1. Consider a separable function \( f(x, y) = f_x(x)f_y(y) \). Show that \( f(x, y) \) is uniquely determined by two of its projections.

2. Consider a projection tomography experiment in which the projections are contaminated by additive noise of spectral power density \( N(f) \) for each projection at angle \( \theta \). How does the ramp filter affect the noise in the filtered back projection algorithm? How does the filtered noise affect the image?

3. A 3D function \( f(x, y, z) \) is sampled in the \( z \)-direction at Nyquist frequency \( 1/\Delta z \), such that slices \( f(x, y, k\Delta z) \) are defined. A helical scan system provides the following projections

\[
p_o(t) = \int_{-\infty}^{\infty} f(t \cos \theta - r \sin \theta, t \sin \theta + r \cos \theta, \theta/\Delta z)dr
\]

Is it possible to reconstruct the object \( f(x, y, z) \) from these projections? If the answer is positive, provide an algorithm; if the answer is negative, explain why and how you would obtain the required additional data.

4. The correlation between two functions is provided as

\[
h(x, y) * g(x, y) = \exp[-\pi(x^2 + y^2)].
\]

a. Is there a unique solution for \( h(x, y) \) and \( g(x, y) \)?

b. Suppose that the following additional conditions are provided

\[
G(f_x, f_y) = 0 \text{ if } f_x^2 + f_y^2 \geq 10
\]

\[
H(f_x, f_y) = 0 \text{ if } f_x^2 + f_y^2 \geq 10
\]

Does the new problem have an exact solution? If yes, provide it. If not, does the problem have an approximate solution? If it has calculate it numerically, if not explain why.