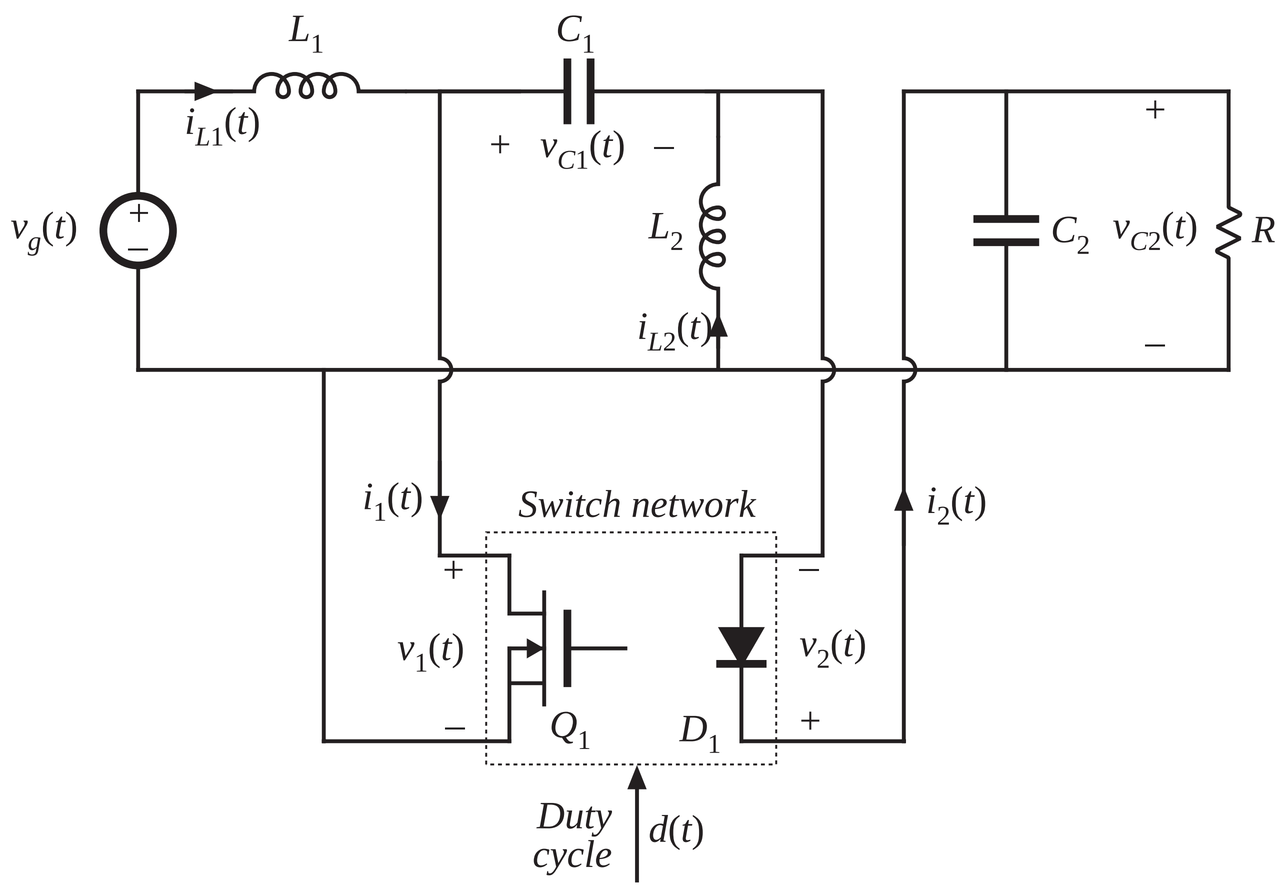
Appendix 3: Averaged switch modeling of a CCM SEPIC

SEPIC example: write circuit with switch network explicitly identified



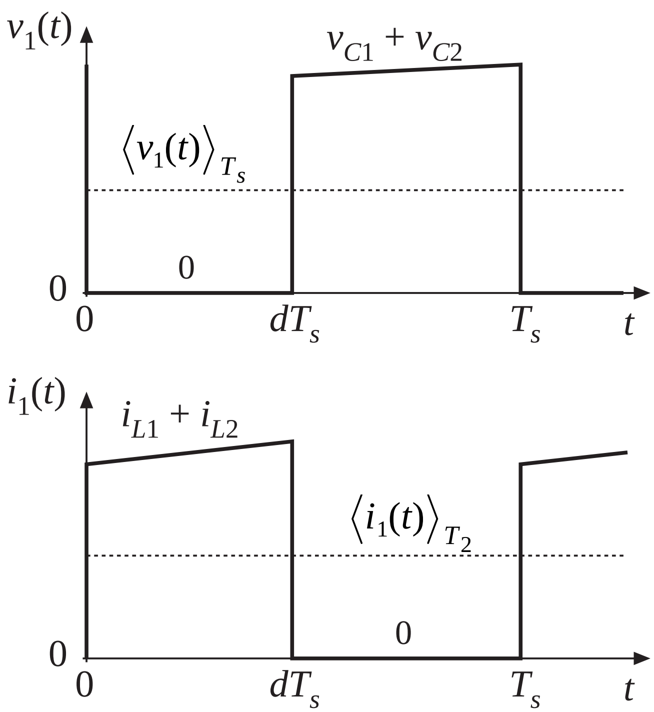
A few points regarding averaged switch modeling

- The switch network can be defined arbitrarily, as long as
  - its terminal voltages and currents are independent, and
  - the switch network contains no reactive elements.
- It is ***not*** necessary that some of the switch network terminal quantities
  - coincide with inductor currents or capacitor voltages of the converter, or
  - be nonpulsating.
- The object is simply to write the averaged equations of the switch network; i.e., to express the average values of half of the switch network terminal waveforms as functions of
  - the average values of the remaining switch network terminal waveforms, and
  - the control input.

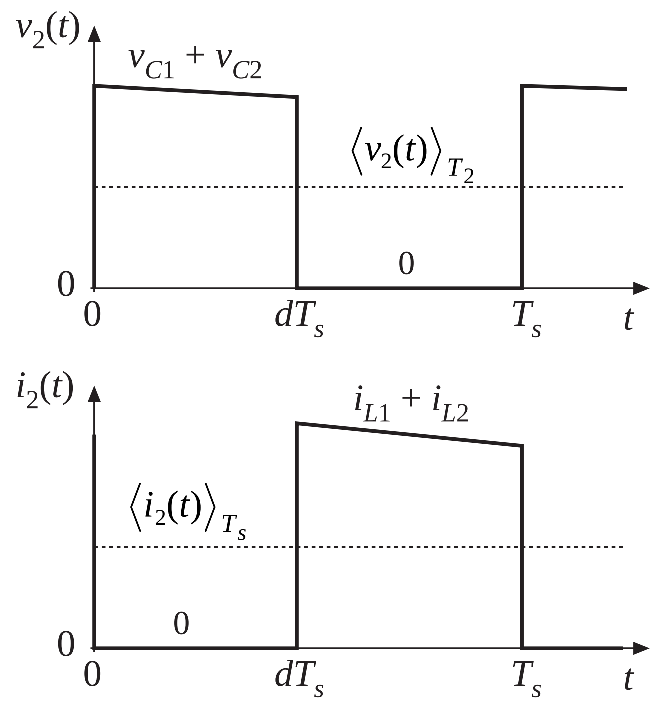
SEPIC CCM waveforms

Sketch terminal waveforms of switch network

Port 1



Port 2



Expressions for average values of switch network terminal waveforms

Use small ripple approximation

$$\left\langle v_1(t) \right\rangle_{T_s} = d'(t) \left(\left\langle v_{C1}(t) \right\rangle_{T_s} + \left\langle v_{C2}(t) \right\rangle_{T_s} \right)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left(\left\langle i_{L1}(t) \right\rangle_{T_s} + \left\langle i_{L2}(t) \right\rangle_{T_s} \right)$$

$$\left\langle v_2(t) \right\rangle_{T_s} = d(t) \left(\left\langle v_{C1}(t) \right\rangle_{T_s} + \left\langle v_{C2}(t) \right\rangle_{T_s} \right)$$

$$\left\langle i_2(t) \right\rangle_{T_s} = d'(t) \left(\left\langle i_{L1}(t) \right\rangle_{T_s} + \left\langle i_{L2}(t) \right\rangle_{T_s} \right)$$

Need next to eliminate the capacitor voltages and inductor currents from these expressions, to write the equations of the switch network.

Derivation of switch network equations (Algebra steps)

We can write

$$\left\langle i_{L1}(t)\right\rangle_{T_s} + \left\langle i_{L2}(t)\right\rangle_{T_s} = \frac{\left\langle i_1(t)\right\rangle_{T_s}}{d(t)}$$

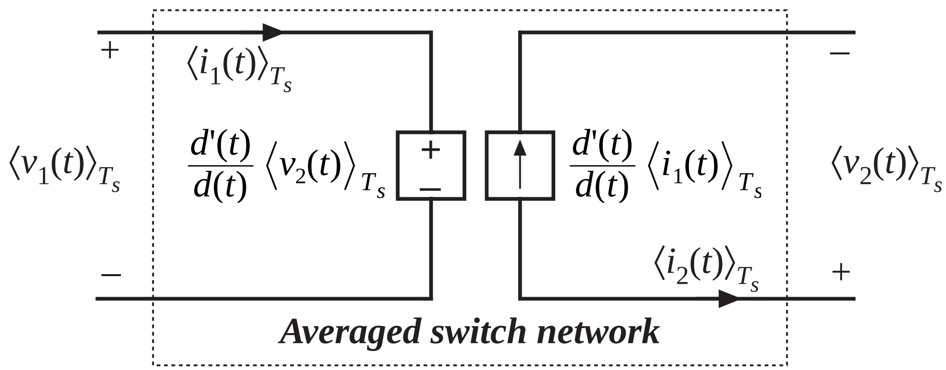
$$\left\langle v_{C1}(t)\right\rangle_{T_s} + \left\langle v_{C2}(t)\right\rangle_{T_s} = \frac{\left\langle v_2(t)\right\rangle_{T_s}}{d(t)}$$

Hence

$$\left\langle v_1(t)\right\rangle_{T_s} = \frac{d'(t)}{d(t)} \left\langle v_2(t)\right\rangle_{T_s}$$

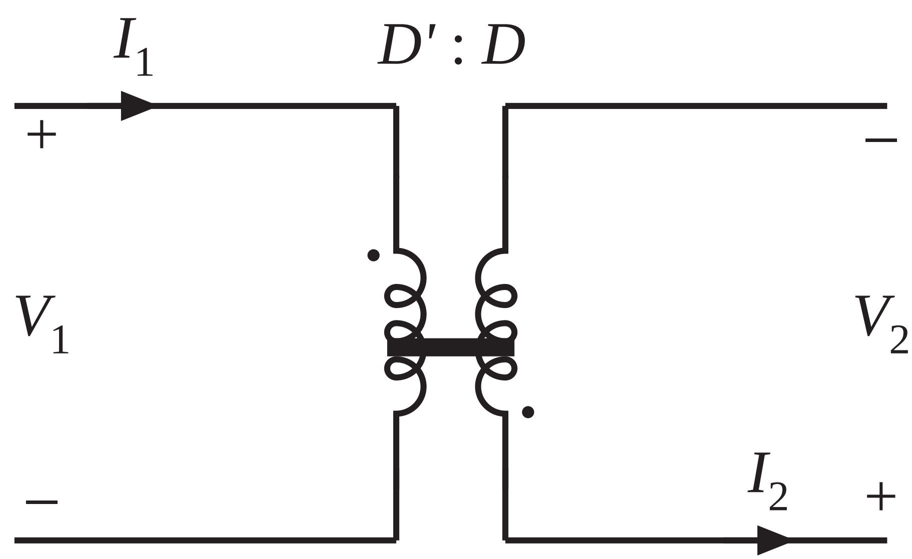
$$\left\langle i_2(t)\right\rangle_{T_s} = \frac{d'(t)}{d(t)} \left\langle i_1(t)\right\rangle_{T_s}$$

Result



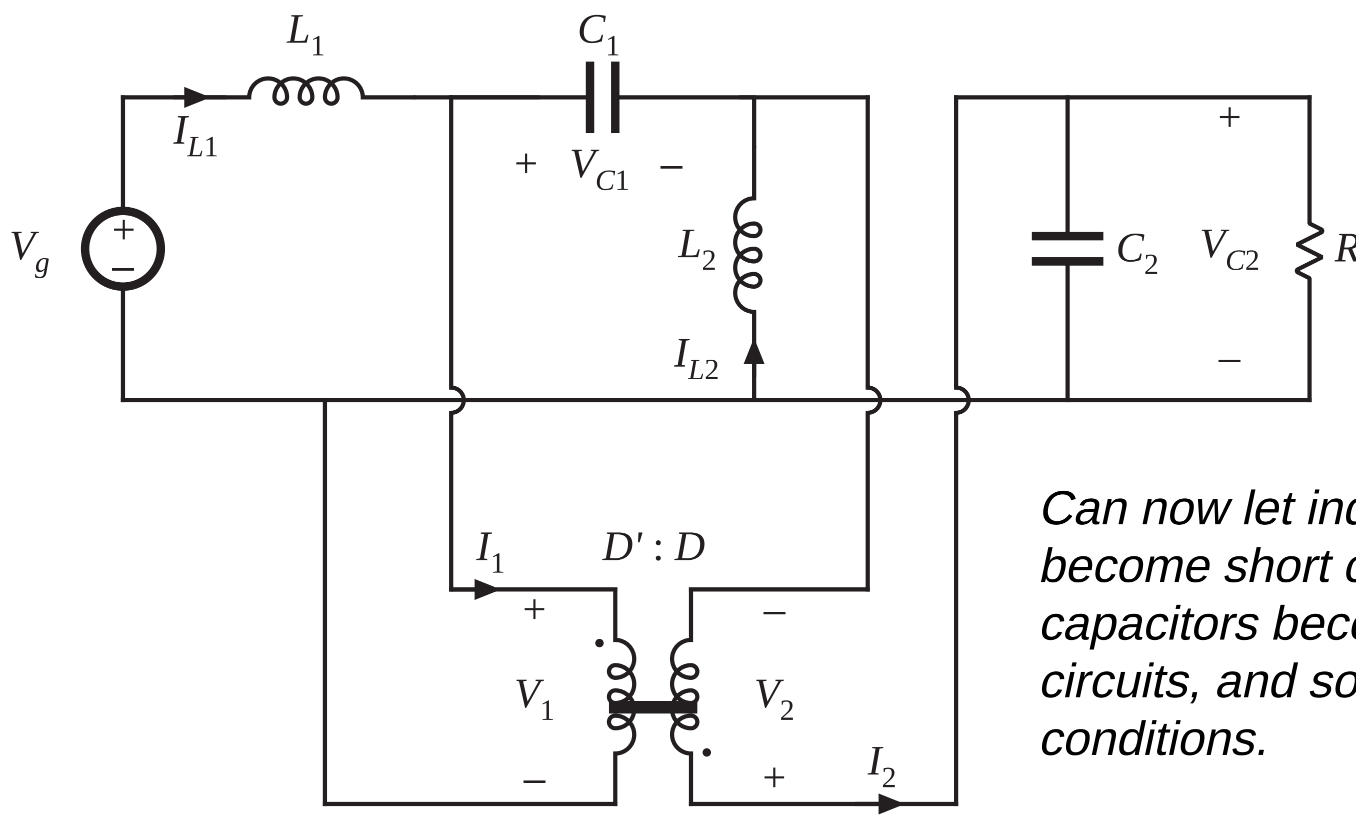
Modeling the switch network via averaged dependent sources

Steady-state switch model: Dc transformer model



Steady-state CCM SEPIC model

Replace switch network with dc transformer model



Can now let inductors become short circuits, capacitors become open circuits, and solve for dc conditions.

Small-signal model

Perturb and linearize the switch network averaged waveforms, as usual:

$$d(t) = D + \hat{d}(t)$$
$$\left\langle v_1(t) \right\rangle_{T_s} = V_1 + \hat{v}_1(t)$$
$$\left\langle i_1(t) \right\rangle_{T_s} = I_1 + \hat{i}_1(t)$$
$$\left\langle v_2(t) \right\rangle_{T_s} = V_2 + \hat{v}_2(t)$$
$$\left\langle i_2(t) \right\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

Voltage equation becomes

$$\left(D + \hat{d}\right)\left(V_1 + \hat{v}_1\right) = \left(D' - \hat{d}\right)\left(V_2 + \hat{v}_2\right)$$

Eliminate nonlinear terms and solve for v_1 terms:

$$\left(V_1 + \hat{v}_1\right) = \frac{D'}{D}\left(V_2 + \hat{v}_2\right) - \hat{d}\left(\frac{V_1 + V_2}{D}\right)$$
$$= \frac{D'}{D}\left(V_2 + \hat{v}_2\right) - \hat{d}\left(\frac{V_1}{DD'}\right)$$

Linearization, continued

Current equation becomes

$$\left(D + \hat{d}\right)\left(I_2 + \hat{i}_2\right) = \left(D' - \hat{d}\right)\left(I_1 + \hat{i}_1\right)$$

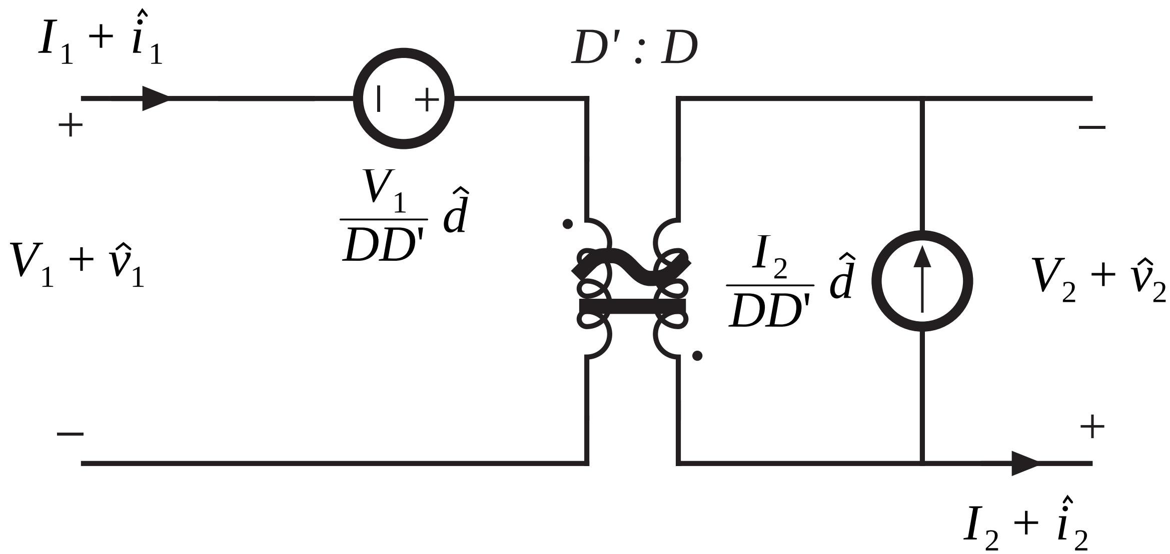
Eliminate nonlinear terms
and solve for i_2 terms:

$$\left(I_2 + \hat{i}_2\right) = \frac{D'}{D}\left(I_1 + \hat{i}_1\right) - \hat{d}\left(\frac{I_1 + I_2}{D}\right)$$

$$= \frac{D'}{D}\left(I_1 + \hat{i}_1\right) - \hat{d}\left(\frac{I_2}{DD'}\right)$$

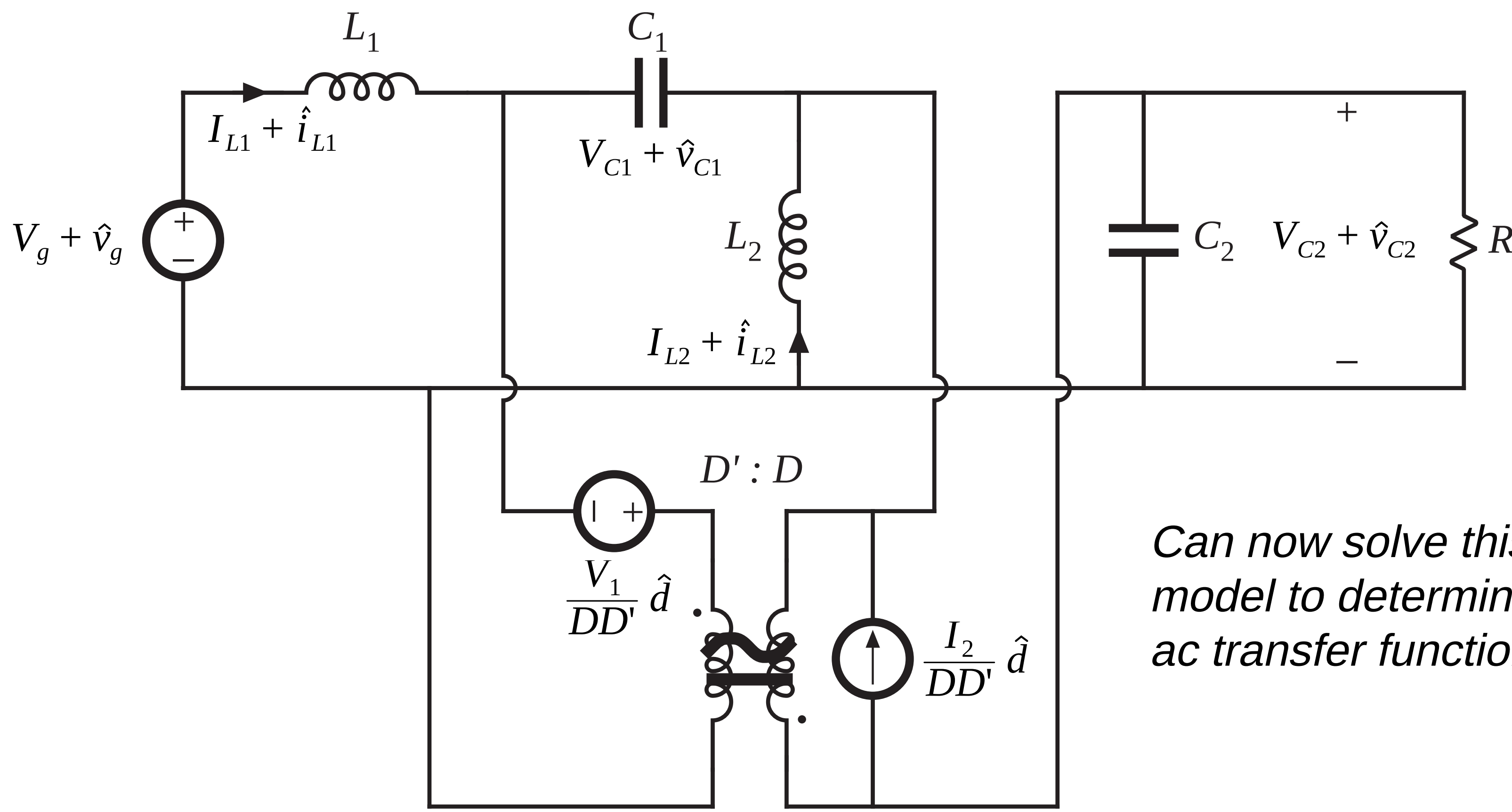
Switch network: Small-signal ac model

Reconstruct equivalent circuit in the usual manner:



Small-signal ac model of the CCM SEPIC

Replace switch network with small-signal ac model:



Can now solve this model to determine ac transfer functions