can’t see the light by placing a screen at the plane where it is in focus, then it is virtual.
Equivalently, an image is virtual if you need another lens (e.g. an eye) to make the image real.
Equivalently, real (virtual) objects are to the left (right) of the surface and real (virtual) images are to the right (left).
Tracing mirrors

Mirror system

Equivalent lens system
Roughly a sphere of ~12 mm radius

Typical extreme range of vision is 380 nm to 740 nm

The rods are sensitive to weak light, inoperative in strong light, and have maximum sensitivity at about 507 nm. Rods cover the retina.

The cones are sensitive to strong light, insensitive to weak light, and have a maximum sensitivity at 555 nm. Cones occupy only the fovea.

Cones and rods on retina are waveguides. Cats back these with a reflective tapetum to get double pass, but eyes become cat's eye retroreflectors.

Pupil diameter changes from 4 to 8 mm, many times less that ~10^6 dynamic range of eye. Reason is not light reduction but aberration reduction by “stopping down the system”. At any one time, dynamic range of eye is ~10^3.

Spacing of rods on fovea is about equal to diffraction-limited spot size of the pupil at the minimum diameter.

Most refraction occurs at the cornea (large index contrast) while the lens adjusts via change of shape to change total power.

Typical visual resolution is about 6 minutes of arc. 20/20 vision = ability to resolve 5 arc minute features at 20 feet.

http://www.du.edu/~jcalvert/optics/colour.htm#Eyes
20/20

- A measure of *Visual Acuity*.
- 20 / XX implies that a subject can identify a letter at 20’ what a standard observer can at XX feet.
  - 20 / 10 *GOOD*
  - 20 / 40 *BAD*
- The retina can support better than 20 / 10
The eye
A simple optical model for $\infty$ focus

\[ z = \frac{x^2 + y^2 + pz^2}{2r} \]

- f = \frac{1}{60 \text{ Diopters}} = 16.67 \text{ mm}
- \frac{n_{VH}}{t'} = -\frac{1}{\infty} + \frac{1}{f}
- t' = n_{VH} \cdot f = 22.22 \text{ mm}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emmetropic wavelength of the model, $\lambda_D$</td>
<td>5.80 mm</td>
</tr>
<tr>
<td>Refractive index for emmetropic wavelength, $n_D$</td>
<td>1.333</td>
</tr>
<tr>
<td>Refractive index for wavelength $\lambda$ in microns, $n(\lambda) = a + b/(\lambda - c)$</td>
<td>$a = 1.320535$, $b = 0.004685$, $c = 0.214102$</td>
</tr>
<tr>
<td>Constriction of ocular medium, $F_p = (n_p - 1)/(n_p - n_c)$</td>
<td>50.25</td>
</tr>
<tr>
<td>Apex radius of curvature, $r$</td>
<td>5.55 mm</td>
</tr>
<tr>
<td>Shape parameter $p$</td>
<td>0.6</td>
</tr>
<tr>
<td>Elliptical eccentricity $e = \sqrt{1 - p}$</td>
<td>0.6325</td>
</tr>
<tr>
<td>Axial position of the physical pupil from the apex, $z$</td>
<td>2.55 mm</td>
</tr>
<tr>
<td>Axial position of the entrance pupil from the apex</td>
<td>2.16 mm</td>
</tr>
<tr>
<td>Focal power for emmetropic wavelength $f_0 = (n_p - 1)/r$, 60 diopters</td>
<td></td>
</tr>
<tr>
<td>Anterior focal length for emmetropic wavelength $f_0 = 1/(n_p)$</td>
<td>16.67 mm</td>
</tr>
<tr>
<td>Posterior focal length for emmetropic wavelength $f_0' = (n_p/n_0)$</td>
<td>22.22 mm</td>
</tr>
</tbody>
</table>

Source: http://research.opt.indiana.edu/Library/Ch4_ModelEye/Ch4_ModelEye.pdf
Most important quantity
Retinal magnification factor

aka focal length!

\[ s = f\theta = 16.6\theta \]

\( f, s \) in mm, \( \theta \) in radians
Accommodation

<table>
<thead>
<tr>
<th>Object Distance [m]</th>
<th>Focal Length [mm]</th>
<th>Power [diopters=1/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>15.9</td>
<td>62.8</td>
</tr>
<tr>
<td>1</td>
<td>16.7</td>
<td>59.9</td>
</tr>
<tr>
<td>3</td>
<td>16.9</td>
<td>59.2</td>
</tr>
<tr>
<td>100</td>
<td>17.0</td>
<td>58.8</td>
</tr>
<tr>
<td>Infinity</td>
<td>17.0</td>
<td>58.8</td>
</tr>
</tbody>
</table>

- Power of accommodation = 4 diopters in young, decreases with age.
- Near point $D_{np}$ is 25 cm in young and increases with age as power of accommodation decreases
Thin lens equations
+ derived forms and quantities

Via similar triangles…
transverse magnification

Take derivative of thin lens equation

Longitudinal magnification

Via similar triangles…

Newton’s lens equation
(applies to thick lenses as well)

\[ M \equiv \frac{y'}{y} = \frac{t'}{t} \]

\[ 0 = -\frac{dt}{t^2} + \frac{dt'}{t'^2} \]

\[ M_l \equiv \frac{dt'}{dt} = \frac{t'^2}{t^2} = M^2 \]

\[ z = -y \frac{f}{y'} = \frac{f}{M} \]

\[ z' = -y' \frac{f}{y} = -f M \]

\[ zz' = -f^2 \]
Angular magnification

From previous slide

\[ M \equiv \frac{y'}{y} = \frac{t'}{t} \]

Angular magnification is ratio of two axial rays (rays that cross the axis at the object and image):

\[ M_\theta \equiv \frac{u'}{u} = \frac{-h/t'}{h/(-t)} = \frac{t}{t'} = \frac{1}{M} \]

\( h \) is ray height at lens.

Remember the radiometric unit \( L = \) radiance (or photometric “brightness”) in units of \([W/(sr \, \text{m}^2)]\)? We just found that as size of an object goes up, it’s angular extent decreases by the same amount. Brightness is conserved.
The eye continued

+ Simple instrument: single lens magnifier

\[ \frac{1}{D_{np}} + \frac{n_e}{t_e} = \frac{1}{f_e} \]

\[ h_e = \left( \frac{t_e}{-n_eD_{np}} \right) h \]

Gaussian lens & mag eqs.

- \[ -t = D_{np} \]

\[ f_M \]

Define visual magnification, \( M_v \), as

\[ M_v h_e = \left( \frac{t_e}{n_e t_M} \right) h \]

Visual magnification of single lens at near point of eye

\[ M_v = \frac{D_{np}}{-t_M} \]

\[ = 1 + \frac{D_{np}}{f_M} \]
The magnifier (again)
via angles – useful for infinite conjugates

For a equal focal lengths, \( f_e \), visual magnification should be proportional to ratio of angles via similar triangles

\[
M_v = \frac{\beta}{\alpha} = \frac{D_{np}}{-t_M} = 1 + \frac{D_{np}}{f_M}
\]

For an object at infinity, this becomes \( M_v = \frac{D_{np}}{f_M} \)}
Scheimpflug condition
Imaging from a tilted plane

Paraxial (first-order) optical systems image lines to lines and planes to planes, even when the objects are not normal to the optical axis.

Proof: Draw ray $ABA'B'$ using a ray parallel to $AB$ through the front focal plane. Then draw rays $AA'$ and $BB'$
**Throw**

Assume you need to deliver an image a fixed distance, \( T \) from the object

\[
T \equiv -t + t' \quad \text{Eq 1: Throw is sum of object and image dists}
\]

\[
-\frac{1}{t} + \frac{1}{t'} = \frac{1}{f} \quad \text{Eq 2: Gaussian thin lens equation in air}
\]

\[
t^2 + tT + Tf = 0
\]

\[
t = \frac{1}{2} \left[ -T \pm \sqrt{T^2 - 4Tf} \right]
\]

Eliminate \( t' \) and solve for \( t \)

Two solutions (\( t \) and \( t' \) interchange roles) with the minimum throw being \( T = 4f \) and \( t = t' = 2f \).
Single lens design problem
Summary

Possible unknowns: \( t, t', T, M, f. \)

Equations:
\[
T \equiv -t + t' \quad M = \frac{t'}{t} \quad \frac{1}{t'} = \frac{1}{t} + \frac{1}{f}
\]

So given any two variables, we can solve for the remainder.
The Newtonian form of the thin lens equation can be handy if \( f \) and \( M \) are given:
\[
z \equiv t + f = \frac{f}{M} \\
z' \equiv t' - f = -f M
\]

Commonly in camera sorts of applications, \( T \) and \( M \) are given.
\[
T \equiv -t + t' = f \left(1 - \frac{1}{M}\right) + f (1 - M) \\
f = \frac{T}{\left(1 - \frac{1}{M} + 1 - M\right)} \\
= -TM / (M - 1)^2
\]

Work through the examples 3.2 and 3.3 to be sure you are comfortable with the sign conventions. Do graphical sketches to confirm the equations.