A Nonlinear Resonant Switch


Abstract—A new resonant switch is introduced which employs nonlinear tank elements. Zero current switching is obtained yet the peak transistor voltage and current stresses can approach those of an equivalent ideal pulse-width-modulated converter. Reduced switching loss without a substantial increase in conduction loss is therefore possible. An approximate analysis is outlined, and transistor peak voltage and current stresses are shown to be much lower than those of linear resonant switch technologies. Single-transistor implementations of the buck, boost, and buck-boost nonlinear resonant switch converters are given. The validity of the nonlinear resonant switch concept, as well as that of the approximate analysis, is proven experimentally.

I. INTRODUCTION

THE OBJECTIVE of high-frequency operation in dc-dc converters is the reduction of transformer, filter inductor, and filter capacitor size and cost. As in any power application, high efficiency is essential, and hence increase of the switching frequency to 500 kHz or more can be problematic because of the direct dependence of transistor switching loss on frequency. The use of resonant switching techniques is an attempt to substantially reduce transistor switching loss and hence attain high efficiency at increased frequency. Other sources, such as transformer core loss, then become the limitation on maximum switching frequency.

A class of resonant converters that has received much recent attention is the resonant switch, or quasi-resonant, converters [1]-[5]. Resonant tank elements are connected to the semiconductor switch components of a PWM converter such that the switch waveform is initially zero, rings positive, and then again reaches zero. The transistor turn-on and turn-off transitions naturally occur at zero voltage [2]-[4] or zero current [1], [3], [5], and hence switching loss is reduced substantially or eliminated. Since resonant switch versions of any previously known PWM converter are possible, the concept is an extremely general one, applicable to a wide variety of applications.

Although switching losses are nearly eliminated in quasi-resonant converters, the peak transistor stresses, and hence conduction losses, are substantially increased. Typical ideal full-load transistor current waveforms of conventional pulse-width-modulated (PWM) and zero-current resonant switch (ZCS) [1] converters are compared in Figs. 1(a) and 1(b) and the corresponding circuits are given in Figs. 2(a) and 2(b). Because of the quasiusinusoidal nature of the ZCS waveform, its peak and rms values are significantly higher than those of a PWM converter for the same output power, and hence conduction losses occur. In the case of the zero-voltage resonant switch (ZVS) [2], the transistor blocking voltage is substantially increased. This necessitates use of high-voltage MOSFET's with significantly increased on-resistance, and hence also increased conduction loss. The multiresonant switch (MRS) [4] allows a further reduction in switching loss and a compromise between the ZCS and ZVS in peak switch voltage and current stresses, but its conduction loss is nonetheless significantly higher than in the PWM case.
A further problem arises when a wide range of load currents and/or input voltages must be tolerated. By skilled choice of the tank characteristic impedance, it is possible to minimize peak transistor stresses at a single operating point. However, these stresses can increase significantly at other operating points. In the case of the zero-voltage switch, a 10:1 load current variation can result in a similar variation of transistor peak voltage; extremely high switch-blocking voltage ratings are then necessary. In the case of a zero-current switch buck converter, input voltage variations directly affect the peak transistor current. In addition, the peak switch current does not depend strongly on load current, leading to poor efficiency at light load.

In consequence of these problems, the overall efficiency of any of the presently known linear resonant switch converters, operating at any switching frequency, must always be significantly lower than that of an equivalent 20-kHz PWM converter. The significant reduction in switching loss obtained through resonant switching is offset by increases in conduction loss. It has been estimated that the increased conduction loss of the zero-current switch is approximately equal to the switching loss of an equivalent 500-kHz PWM switch [6], hence the overall efficiency of the ZCS is lower than the PWM switch at frequencies below approximately 500 kHz. A similar result holds for the other types of resonant switches. Therefore the goal of increasing the switching frequency without reducing efficiency has not been achieved.

It is apparent that the zero-current or zero-voltage switching property must be obtained in a way that does not significantly increase transistor peak current and voltage stress. To accomplish this, a new nonlinear resonant switch [7] has been developed which exhibits zero-current switching. Its peak transistor voltage is identical to, and the peak current is typically 20 percent greater than, that of an equivalent PWM converter. The efficiency of a nonlinear resonant switch (NRS) converter can therefore approach that of a low-frequency PWM converter. Furthermore, the nonlinearity can be controlled such that the peak transistor current is closely related to the load current; improved efficiency at light load is then obtained.

These advantages are realized through the use of nonlinear tank elements whose natural response is quasi-trapezoidal, as in Fig. 1(c), rather than quasi-sinusoidal. As described in Section II, a biased saturating inductor is used as the tank element in a zero-current resonant switch. A buck converter implemented with a half-wave nonlinear resonant switch is shown in Fig. 2(c). The tank inductor $L$ is normally biased into saturation by the dc current $I_T$ in a secondary winding; the tank inductance is then similar in value to that of a linear zero-current resonant switch. When the tank current $i_L$ becomes sufficiently large, the tank inductor desaturates, which causes its inductance to increase and hence reduces the peak value of the switch current. Near the end of the interval, the tank inductor resaturates and the current rings back to zero.

As in the case of linear resonant switches, the nonlinear resonant switch cell can be inserted into most of the common converter topologies. The properties of such converters can be found by analyzing the nonlinear resonant switch to cell to find its average terminal waveforms, then inserting the results into the converter equations. Such an analysis is performed in Section III using a simple piecewise-constant model for the tank inductor. The averaged switch waveforms are found for the half-wave and full-wave cases, and the results are tabulated for the buck, boost, and buck-boost converters. The addition of a third winding to a nonlinear tank inductor is also considered; this allows another degree of control of the switch characteristics. The range of switching frequency variations can then be reduced, and constant frequency operation is feasible if the range of operating points is sufficiently limited.

The peak switch stresses for a 100-W 50-V input dc-dc converter utilizing PWM and various resonant switches are compared in Section IV. A tradeoff between NRS peak switch current, transistor switching times, and minimum tank inductance is also discussed. The transistor peak current can approach the ideal PWM value, but this requires an instantaneous switching time and a zero minimum tank inductance. Nonetheless, full-load peak switch currents which exceed the PWM value by only 20 percent are feasible.

The analysis and discussion results are applied in Section V to the design of a 50-W full-wave buck converter. Experimental verification of the nonlinear resonant switch concept and the analysis of Section III is presented in Section VI. The results are summarized in Section VII.

II. INTRODUCTION OF NONLINEARITY INTO ZERO CURRENT SWITCH

The objective of reducing and controlling the peak switch current of the zero-current resonant switch can be attained through the use of nonlinear tank elements. By proper variation of the tank inductance characteristic, a quasi-trapezoidal tank current natural response can be obtained. In particular, the incremental inductance must increase with current thereby yielding the electrical analog of a hard spring. A practical implementation of this which employs a biased saturating inductor is described here.

In the zero-current resonant switch with linear tank capacitance, the tank capacitor must charge twice the applied input voltage during every ringing interval. This requires that the capacitor current $i_c$ be positive during the first portion of the ringing interval, where

$$i_c = i_i - i_T.$$  \hspace{1cm} (1)

Hence one cannot expect to limit the peak $i_I$, or the peak switch current, to less than $I_T$, the current conducted by an equivalent PWM switch.

However, it is possible to limit the peak $i_I$ to a value slightly higher than $I_T$. This is done by increasing the incremental tank inductance $L(i_I)$, when $i_I$ exceeds $I_T$, such that $i_I$ does not change very rapidly but the capacitor con-
continues to charge. Hence it is desirable that the incremental inductance vary as shown in Fig. 3(a).

This nonlinear function $L(i_L)$ can be realized using a biased saturable reactor, as shown in Fig. 3(b). In addition to the primary tank inductance winding, one or more secondary bias windings are placed on the ungapped core. These secondary windings are driven by dc bias currents such that the core is saturated when $i_L = 0$ and hence $L(i_L)$ is relatively small. However, if $i_L$ increases to a value such that the primary-induced magnetomotive force is approximately equal and opposite to the total magnetomotive forces induced by the secondary bias windings, then the core enters the unsaturated state. The incremental inductance then increases by several orders of magnitude.

Thus it is desirable to maintain the desaturation point to be always slightly greater than $I_T$ such that the switch peak currents are as low as possible for all values of load current. This can be accomplished by placing a tank-inductor secondary-bias winding in series with the converter low-frequency filter inductor, as shown in Fig. 2. If the turns ratio $N$ is chosen to be greater than 1, then the tank inductor will desaturate when $i_L$ is approximately $NI_T$. Another case of interest is the choice $N = 1$, with additional bias $I_B$ provided by a third winding as in Fig. 4. As shown in the next section, the switch average tank voltage for full-wave operation is then independent of load current. A limited amount of output voltage control can also be exercised by varying $I_B$ while maintaining a constant switching frequency.

Regardless of the bias scheme, the incremental inductance using the piecewise model can be expressed as

$$L(i_L) = \begin{cases} L_1, & i_L < I_T + I_{sat} \\ L_2, & i_L > I_T + I_{sat} \end{cases}$$

where $I_{sat} = (N - 1)I_T + I_B$. Since the converter characteristics depend strongly on the value of $L_1$, and to a lesser extent on $L_2$, it may be desirable to accurately control these values by introducing gapped inductors in series ($L_1$) and parallel ($L_2$) with the saturating inductor.

Thus peak transistor current is limited by causing the tank inductance to increase at high tank current. The nonlinear tank inductance characteristic is realized by biasing a saturating inductor with dc currents in one or more secondary windings.

III. Analysis

The principal steps in the analysis of the nonlinear resonant switch cell of Fig. 4 with piecewise constant tank inductance are outlined here. It is assumed that the converter imposes an essentially constant current $I_T$ on switch terminal B as shown; this generally occurs because a converter low-frequency filter inductor is connected there. Likewise, the converter imposes an essentially constant voltage $V_T$ between terminals A and C through the connection of the converter dc input voltage or a low-frequency filter capacitor.

It is convenient to normalize all variables with respect to the following base quantities:

$$V_{base} = V_T$$

$$R_{base} = R_o = \sqrt{L_1/C}$$

$$I_{base} = V_T/R_o$$

$$\omega_o = \frac{1}{\sqrt{L_1C}}$$

$$f_o = \frac{\omega_o}{2\pi}$$

The normalized tank waveforms

$$i_t = i_L/I_{base}$$

$$m_r = r_r/V_{base}$$

are plotted versus normalized time in Fig. 5, and in the phase plane in Fig. 6.

Interval 1, $0 \leq \omega_o t \leq \alpha$: Switch S1 and diode D2 conduct during this interval, and the tank inductance is sat-
The interval ends when $i_t = I_T$ and $\omega_{st} = \alpha$. Hence
\[
J_T = I_T / I_{base} = \alpha.
\] (7)

Interval 2, $\alpha \leq \omega_{st} \leq \alpha + \beta$: Switch S1 conducts during this interval. The tank elements are excited by the essentially constant quantities $V_T$ and $I_T$ and ring sinusoidally. The normalized solution is
\[
j_L = J_T + \sin (\omega_{st} - \alpha)
\] (8)
\[
m_c = 1 - \cos (\omega_{st} - \alpha). \tag{9}
\]

Interval 2 ends when the tank inductor comes out of saturation. This occurs when the tank current $i_t$ reaches the critical value $I_{crit} + I_T$, corresponding to the bottom of the core $B-H$ loop. The length of this interval is found by substitution of $\omega_{st} = \alpha + \beta$ into (8), resulting in
\[
\beta = \sin^{-1} (J_{crit}) \tag{10}
\]

where
\[
J_{crit} = \frac{I_{crit}}{I_{base}}.
\]

The capacitor voltage at the end of the interval is
\[
m_c(\alpha + \beta) = 1 - \cos (\beta) = 1 - \sqrt{1 - J_{crit}^2}. \tag{11}
\]

Interval 3, $\alpha + \beta \leq \omega_{st} \leq \alpha + \beta + \delta$: The tank inductor is unsaturated during this interval, and hence the effective tank inductance is $L_2$, a quantity much larger than $L_1$ in a well-designed converter. The normalized state equations for this interval are
\[
\frac{1}{\omega_1} \frac{dj_L}{dt} = k(1 - m_c) \quad \frac{1}{\omega_1} \frac{dm_c}{dt} = \frac{1}{k} (j_L - J_T) \tag{12}
\]

where
\[
\omega_1 = \frac{1}{\sqrt{L_2 C}} \quad k = \frac{\omega_1}{\omega_o} = \frac{\sqrt{L_1}}{\sqrt{L_2}} \ll 1.
\]

All quantities are again normalized with respect to the base values of (3). The solution is an ellipse in the phase plane, of the form
\[
m_c = 1 - A \cos \varphi + \frac{B}{k} \sin \varphi
\]
\[
j_L = J_T + kA \sin \varphi + B \cos \varphi \tag{13}
\]

where
\[
\varphi = \omega_1 t - k(\alpha + \beta)
\]
\[
A = \cos \beta = \sqrt{1 - J_{crit}^2}
\]
\[
B = \sin \beta = J_{crit}.
\]

Note that the solution can be manipulated into the following form:
\[
k^2 (m_c - 1)^2 + (j_L - J_T)^2 = (kA)^2 + B^2. \tag{14}
\]

This describes an ellipse centered at $m_c = 1$, $j_L = J_T$, which is symmetrical about its horizontal and vertical axes. The vertical (semiminor) axis is of length
\[ \sqrt{(kA)^2 + B^2}, \text{ and hence the peak normalized tank current is} \]
\[ J_{pk} = J_T + \sqrt{(kA)^2 + B^2}. \quad (5) \]

The interval ends when the tank inductance resaturates, i.e., when \( j_L = J_{cent} + J_T \) again. Because of the symmetry of the ellipse, this occurs at a normalized capacitor voltage of
\[ m_c(\alpha + \beta + \delta) = 1 + \sqrt{1 - J_{cent}^2}. \quad (6) \]
Substitution of this relation into the waveform expression and solution for the interval length \( \delta \) yields
\[ \delta = \frac{2}{k} \tan^{-1}\left( \frac{kA}{B} \right). \quad (7) \]

**Interval 4**, \( \alpha + \beta + \delta \leq \omega_o t \leq \alpha + \beta + \delta + \xi \): The tank inductance is again saturated, and the circuit is the same as during interval 2. The solutions are again sinusoidal and lead to a circular arc in the phase plane (Fig. 6) centered at \( m_c = 1, J_L = J_T \) with radius 1. This interval ends at the first (half-wave) or second (full-wave) zero crossing of the tank inductor current.

This interval can be solved by simple geometry. As shown on the phase plane diagram, the final value of tank capacitor voltage is
\[ m_c(\alpha + \beta + \delta + \xi) = M_{cl} = \begin{cases} 
1 + \sqrt{1 - J_T^2} & \text{half-wave} \\
1 - \sqrt{1 - J_T^2} & \text{full wave} 
\end{cases} \quad (8) \]
subject to \( J_T \leq 1 \). The length of this interval is given by the angle subtended by the arc, or
\[ \xi = \begin{cases} 
\beta + \sin^{-1} J_T & \text{half-wave} \\
\beta + \pi - \sin^{-1} J_T & \text{full wave} 
\end{cases} \quad (9) \]
with \( 0 \leq \sin^{-1} J_T \leq \pi/2 \).

**Interval 5**, \( \alpha + \beta + \delta + \xi + \zeta \leq \omega_o t \leq \alpha + \beta + \delta + \xi + \zeta + \xi \): During this interval all transistors and diodes are in the off state. The tank capacitor is discharged linearly by the filter inductor current \( I_T \), and the tank inductor current is zero. The expression for the normalized capacitor voltage is
\[ m_c = M_{cl} - J_T(\omega_o t - (\alpha + \beta + \delta + \xi)). \quad (10) \]

This interval ends when the tank capacitor voltage reaches zero and diode D2 turns on. Substitution of \( m_c = 0 \) into the preceding expression allows one to solve for the length of this interval:
\[ \xi = \frac{M_{cl}}{J_T}. \quad (11) \]

**Interval 6**, \( \alpha + \beta + \delta + \xi + \zeta \leq \omega_o t \leq \alpha + \beta + \delta + \xi + \zeta + \xi + \alpha \): During this interval, diode D2 conducts for the remainder of the switching period. The tank states remain at zero.

**Circuit Averaging**

The static and low-frequency dynamic characteristics of converters containing a nonlinear resonant switch can be found by averaging the switch terminal waveforms (tank inductor current \( i_T \) and capacitor voltage \( v_c \)) over one switching period. This can be done using the solutions derived in the preceding. The result for the average normalized tank voltage is
\[ \frac{\langle v_c \rangle}{V_T} = \mu = \langle m_c \rangle = FP(J_T, J_{cent}) \quad (12) \]
with
\[ F(J_T, J_{cent}) = \frac{1}{2\pi} \left\{ \frac{1}{2} J_T + \frac{M_{cl}}{J_T} \beta + \delta + \xi \right\} \]
and \( F = f_s/f_o = \) normalized switching frequency. To the extent that losses can be neglected, the resonant switch input and output powers are equal since the switch does not contain dynamic states. In normalized form, this means that
\[ j_L \cdot 1 = \langle m_c \rangle \cdot J_T \quad (13) \]
so
\[ \langle j_L \rangle = F(J_T, J_{cent}) \quad (14) \]
Equations (22) and (24), and the use of circuit averaging techniques \([9], [10]\) are sufficient to model converters incorporating nonlinear resonant switches. For example, in the case of the buck converter,
\[ V_T = V_S \quad I_T = I_f \quad (15) \]
and the overall dc conversion ratio is
\[ \frac{V_{out}}{V_S} = \mu = FP(I_f R_o/V_T, I_{cent} R_o/V_S). \quad (16) \]

The averaged function \( P(J_T, J_{cent}) \) is plotted in Fig. 7 for the half-wave nonlinear resonant switch with \( J_B = 0 \). The switch conversion ratio \( \mu = FP \) is dependent on \( J_T \) and load current in this case. A good approximation when \( N > 1 \) and \( J_B = 0 \) is
\[ P = \frac{N}{(N - 1) \pi J_T}. \quad (17) \]

This is valid for \( k \ll (N - 1) J_T \), and is within 10 percent of the exact value for any allowed load current when \( N = 1.2 \) and \( k = 0 \). The dc conversion ratio \( M \) for a half-wave buck converter is plotted in Fig. 8 for various values of \( Q = R/R_o \).

The averaged function \( P(J_T, J_{cent}) \) is plotted in Fig. 9 for the full-wave switch with \( N = 1 \). The switch conversion ratio \( \mu = FP \) is then essentially independent of \( J_T \) and load current. The converter output voltage depends on switching frequency and bias current. A good approximation for \( P \) in this case is
\[ P(J_{cent}) = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} J_{cent} + \frac{1}{\pi k} \tan^{-1}\left( \frac{k \sqrt{1 - J_{cent}^2}}{J_{cent}} \right) \quad (18) \]
Fig. 7. Switch conversion factor \( P(J_T) \) for half-wave nonlinear resonant switch with \( J_B = 0 \) and \( N > 1 \).

Fig. 8. Voltage conversion ratio \( M = V/V_x \) for half-wave buck converter, for various values of load resistance \( R \). Note: \( Q = R/R_c \).

Fig. 9. Switch conversion factor \( P(J_B) \) for full-wave nonlinear resonant switch with \( N = 1 \), for various values of \( k \) with \( J_{cm} = (N - 1)J_T + J_B \). When \( N = 1 \) and \( k \ll J_B \), this further reduces to

\[
P(J_B) = \frac{1}{2} + \frac{1}{\pi J_B} + \frac{J_B}{2\pi}.
\]

When the inequality \( k \ll J_B \) is not satisfied, \( P(J_B) \) is reduced in value but is nonetheless independent of load current and \( J_T \). Hence the voltage conversion ratio \( V/V_x \) of the buck converter in this case is a linear function of switching frequency, a nonlinear function of bias current, and independent of load current.

Full-wave nonlinear resonant switch versions of the buck, boost, and buck-boost converters are shown in Fig.

Fig. 10. Nonlinear resonant switch implementations of three basic converters. (a) Buck. (b) Boost. (c) Buck-boost.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>STEADY-STATE CONVERTER EQUATIONS</th>
</tr>
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<tbody>
<tr>
<td>Converter</td>
<td>( M = V/V_x )</td>
</tr>
<tr>
<td>buck</td>
<td>( I_T R_c/V_x )</td>
</tr>
<tr>
<td>boost</td>
<td>( 1/1 - \mu )</td>
</tr>
<tr>
<td>buck-boost</td>
<td>( \mu/(1 - \mu) )</td>
</tr>
</tbody>
</table>

\( \mu = F P(J_T, J_B); J_x = M/Q = I_T R_c/V_T, Q = R/R_c. \)

10. For each case, the relation between \( V_T \) and the average tank capacitor voltage \( \langle V_C \rangle \) is given by (22). However, \( V_T \) does not in general coincide with the input voltage: \( V_T = V \) for the boost converter, and \( V_T = V_Q + V \) for the buck-boost. Expressions for \( M, J_T, \) and \( J_B \) are given in Table I.

As for the linear zero-current resonant switch, there are two basic restrictions for operation in the mode described here. First, the normalized output filter inductor current \( I_T \) must not exceed 1; otherwise the phase plane trajectory during the fourth interval cannot reach the \( j_L = 0 \) axis because the radius of the circle is too small (see Fig. 6). Attempting to operate at currents greater than this value results in high transistor turn-off stress. The curves of Fig. 8 end at \( M = Q \) for \( Q < 1 \) because larger output voltages would violate the \( J_T < 1 \) condition.

Second, the length of the sixth interval must be positive, and this limits the maximum switching frequency.
This limit can be expressed as

$$\omega_n T_s = \frac{2\pi}{F} \geq \alpha + \beta + \delta + \xi + \zeta. \quad (30)$$

IV. DISCUSSION

As in the linear zero-current switch, the energy stored in the MOSFET drain-to-source capacitance is lost during the transistor switching transitions and results in switching loss [6]. The recovery time and stored charge of diode D1 can also contribute to switching loss. These mechanisms can limit the maximum switching frequency of the linear zero-current switch to roughly 1 MHz with current component technology. In the nonlinear resonant switch, this frequency limit is similar but somewhat higher because the reduced conduction loss allows the use of smaller MOSFET devices with higher on-resistance and lower output capacitance.

The nonlinear resonant switch is also somewhat less tolerant of long transistor switching times and large transformer leakage inductance. However, a reasonable trade-off between these quantities, switching frequency, and peak transistor current can be obtained. Consider a full-wave buck converter example. In the linear zero-current switch, the voltage conversion ratio $M = V_o/V_i$ and the normalized switching frequency $F = f_s/f_r$ are directly related: $M = F$. Also, the transistor peak current $I_{pk}$ always exceeds the output filter inductor current $I_T$ by the amount $V_o/R_o$, hence, $(I_{pk} - I_T)$ is equal to $V_o/R_o$.

For the nonlinear resonant switch, the peak transistor current can be varied continuously from PWM switch value ($I_{pk} = I_T$) to the linear resonant switch value ($I_{pk} = I_T + V_o/R_o$), depending on the design of the tank inductor and the choice of $I_{en}$. Maintaining given converter operating conditions requires that the switching frequency vary also. Equations (15), (22), and (26) can be used to determine the normalized switching frequency of an equivalent nonlinear zero current switch for various values of $(I_{pk} - I_T)$. The results are plotted in Fig. 11. It can be seen that a design in which the peak transistor current $I_{pk}$ exceeds the output current $I_T$ by 0.2 $V_o/R_o$ results in $F = 0.5 M$, or $P = 2$. For the same voltage conversion ratio $M$ and switching frequency $f_s$, this nonlinear design requires a tank frequency $f_r$ twice as high as in the linear case. Hence the tank inductance and capacitance must be smaller by a factor of two, and the transistor switching times must be half as long. This is a reasonable price to pay for the greatly reduced conduction loss of the nonlinear resonant switch.

Estimated peak transistor current $I_{pk}$ and peak transistor voltage $V_{pk}$ of various switch technologies are compared in Table II for the following nonsaturated buck converter application:

- input voltage $V_i = 50 \text{ V} \pm 20\%$
- output voltage $V = 25 \text{ V}$
- output power = 10 W - 100 W.

<table>
<thead>
<tr>
<th>Switch Type</th>
<th>Current, A</th>
<th>Voltage, V</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM</td>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>NRS</td>
<td>4.8</td>
<td>60</td>
</tr>
<tr>
<td>ZCS</td>
<td>11.5</td>
<td>60</td>
</tr>
<tr>
<td>ZVS</td>
<td>4</td>
<td>810</td>
</tr>
<tr>
<td>MRS</td>
<td>6.4</td>
<td>200</td>
</tr>
</tbody>
</table>

The zero-current nonlinear resonant switch (NRS) was designed to operate at a maximum $I_T$ of 0.8 and a full load $I_R$ of 0.125. The linear zero current switch (ZCS) and linear zero-voltage switch (ZVS) were designed with similar margins. The linear multiresonant switch (MRS) values were derived from results reported in [4], [8].

It can be seen that the NRS case attains zero-current switching without substantially increasing the peak transistor current or voltage above the PWM values. The linear ZCS results in significantly increased conduction loss because of its high peak currents. The linear ZVS and MRS require high-voltage MOSFET switches, with corresponding high on-resistance and high conduction loss.

V. steady-state design example

Consider the design of a 50-W buck converter to meet the following specifications:

- input $V_i = 50 \text{ V}$
- output $V = 30 \text{ V}$ at $I = 1.67 \text{ A}$.

The desired dc conversion ratio is $M = 30/50 = 0.6$, at a switching frequency of $f_s = 500 \text{ kHz}$. The following identities hold for the buck converter:

$$V_T = V_i = 50 \text{ V}$$
$$I_T = I = 1.67 \text{ A}.$$

As discussed previously, the normalized current $I_T$ must not exceed unity. Let us choose $I_T = 0.75$ at full load.
Hence
\[ R_n = JV_t/I_T = (0.75) (50 \text{ V})/(1.67 \text{ A}) = 22.5 \Omega. \] (31)

To obtain a control characteristic which is independent of load current variations, i.e., in which variations in \( I_T \) do not require an adjustment of switching frequency of bias current, choose the full-wave mode of operation with \( N = 1 \) and with a bias winding current \( I_B \). Then
\[ I_{\text{ctrl}} = (N - 1) I_T + I_B = I_B. \] (32)

Let us design for a full-load peak transistor current \( I_{pk} \) which is 20 percent greater than the output current \( I \):
\[ I_{pk} = 1.2I = 2 \text{ A} \]
\[ J_{pk} = I_{pk} R_o/V_T = 0.9. \] (33)

Equation (15) has no solution for positive \( J_{\text{ctrl}} \) unless the quantity \( k \) is less than \( J_{pk} - J_F = 0.15 \). The choice \( k = 0.1 \) is therefore acceptable. Equation (15) can be used to solve for the normalized bias current \( J_B = J_{\text{ctrl}} \):
\[ J_B = (J_B - J_F)^2 - k^2 \]
\[ I_B = I_B V_T/R_o = 0.25 \text{ A}. \] (34)

This value of bias winding current can be reduced by increasing the bias winding turns ratio.

The converter solution is given by (26) in conjunction with (22), (10), (17), (18), and (19); evaluation of these expressions yields
\[ P = 2.837 \]
\[ F = M/P = 0.211. \] (35)

The required tank resonant frequency is
\[ f_o = f_r/F = 2.36 \text{ MHz}. \] (36)

This leads to the following tank component values:
\[ C = 1/2\pi f_o R_o = 3 \text{ nF} \]
\[ L_1 = R_o/2\pi f_o = 1.5 \text{ \mu H} \]
\[ L_2 = L_1/k^2 = 150 \text{ \mu H}. \] (37)

VI. EXPERIMENTAL VERIFICATION

A prototype nonlinear resonant switch buck converter, as in Fig. 10a, was constructed using the following (different) values:

- input voltage \( V_i = 24 \text{ V} \)
- transistor \( Q_i \): IRF 530
- diodes \( D_1, D_2 \): MBR 1045
- tank capacitor \( C = 9.1 \text{ nF} \).

Initially, these components were connected in a full-wave configuration. The nonlinear tank inductor was constructed using a Magnetics, Inc. 80523-1/8-D-MA tape-wound core (1/8 mil Permalloy 80). The primary and secondary windings each contained 12 turns. A third winding, driven by the dc bias current \( I_B \), consisted of 36 turns. The measured primary inductance was 109 \text{ \mu H} with no bias, and was within 20 percent of 0.3 \text{ \mu H} for total bias currents between 130 mA and 1.2 A referred to the bias winding. The tank characteristic impedance is therefore
\[ R_o = \frac{L_o}{\sqrt{C}} = 5.7 \Omega \] (38)
and the base current is
\[ I_{\text{base}} = \frac{V}{R_o} = 4.15 \text{ A}. \] (39)

Full-Wave Results: Constant Bias Current, Variable-Frequency Control

The theoretical curves and measured data points for the full-wave converter using a constant bias current \( I_B \) of 0.54A are shown in Fig. 12. It can be seen that the converter behaves nearly as a voltage source, with output voltage directly dependent on switching frequency and nearly independent of load current. Good agreement between the model and experiment is obtained, except at light load current where the core does not fully saturate. Prediction of the observed behavior at light load requires a more accurate core saturation model.

Full-Wave Results: Constant Frequency, Variable Bias Current Control

Next, the switching frequency was held constant at 500 kHz and the bias current \( I_B \) was varied. The results are given in Fig. 13. The output voltage now depends directly on \( I_B \) and is nearly independent of load current, except at light load or low bias current, where incomplete saturation of the core again causes the converter characteristics to deviate from the model. Hence constant-frequency operation is possible and is desirable provided that the range of voltage conversion ratio variations is not too large.
Constant-frequency regulation for a wide range of voltage variations requires large bias currents and consequently leads to large peak tank currents and lower efficiency. Nonetheless, it is apparently feasible to control voltage variations of a factor of two with this design.

**Half-Wave Results: Variable-Frequency Control, No Bias Current**

The same components were connected in the half-wave configuration (as in Fig. 2(c)), and the nonlinear tank inductor was redesigned. The same 80523-1/8-D-MA tape-wound core was used, but with a 12-turn primary, a 15-turn secondary, and no additional bias winding. Experiment and theory are compared in Fig. 14, and good agreement is obtained. As predicted, the output voltage is strongly load-dependent in this case. No data is shown for load currents under 1 A because the tank inductor saturated during the third (8) interval; additional turns are required to support the applied volt-seconds in this case.

**Results with Ferrite Tank Inductor: Full-Wave Case**

The core loss of the tape-wound core was fairly high at 500 kHz. It is possible to obtain high-efficiency operation and low peak tank currents by use of a ferrite core material. However, the converter characteristics are more difficult to predict because of the gradual saturation characteristic of ferrite.

The saturating inductor of the full-wave converter described in the preceding was rewound as follows. Two TDK H7C4 T4-8-2 cores were stacked and wound with primary and secondary windings of five turns and an additional bias winding of 15 turns. The measured primary inductance with zero bias was 44 μH, and varied from 10 μH to 2.8 μH as the bias was increased from 100 mA to 2 A. Note that even at 2-A bias current, the core relative permeability was over 100.

Because of the significant variation of the "saturated" inductance, the piecewise constant-inductance model of Section III does not directly apply. Nonetheless, the converter worked well: zero-current switching occurred for load currents up to approximately 3 A at \( V_e = 24 \) V, the maximum switching frequency was over 500 kHz, and the peak transistor current was limited. A typical transistor current waveform is shown in Fig. 15 for a load current of 1.95 A, a bias current of 245 mA, an output voltage of 13.5 V, an input of 24 V at 1.26 A, and a switching frequency of 330 kHz. With a 24-V input and bias current of 0.25 A, the peak transistor current was approximately 0.6 A greater than the load current.

**Results with Ferrite Tank Inductor: Half-Wave Case**

Measured tank waveforms for the half-wave case are shown in Fig. 16. The primary was wound with five turns, the secondary contained six turns, and there was no additional bias winding. Two stacked TDK H7C4 T4-8-2 cores were again used. The measured operating conditions were: 24-V input at 1.88 A, 19-V output at 2.15 A, peak transistor current: 2.4 A; switching frequency: 500 kHz. It can be seen that zero-current switching occurs and that the peak transistor current is limited to 0.25 A greater than the load current. The maximum load current for this converter was again approximately 3 A.

**VII. CONCLUSION**

Linear resonant switching exhibit greatly reduced switching loss and can be employed in a wide variety of converter topologies. However, with these advantages come the problems of poor switch utilization and substantially increased conduction losses. High efficiency is essential if the converter power density is to be increased, and poor switch utilization limits the number of useful applications of the resonant switch.

A resonant switch employing nonlinear tank elements has been developed which overcomes these disadvantages. The nonlinearity forces the tank natural response to exhibit a reduced peak value while maintaining the zero-current switching property. Peak switch stresses approaching those of ideal PWM converters can be obtained, yielding low conduction losses and high efficiency.

The nonlinear inductance characteristic is realized using a saturating core, biased by dc currents in one or more secondary windings. By forcing a current proportional to the load through one of the secondary windings, the peak
transistor current can be made to scale directly with load current. Constant-frequency operation can also be achieved by control of the secondary bias current.

A piecewise constant inductance model can be employed to solve for approximate switch terminal characteristics; this works well for square-loop core materials but needs further refinement for prediction of the characteristics obtained using ferrite core material. The resonant switch voltage conversion ratio is equal to the normalized switching frequency \( F \) times a function \( P \), which depends on the load and bias currents. The dependence of \( P \) on load current is essentially removed in the full-wave case with \( N = 1 \); the switch voltage conversion ratio then exhibits an inverse dependence on bias current \( I_B \). The equilibrium characteristics of buck, boost, and buck-boost converters utilizing nonlinear resonant switches have been determined.

This analysis can also be used to examine the tradeoff between peak switch current, saturated tank inductance, and transistor switching time. By proper design, any value of peak switch current between that of a linear resonant switch and an ideal PWM switch can be obtained; however, low peak currents require a small tank inductance and fast transistor switching times. Nonetheless, peak

Fig. 15. Experimentally measured tank current waveform for full-wave nonlinear switch with ferrite core material. Vertical scale: 1 A/div. Horizontal scale: 500 ns/div.

Fig. 16. Experimentally measured tank voltage and current waveform for half-wave nonlinear switch with ferrite core material. Upper trace: capacitor voltage, 26 V/div. Lower trace: inductor current, 1 A/div. Horizontal scale: 1 ps/div.
currents which exceed the ideal PWM value by only 20 percent are feasible and require reduction of transistor switching times by approximately 50 percent.

Extensive experimental verification of the nonlinear resonant switch concept has been performed. Buck converter experimental data is presented here, including results for full-wave switches with frequency and bias current control, and half-wave switches with frequency control. The nonlinear tank inductor has been implemented using both ferrite and square-loop tape-wound core material. The results show that use of the nonlinear resonant switch technique is an effective way to simultaneously obtain low conduction loss, low switching loss, and good switch utilization in high-frequency converters.

**References**


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