1. Consider a thin optical element with the shape of a section of a cylinder of radius \( R \) as shown in figure 1.
   a. Find the amplitude transmittance function of the optical element assuming it is composed of a homogeneous and isotropic transparent material of index of refraction \( n \).
   b. How will the element affect a plane wave propagating normal to the planar surface?

![Figure 1](image1.png)

2. Consider 1-D Fraunhofer diffraction from two slits of equal width \( w \) separated by distance \( 2d \). The slits are normally illuminated by a plane wave of wavelength \( \lambda \). A thin slab of glass is placed in one of the slits to produce a uniform phase shift \( \phi \) relative to the clear slit. Calculate and sketch the intensity of the pattern in the back focal plane of a lens with focal length \( f \).

![Figure 2](image2.png)

3. Consider the optical system depicted in Fig. 3. \( h(x,y) \) is a transparency placed just behind the lens L1. The focal length of the lenses is \( f \).
   a. In what planes do we obtain the FT of \( h(x,y) \) up to a phase factor?
   b. In what planes do we obtain the image of \( h(x,y) \) up to a phase factor?

![Figure 3](image3.png)
4. Find an optical system that performs a 1-D Fourier transform of the input in the $x$ direction and an image of the same input in the $y$ direction. Note: $x$ and $y$ are the transverse coordinates. Find the location of the output, the scaling of the 1-D Fourier transform, and the magnification of the 1-D image. What is the output of the system if the input is $g(x,y) = \text{rect}(x/2w) \text{rect}(y/2w)$?

5. Consider the 4f imaging system depicted in figure 4. The limiting pupil function is described by the function $p(x,y)$.
   a. Find the impulse response and the amplitude transfer function of the system.
   b. Two point sources are located at the input plane of the system separated by the Rayleigh resolution distance. Assume a circular pupil function of radius $R$. Plot a cross-section of the intensity distribution at the imaging plane if
      i. the two point sources are coherent and in phase
      ii. the two point sources are coherent and 180 degrees out of phase

![Figure 4](image)

6. Phase contrast imaging
   a. Find an expression for the image intensity observed when the phase-shifting dot of a Zernike phase-contrast microscope is also partially absorbing with intensity transmittance equal to $\alpha$ ($0 < \alpha < 1$).
   b. Assume an input object described by the transmittance function $t = \exp[jb\sin(2\pi f_0 x)]$ where $b<<2\pi$ and $f_0$ are constants. What is the optimal value of $\alpha$? What happens if $\alpha = 0$?