ECEN 5156 – Physical Optics, Spring Semester 2003, University of Colorado at Boulder

Homework #4

Problems due Tuesday, March 11.

1. Find the shape of a reflecting imaging surface for ideally imaging a single point to a single point that Fermat’s principle gives a constant value for the path length (or time) for rays in the neighborhood of a given ray, instead of a minimum as originally formulated by Fermat.

2. Suppose that we know the eikonal in a given medium is described by $S = n_0 z - \frac{\Delta}{2} (x^2 + y^2)$.
   a. Find the index of refraction of the medium.
   b. Find the ray paths in the medium. Draw the trajectories assuming $\Delta > 0$.
   c. Qualitatively discuss what would happen to a plane wave after entering such a medium.

3. Consider a slab with a gradient refractive index given by $n^2(y) = n_0^2 (1 - \alpha^2 y^2)$, and length $d < \frac{\pi}{2\alpha}$. Show that this slab acts as a cylindrical lens of focal length $f \approx \frac{1}{n_0 \alpha \sin(\alpha d)}$. Sketch the ray trajectories.

4. Consider a dielectric isotropic medium with spherical symmetry, i.e. the index of refraction satisfies $n = n(r)$ where $r$ is the distance from a fixed point in space.
   a. Find the equation of the rays $\theta(r)$ in such a medium (hint: see Born&Wolf pp.130).
   b. An observer at sea level notices a plane that appears to be located at an angle of 80° with respect to the zenith. What is its real location? Assume that the plane is at height 15 km, the earth radius is $r_0 \approx 6380$ km, the sea level temperature $T_0 = 20$° C, and the atmosphere’s index of refraction is approximately
      \[ n \approx 1 + \frac{a \exp(-r/r_i)}{1 + b[T_0 - \alpha(r - r_0)]}, \]
      with $a = 3 \times 10^{-4}$, $b = 4 \times 10^{-3}$° C$^{-1}$, $r_i = 10$ km, and $\alpha = -7$° C/km (this formula assumes, among other things, an atmosphere whose pressure, temperature, and water content only vary with elevation).