Chapter 13. Filter Inductor Design

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13.1. Several types of magnetic devices, their $B-H$ loops, and core vs. copper loss

A key design decision: the choice of maximum operating flux density $B_{\text{max}}$

- Choose $B_{\text{max}}$ to avoid saturation of core, or
- Further reduce $B_{\text{max}}$, to reduce core losses

Different design procedures are employed in the two cases.

Types of magnetic devices:
- Filter inductor
- Ac inductor
- Conventional transformer
- Coupled inductor
- Flyback transformer
- SEPIC transformer
- Magnetic amplifier
- Saturable reactor
Filter inductor

**CCM buck example**

![Diagram of a CCM buck converter with filter inductor](image)

- $i(t)$: Current
- $L$: Inductor
- $B_{sat}$: Saturation flux density
- $H_{c0}$: Initial flux density
- $\Delta H_c$: Change in flux density
- $\Delta i_L$: Change in inductor current

**B-H loop, large excitation**

- **minor B-H loop, filter inductor**
Filter inductor, cont.

- Negligible core loss, negligible proximity loss
- Loss dominated by dc copper loss
- Flux density chosen simply to avoid saturation
- Air gap is employed
- Could use core materials having high saturation flux density (and relatively high core loss), even though converter switching frequency is high
Ac inductor

\[ i(t) \]

\[ \Delta i \]

\[ B \]

\[ B_{\text{sat}} \]

\[ -\Delta H_c \]

\[ \Delta H_c \]

core B-H loop

B-H loop, for operation as ac inductor
Ac inductor, cont.

- Core loss, copper loss, proximity loss are all significant
- An air gap is employed
- Flux density is chosen to reduce core loss
- A high-frequency material (ferrite) must be employed
Conventional transformer

\[ v_1(t) \longrightarrow L_{mp} \longrightarrow v_2(t) \]

\[ i_1(t) \longrightarrow i_{mp}(t) \longrightarrow i_2(t) \]

\[ \lambda_1 \]

\[ H(t) = \frac{n_i_{mp}(t)}{l_m} \]

B-H loop, for operation as conventional transformer

Core B-H loop

\[ \frac{\lambda_1}{2n_1A_c} \]
Conventional transformer, cont.

- Core loss, copper loss, and proximity loss are usually significant
- No air gap is employed
- Flux density is chosen to reduce core loss
- A high frequency material (ferrite) must be employed
Coupled inductor

Two-output forward converter example

\[ \begin{align*}
  n_1 &+ v_1 - n_2 \\
  i_1 &+ v_2 - i_2
\end{align*} \]

\[ v_g \]

\[ I_1 \quad \Delta i_1 \quad i_1(t) \]

\[ I_2 \quad \Delta i_2 \quad i_2(t) \]

minors B-H loop, coupled inductor

\[ B-H \ loop, \ large \ excitation \]

fundamentals of power electronics

Chapter 13: Filter inductor design
Coupled inductor, cont.

- A filter inductor having multiple windings
- Air gap is employed
- Core loss and proximity loss usually not significant
- Flux density chosen to avoid saturation
- Low-frequency core material can be employed
DCM Flyback transformer

B-H loop, for operation in DCM flyback converter

core B-H loop

\[
\frac{n_1 i_{1,pk}}{L_m} \frac{R_c}{R_c + R_g}
\]
DCM flyback transformer, cont.

- Core loss, copper loss, proximity loss are significant
- Flux density is chosen to reduce core loss
- Air gap is employed
- A high-frequency core material (ferrite) must be used
13.2. Filter inductor design constraints

Objective:
Design inductor having a given inductance $L$, which carries worst-case current $I_{max}$ without saturating, and which has a given winding resistance $R$, or, equivalently, exhibits a worst-case copper loss of

$$P_{cu} = I_{rms}^2 R$$
Assumed filter inductor geometry

\[ R_c = \frac{l_c}{\mu_c A_c} \]
\[ R_g = \frac{l_g}{\mu_0 A_c} \]

Solve magnetic circuit:

\[ ni = \Phi \left( R_c + R_g \right) \]

For \( R_c \gg R_g \):

\[ ni \approx \Phi R_g \]
13.2.1. Constraint: maximum flux density

Given a peak winding current $I_{\text{max}}$, it is desired to operate the core flux density at a peak value $B_{\text{max}}$. The value of $B_{\text{max}}$ is chosen to be less than the worst-case saturation flux density of the core material.

From solution of magnetic circuit:

$$ni = BA_c \frac{R_g}{l_g}$$

Let $I = I_{\text{max}}$ and $B = B_{\text{max}}$:

$$nI_{\text{max}} = B_{\text{max}}A_c \frac{R_g}{l_g} = B_{\text{max}} \frac{l_g}{\mu_0}$$

This is constraint #1. The turns ratio $n$ and air gap length $l_g$ are unknown.
13.3.2. Constraint: Inductance

Must obtain specified inductance $L$. We know that the inductance is

$$L = \frac{n^2}{R_g} = \frac{\mu_0 A_c n^2}{l_g}$$

This is constraint #2. The turns ratio $n$, core area $A_c$, and air gap length $l_g$ are unknown.
13.3.3. Constraint: Winding area

Wire must fit through core window (i.e., hole in center of core)

Total area of copper in window:
\[ nA_w \]

Area available for winding conductors:
\[ K_u W_A \]

Third design constraint:
\[ K_u W_A \geq nA_w \]
The window utilization factor $K_u$ also called the “fill factor”

$K_u$ is the fraction of the core window area that is filled by copper.

Mechanisms that cause $K_u$ to be less than 1:

- Round wire does not pack perfectly, which reduces $K_u$ by a factor of 0.7 to 0.55 depending on winding technique.
- Insulation reduces $K_u$ by a factor of 0.95 to 0.65, depending on wire size and type of insulation.
- Bobbin uses some window area.
- Additional insulation may be required between windings.

Typical values of $K_u$:
- 0.5 for simple low-voltage inductor.
- 0.25 to 0.3 for off-line transformer.
- 0.05 to 0.2 for high-voltage transformer (multiple kV).
- 0.65 for low-voltage foil-winding inductor.
13.2.4 Winding resistance

The resistance of the winding is

\[ R = \rho \frac{l_b}{A_W} \]

where \( \rho \) is the resistivity of the conductor material, \( l_b \) is the length of the wire, and \( A_W \) is the wire bare area. The resistivity of copper at room temperature is \( 1.724 \times 10^{-6} \Omega\text{-cm} \). The length of the wire comprising an \( n \)-turn winding can be expressed as

\[ l_b = n (MLT) \]

where \( (MLT) \) is the mean-length-per-turn of the winding. The mean-length-per-turn is a function of the core geometry. The above equations can be combined to obtain the fourth constraint:

\[ R = \rho \frac{n (MLT)}{A_W} \]
13.3 The core geometrical constant $K_g$

The four constraints:

\[ nI_{\text{max}} = B_{\text{max}} \frac{l_g}{\mu_0} \]
\[ K_u W_A \geq n A_W \]
\[ L = \frac{\mu_0 A_c n^2}{l_g} \]
\[ R = \rho \frac{n (MLT)}{A_W} \]

These equations involve the quantities $A_c$, $W_A$, and $MLT$, which are functions of the core geometry,

$I_{\text{max}}$, $B_{\text{max}}$, $\mu_0$, $L$, $K_u$, $R$, and $\rho$, which are given specifications or other known quantities, and

$n$, $l_g$, and $A_W$, which are unknowns.

Eliminate the three unknowns, leading to a single equation involving the remaining quantities.
Core geometrical constant $K_g$

Elimination of $n$, $l_g$, and $A_W$ leads to

$$\frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

- Right-hand side: specifications or other known quantities
- Left-hand side: function of only core geometry

So we must choose a core whose geometry satisfies the above equation.

The core geometrical constant $K_g$ is defined as

$$K_g = \frac{A_c^2 W_A}{(MLT)}$$
Discussion

\[ K_g = \frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} \]

\( K_g \) is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where the following quantities are specified:

- Copper loss
- Maximum flux density

How specifications affect the core size:

A smaller core can be used by increasing

- \( B_{max} \) \( \Rightarrow \) use core material having higher \( B_{sat} \)
- \( R \) \( \Rightarrow \) allow more copper loss

How the core geometry affects electrical capabilities:

A larger \( K_g \) can be obtained by increase of

- \( A_c \) \( \Rightarrow \) more iron core material, or
- \( W_A \) \( \Rightarrow \) larger window and more copper
13.4 A step-by-step procedure

The following quantities are specified, using the units noted:

- Wire resistivity $\rho$ (\(\Omega\)-cm)
- Peak winding current $I_{\text{max}}$ (A)
- Inductance $L$ (H)
- Winding resistance $R$ (\(\Omega\))
- Winding fill factor $K_u$
- Core maximum flux density $B_{\text{max}}$ (T)

The core dimensions are expressed in cm:

- Core cross-sectional area $A_c$ (cm\(^2\))
- Core window area $W_A$ (cm\(^2\))
- Mean length per turn $MLT$ (cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.
Determine core size

\[ K_g \geq \frac{\rho L^2 I_{\text{max}}^2}{B_{\text{max}}^2 R K_u} \times 10^8 \text{ (cm}^5\text{)} \]

Choose a core which is large enough to satisfy this inequality (see Appendix 2 for magnetics design tables).

Note the values of \( A_c \), \( W_A \), and \( MLT \) for this core.
Determine air gap length

\[ l_g = \frac{\mu_0 L I_{max}^2}{B_{max}^2 A_c} \times 10^4 \, \text{(m)} \]

with \( A_c \) expressed in cm\(^2\). \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \).

The air gap length is given in meters.

The value expressed above is approximate, and neglects fringing flux and other nonidealities.
Core manufacturers sell gapped cores. Rather than specifying the air gap length, the equivalent quantity $A_L$ is used.

$A_L$ is equal to the inductance, in mH, obtained with a winding of 1000 turns.

When $A_L$ is specified, it is the core manufacturer’s responsibility to obtain the correct gap length.

The required $A_L$ is given by:

$$A_L = \frac{10B_{\text{max}}^2 A_c^2}{LI_{\text{max}}^2} \quad \text{(mH/1000 turns)}$$

Units:

- $A_c$: cm$^2$,
- $L$: Henries,
- $B_{\text{max}}$: Tesla.

$$L = A_L n^2 10^{-9} \quad \text{(Henries)}$$
Determine number of turns $n$

$$n = \frac{LI_{max}}{B_{max}A_c} \times 10^4$$
Evaluate wire size

\[ A_w \leq \frac{K_u W_A}{n} \quad (\text{cm}^2) \]

Select wire with bare copper area \( A_w \) less than or equal to this value. An American Wire Gauge table is included in Appendix 2.

As a check, the winding resistance can be computed:

\[ R = \frac{\rho n (MLT)}{A_w} \quad (\Omega) \]
13.5 Summary of key points

1. A variety of magnetic devices are commonly used in switching converters. These devices differ in their core flux density variations, as well as in the magnitudes of the ac winding currents. When the flux density variations are small, core loss can be neglected. Alternatively, a low-frequency material can be used, having higher saturation flux density.

2. The core geometrical constant $K_g$ is a measure of the magnetic size of a core, for applications in which copper loss is dominant. In the $K_g$ design method, flux density and total copper loss are specified.