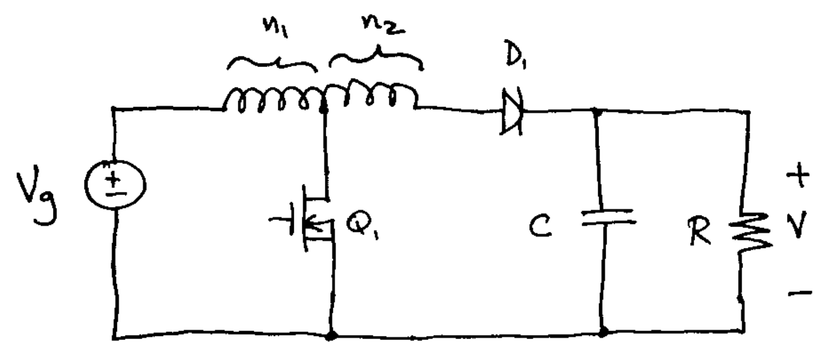
R.W. Erickson
Fundamentals of
Power Electronics

Solution to Problem 6.1

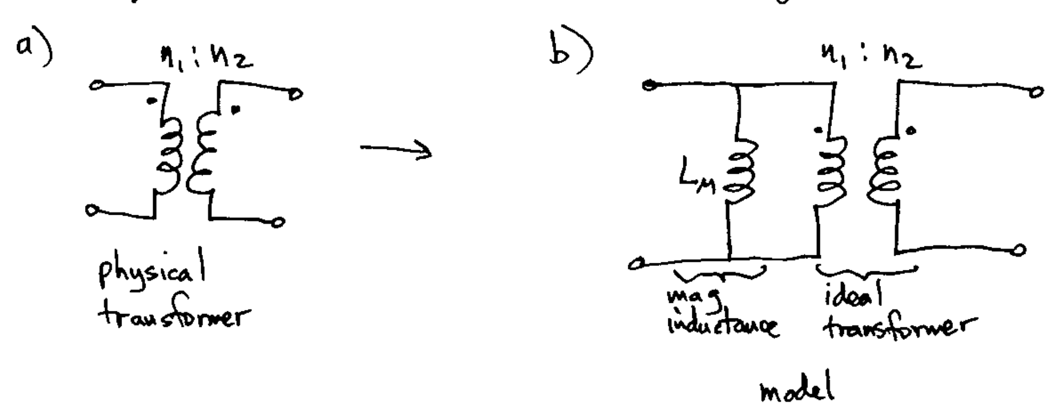
Analysis of tapped-inductor boost converter



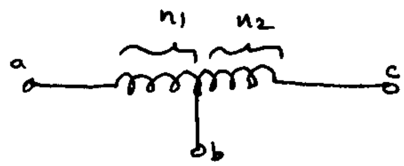
Inductance of entire (n_1+n_2) turn inductor is L

a) Sketch an equivalent circuit model for the tapped inductor, which includes an ideal transformer and a magnetizing inductance.

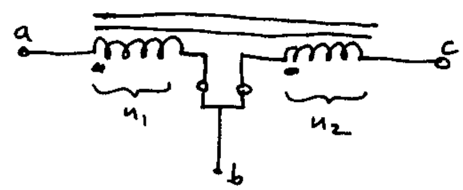
The equivalent circuit model of Fig. 6.17:

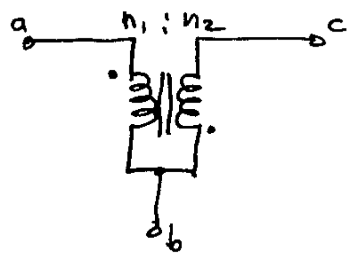


The tapped inductor can be viewed as a two-winding transformer:



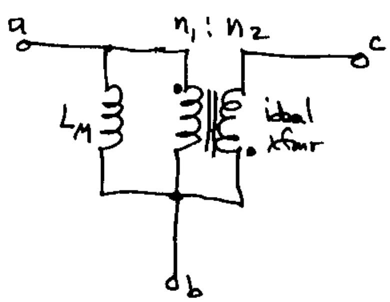
tapped inductor





two-winding transformer

Use two-winding transformer model:

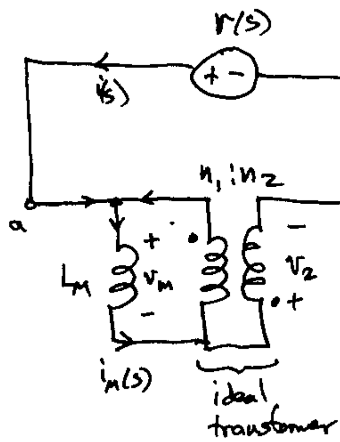


What is the value of L_M?

We are given that the inductance of the entire (n_1+n_2) turn winding is L. So we should choose the value of L_M such that the inductance between terminals a and c, with terminal b open, is L.

Measurement of inductance between terminals a and c:

inject ac voltage v, measure current i. Impedance is $Z(s) = sL = \frac{v(s)}{i(s)}$



- current in n_2 winding is $i(s)$, going into dot

$\Rightarrow$ current in n_1 winding is $\frac{n_2}{n_1} i(s)$, coming out of dot

$\Rightarrow$ node equation for magnetizing current $i_M(s)$:

$$i_M(s) = i(s) + \frac{n_2}{n_1} i(s) = \frac{n_1+n_2}{n_1} i(s)$$

$\Rightarrow$ voltage across inductor is

$$v_M(s) = sL_M i_M(s) = sL_M \frac{n_1+n_2}{n_1} i(s)$$

$\Rightarrow$ voltage across n_2 winding (positive at dot) is $v_2 = \frac{n_2}{n_1} v_M(s) = \frac{n_2(n_1+n_2)}{n_1^2} sL_M i(s)$

$\Rightarrow$ voltage $v(s)$ is

$$v(s) = v_M(s) + v_2(s) = sL_M i(s)\left(\frac{n_1+n_2}{n_1} + \frac{n_2(n_1+n_2)}{n_1^2}\right)$$

$$= s\left(\frac{n_1+n_2}{n_1}\right)^2 L_M \, i(s)$$

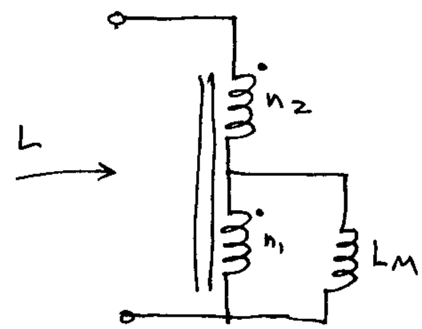
So the inductance between terminals a and c is

$$L = \left(\frac{n_1+n_2}{n_1}\right)^2 L_M$$

$$\Rightarrow \quad L_M = \left(\frac{n_1}{n_1+n_2}\right)^2 L$$

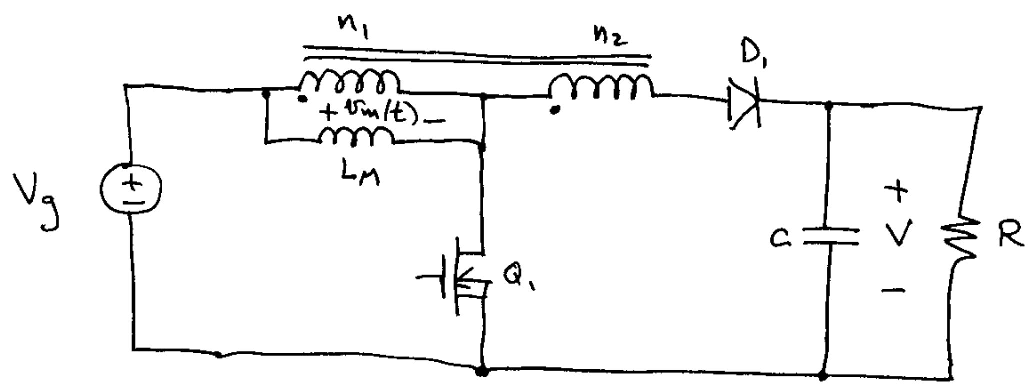
---

an easier way: reflect L_M through auto transformer



$$L = \left(\frac{n_1+n_2}{n_1}\right)^2 L_M$$

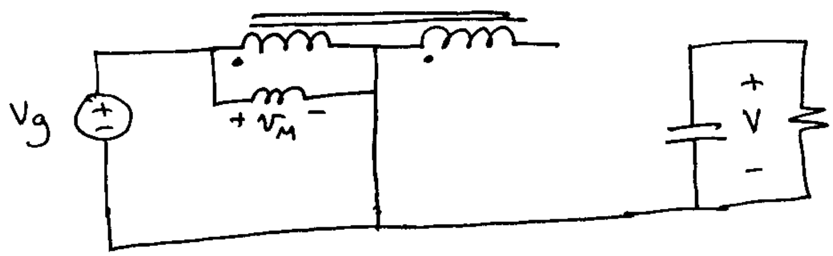
So the converter circuit becomes



b) Determine an analytical expression for $M(D) = \frac{V}{V_g}$ in CCM, no losses.

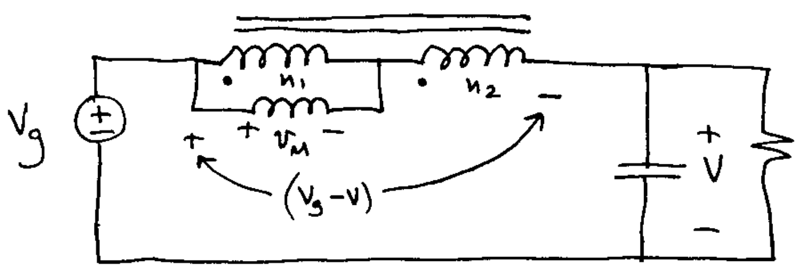
Apply volt-second balance to L_M

$0 < t < DT_s$ Q_1 on, D_1 off

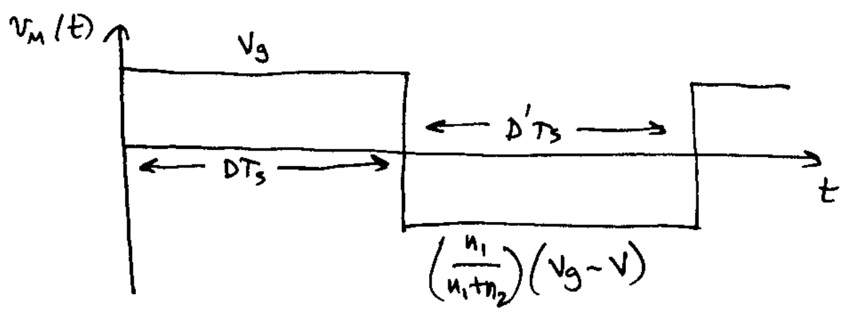


$$v_M = V_g$$

$DT_s < t < T_s$ Q_1 off, D_1 on



$$v_M = \frac{n_1}{n_1+n_2}(V_g - V)$$



$$\langle v_M \rangle = 0 = D V_g + D' \left(\frac{n_1}{n_1+n_2}\right)(V_g - V)$$

Solve for V:

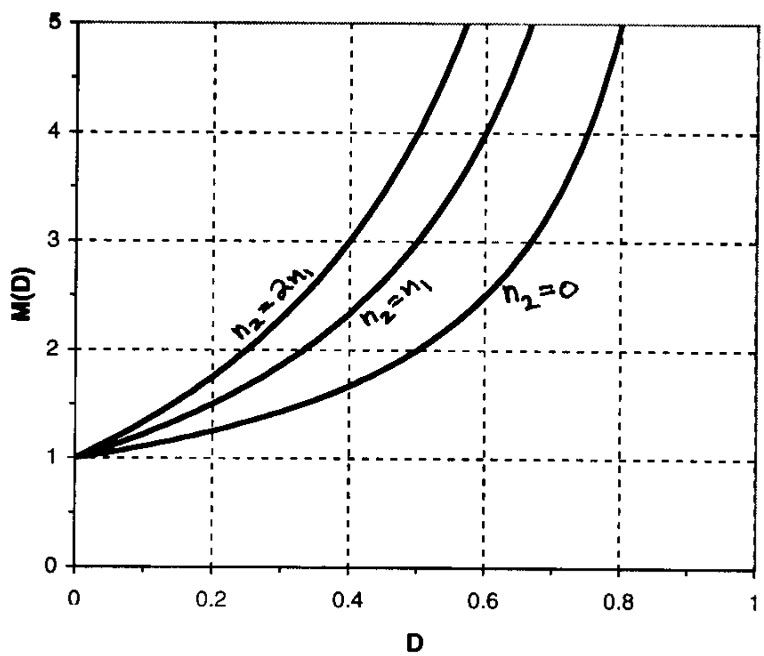
$$V\, D'\left(\frac{n_1}{n_1+n_2}\right) = D V_g + D' V_g \frac{n_1}{n_1+n_2}$$

$$\Rightarrow \frac{V}{V_g} = \frac{D + D'\left(\frac{n_1}{n_1+n_2}\right)}{D'\left(\frac{n_1}{n_1+n_2}\right)} = \frac{1}{D'}\cdot\left[D\,\frac{n_1+n_2}{n_1} + D'\right]$$

$$M(D) = \left(\frac{1}{D'}\right)\left(1+\frac{n_2}{n_1}D\right)$$

which differs from the conventional untapped ($n_2=0$) boost $M(D)$ by a factor of $\left(1+\frac{n_2}{n_1}D\right)$.

c)



Tapped inductor boost produces increased output voltage.