1. Prove
   a. \(\text{comb}(ax)\text{comb}(by) = \frac{1}{ab} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - n/a, y - m/b).\)
   b. \(F\{g(x,y)\} = g(-x,-y),\) where \(F\) is the Fourier transform
   c. \(F\{\nabla^2 g(x,y)\} = -4\pi^2 (f_x^2 + f_y^2) F\{g(x,y)\}\)

2. Solve
   a. \(F\{\text{rect}(x)\text{rect}(y)\}\)
   b. \(F\{\Lambda(x)\Lambda(y)\}\)

3. (Goodman 2-4) Let us define the transform operators
   \[
   F_A\{g\} = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\xi,\eta) \exp\left[-j \frac{2\pi}{a} (f_x \xi + f_y \eta)\right] d\xi d\eta
   \]
   \[
   F_B\{g\} = \frac{1}{b} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\xi,\eta) \exp\left[-j \frac{2\pi}{b} (f_x \xi + f_y \eta)\right] d\xi d\eta
   \]
   a. Find a simple interpretation for \(F_B\{F_A\{g(x,y)\}\}\).
   b. Interpret the result for \(a>b\) and \(a<b\).

4. Consider the Fourier pair \(g, G\). Define \((\Delta x)^2\) as the variance of \(|g(x)|^2\) and \((\Delta f)^2\) the variance of \(|G(f)|^2\). Prove the uncertainty relation \(\Delta x \Delta f \geq \frac{1}{4\pi}\).

5. Prove that if
   \[
g(r) = \begin{cases} 
   1 & \text{if } a \leq r \leq 1 \\
   0 & \text{otherwise}
   \end{cases}
   \]
   the zero order Hankel transform of \(g\) is
   \[
   H\{g(r)\} = \frac{J_1(2\pi \rho) - a J_1(2\pi a \rho)}{\rho}
   \]
   \(\text{(Hint: } xJ_0'(x) = \frac{d}{dx}[xJ_1(x)]\) )

6. (Goodman 2-8) Suppose that a sinusoidal input \(g(x,y) = \cos[2\pi(f_x x + f_y y)]\) is applied to a linear system. Under what (sufficient) conditions is the output a real sinusoidal function of the same spatial frequency as the input? Express the amplitude and phase of that output in terms of an appropriate characteristic of the system.
7. Prove the following properties of the convolution
   a. Commutative \( f * g = g * f \)
   b. Associative \( f * (g * h) = (f * g) * h \)
   c. Distributive \( f * (g + h) = f * g + f * h \)

8. Prove that the autocorrelation function is hermitian, that is \( C(-u) = C^*(u) \).

9. Consider the truncated exponential function
   \[ E(x) = \begin{cases} \exp(-x) & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases} \]
   Calculate the convolution \( E(ax) * E(bx) \). Sketch each of the functions and the result.

10. Perform the following convolutions graphically: