1. Rayleigh resolution for coherent and incoherent light

Consider the 4f imaging system depicted in figure 1. The limiting pupil function is described by the function \( p(x,y) \).

a. Find the impulse response and the amplitude transfer function of the system.

b. Find the OTF of the system.

c. Two point sources are located at the input plane of the system separated by the Rayleigh resolution distance. Assume a circular pupil function of radius \( R \). Plot a cross-section of the intensity distribution at the imaging plane if

i. the two point sources are coherent and in phase

ii. the two point sources are coherent and 180 degrees out of phase

iii. the two point sources are incoherent

Compare the three cases.

![Figure 1](image)

2. Fourier-transform lens with incoherent light

Quasi monochromatic spatially incoherent light of uniform intensity illuminates a transparency of intensity transmittance \( f(x,y) \) at the front focal plane of a lens. Determine an expression for the intensity of the observed light at the back focal plane. Compare with the case of coherent light illumination.

3. Coherence of light transmitted through a Fourier-Transform Optical System

Light from a quasi-monochromatic spatially incoherent source with uniform intensity is transmitted through a thin slit of width \( 2a \) and travels between the front and back focal planes of a lens. Determine an expression for the normalized mutual intensity in the back focal plane.
4. **3D sampling**

a. Prove the Nyquist-Shannon sampling theorem for 3D sampling on a cubic lattice.

b. Let \( s(x,y,z) \) be a function whose Fourier transform satisfies
\[
S(f_x, f_y, f_z) = 0 \quad \text{if} \quad f_x^2 + f_y^2 + f_z^2 > 1/2.
\]
Derive a sinc-interpolation formula for samples of \( s(x,y,z) \) on a cubic lattice. Why is this better than 3D sinc interpolation?

c. What would be the optimal sampling lattice in 3D?