A Multiple-Winding Magnetics Model Having Directly Measurable Parameters

Robert W. Erickson and Dragan Maksimovic
Colorado Power Electronics Center
University of Colorado at Boulder, USA

http://ece-www.colorado.edu/~pwrelect

Modeling Multiple-Output Converters
Cross regulation, CCM/DCM boundaries, dynamics

**Flyback converter**

**Forward converter**
Modeling Multiple-Output Converters

Multiple-output converters are more than simple extensions of parent single-output nonisolated converters:

- Imperfect coupling between windings leads to problems in cross regulation, small-signal dynamics, and multiple operating modes, which have not been fully explored in the literature
- These phenomena are governed primarily by the transformer leakage inductance parameters

Need a suitable multiple-winding transformer model

- that predicts observed waveforms
- that yields insight into converter cross regulation, CCM/DCM boundaries, and dynamics
- that explains how converter performance depends on winding geometry
- that is useful in computer simulation
Approaches to Multiple-Winding Transformer Modeling

Inductance matrix
- General
- Reduces the circuit to matrix equations
- Numerically ill-conditioned in tightly-coupled case

Reduced-order equivalent circuit
- Physically based
- Not general—Does not predict observed waveforms of flyback converter
- Difficult to apply to some geometries (for ex., toroidal)

Full-order equivalent circuits
- Allow circuit-oriented analysis of converter
- General
Physical-Based Reduced-Order Model

4 winding transformer example

Four-winding transformer example

Physical modeling approach: equivalent circuit contains series-connected leakage inductances

Equivalent circuit proposed in [12]:
An Electrically-Equivalent Form
of the reduced-order model

- Contains seven independent parameters
- Inductance matrix of four-winding transformer contains ten independent parameters
- Is this model sufficient?
Apply a voltage to winding 1, short windings 2, 3, and 4. Measure short-circuit currents in each winding.

Model predicts that $i_3$ and $i_4$ are zero.
A Full-Order Model
Extended Cantilever Model

Include “leakage inductances” between each winding
Again apply a voltage to winding 1, short windings 2, 3, and 4. Measure short-circuit currents in each winding.

Model predicts nonzero $i_3$ and $i_4$. 
Discussion

- It is always possible to connect a transformer such that a reduced-order model does not predict the actual waveforms.
- Are such connections actually encountered in multiple-output converters?
Flyback converter circuit

Three outputs

Primary-side voltage-clamp snubber

Cross-regulation is strongly influenced by *commutation interval*, when transistor turns off and magnetizing current shifts to secondary windings
Commutation Interval
Flyback converter example—similar to thought experiment

- Reflected output voltages are nearly identical
- $l_{23}$, $l_{24}$, $l_{34}$ are irrelevant
- Magnetizing current divides between the output windings according to $l_{12}$, $l_{13}$, $l_{14}$
Commutation Interval
Flyback converter example—similar to thought experiment

- Reflected output voltages are nearly identical
- $l_{23}, l_{24}, l_{34}$ are irrelevant
- Magnetizing current divides between the output windings according to $l_{12}, l_{13}, l_{14}$
Conclusion: Reduced-Order Modeling

- Approximate reduced-order model derived via physical approach does not correctly predict behavior of multiple-output flyback converter
- Approximations must not be based solely on winding geometry
- Application and circuit behavior must be considered before attempting to reduce the order of the model
- Need a suitable full-order model
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Transformer Models

Two-winding transformer models

Extension of cantilever model to four-winding case
$N$-winding transformer models used here

$n$-winding transformer

Extended cantilever model

$n$-port model
Relationship between inductance matrix and extended cantilever model

The inductance matrix:
\[ v = sL_i \]
\[ L = \{L_{jk}\} \text{ inductance matrix} \]
\[ B = L^{-1} = \{b_{jk}\} \text{ inverse inductance matrix} \]

For an \( N\)-winding transformer, contains \( N(N + 1)/2 \) independent parameters.

Extended cantilever model also contains \( N(N + 1)/2 \) independent parameters, related to the inductance matrix as follows:

\[ L_{11} = L_{11} \]
\[ n_j = \frac{L_{1j}}{L_{11}} \]
\[ l_{jk} = -\frac{1}{n_j n_k b_{jk}} \]
Measurement of Leakage Inductance Parameters

To measure leakage inductance parameter $l_{34}$

- Inject ac voltage at winding 3
- Short all other windings
- Measure current in winding 4
- $l_{34}$ is given by
  $$l_{jk} = \frac{v_j(s)}{sn_j n_k i_k(s)}$$
- Must carefully observe polarities, since $l_{jk}$ can be negative

- Measurement frequency must be sufficiently high, so that leakage reactance $>>$ winding resistance
Measurement of Effective Turns Ratios

To measure effective turns ratio $n_3$

- Inject ac voltage at winding 1
- Open-circuit all other windings
- Measure voltage in winding 3
- $n_3$ is given by

\[ n_k = \frac{v_k}{v_1} \]
The \( N \)-Port Transformer Model

- Useful in deriving expressions for current ripples and zero-ripple condition, and for computer simulation
- Primary winding is represented by its current-controlled Norton equivalent
- Each secondary is modeled by a voltage-controlled Thevenin equivalent
- Secondary winding output inductance:
  \[
  L_{ok} = n_k^2 \left( l_{1k}l_{2k}\cdots l_{(k-1)k}l_{(k+1)k}\cdots l_{Nk} \right)
  \]
- Secondary winding controlled voltage source:
  \[
  v_{Tk} = \frac{L_{ok}}{n_kl_{1k}} v_1 + \frac{L_{ok}}{n_kn_2l_{2k}} v_2 + \cdots + \frac{L_{ok}}{n_kn_{k-1}l_{(k-1)k}} v_{k-1} + \frac{L_{ok}}{n_kn_{k+1}l_{(k+1)k}} v_{k+1} + \cdots + \frac{L_{ok}}{n_kn_Nl_{Nk}} v_N
  \]
Flyback Transformer Example

Application specifications:
- Input: 30 V (winding \( W_1 \))
- Output: +12 V (winding \( W_2 \))
- Output: –12 V (winding \( W_3 \))
- Output: +3.3 V (winding \( W_4 \))

Experimental example:
Three-output flyback converter

Winding and core geometry

- \( W_1: \) 36T #18AWG
- \( W_2: \) 15T #18AWG
- \( W_3: \) 15T #22AWG
- \( W_4: \) 5T #16AWG
- EC41-3C80 core

Air gaps

Measured Model
Flyback transformer extended cantilever model
Directions of induced winding currents, when winding $W_3$ is driven and windings $W_1$, $W_2$, and $W_4$ are shorted

Negative $l_{34}$ indicates reversal of polarity of induced current $i_4$

Side-by-side winding geometry leads to negative leakage parameter
Measured $n$-port parameter model
Flyback transformer example

$N$-port parameters are computed from extended cantilever model parameters as follows:

**Winding output impedance**

\[
L_{ok} = n_k^2 \left( l_{1k} \parallel l_{2k} \parallel \cdots \parallel l_{(k-1)k} \parallel l_{(k+1)k} \parallel \cdots \parallel l_{Nk} \right)
\]

**Voltage-controlled voltage source**

\[
v_{Tk} = \frac{L_{ok}}{n_k l_{1k}} v_1 + \frac{L_{ok}}{n_k n_2 l_{2k}} v_2 + \cdots + \frac{L_{ok}}{n_k n_{(k-1)k} l_{(k-1)k}} v_{k-1} + \frac{L_{ok}}{n_k n_{(k+1)k} l_{(k+1)k}} v_{k+1} + \cdots + \frac{L_{ok}}{n_k n_{Nk} l_{Nk}} v_N
\]

Alternatively, these parameters could be directly measured
Flyback converter circuit

Three outputs
Primary-side voltage-clamp snubber
Cross-regulation is strongly influenced by commutation interval, when transistor turns off and magnetizing current shifts to secondary windings
Measured and predicted waveforms
Flyback converter example

Winding 2: CCM
Winding 3: DCM
Winding 4: CCM with inverted (negative) ripple
Secondary current waveforms
Commutation interval

- Magnetizing current commutes from primary winding to secondary windings
- Reflected output winding voltages are nearly equal
- Essentially zero voltage is applied across $l_{23}$, $l_{23}$, and $l_{23}$
- Large voltage is applied across $l_{12}$, $l_{13}$, and $l_{14}$
- Magnetizing current divides between secondary windings according to relative values of $l_{12}$, $l_{13}$, and $l_{14}$
Secondary Winding Current Waveforms

During commutation interval:
- Slopes of secondary winding currents depend on $l_{12}$, $l_{13}$, and $l_{14}$
- Magnetizing current divides between secondary windings according to relative values of $l_{12}$, $l_{13}$, and $l_{14}$
- At end of commutation interval, secondary winding currents depend on magnetizing current and on $l_{12}$, $l_{13}$, and $l_{14}$, but not directly on load

Remainder of diode conduction interval:
- Increased output voltage reduces slope of winding current waveform, leading to reduced average output current
- Decreased output voltage increases slope of winding current waveform, leading to increased average output current
Prediction of small-signal dynamics

The extended cantilever model can also be used to predict the converter control-to-output transfer function

- Depends on operating modes of auxiliary windings
- Significant changes observed when auxiliary winding changes from DCM to CCM

Computer modeling method is described in reference [15]

- Small-signal frequency response is generated by Mathematica, based on converter impulse responses generated by SPICE or PETS
- Approach automatically accounts for changes in operating mode
- Transformer was simulated using $N$-port model
- Simulations converged quickly and easily, even though system contained eight states
Measured and predicted transfer functions
Flyback converter example

Small-signal CCM duty-cycle to $W_4$ output transfer functions
(a) with $W_2$ and $W_3$ outputs operating in DCM
(b) with $W_2$ and $W_3$ outputs operating in CCM

Measurements
Summary

- Extended cantilever model, and $N$-port model, correctly predict observed waveforms of multiple-output converters
- These models are full-order: the number of independent parameters is the same as in the inductance matrix, and the parameters are directly related to the entries of the inverse inductance matrix
- Each model parameter can be directly measured, and the model can be checked using several other measurements
- The mechanisms of cross-regulation in flyback converters can be explained using the extended cantilever model
- Small-signal dynamics are correctly predicted
- Reduced-order models generally do not predict the observed phenomena of multiple-output converters