Direct modelling of envelope dynamics in resonant inverters

Y. Yin, R. Zane, R. Erickson and J. Glaser

A direct dynamic modelling approach is proposed for envelope signals in resonant inverters. Through decomposition of the modulated input, the small signal transfer functions for envelope signals are then derived, which link the transfer functions of envelope signals to the transfer functions of the resonant tank and simplify analysis and controller design.

Introduction: High-frequency DC-AC inverters as the LCC inverter shown in Fig. 1 are used in a variety of applications including electronic ballasts and induction heating. Input variations in both the bus voltage (amplitude modulation, AM) and the switching frequency (frequency modulation, FM) result in amplitude variations at the load. A common objective is to model the AC dynamics from input modulation to output envelopes to facilitate optimised controller design [1–3]. In this Letter, general expressions are provided to model the envelope dynamics in a resonant inverter. The final transfer functions obtained through this approach retain a strong physical relationship to the transfer function of the original resonant tank and significantly simplify the controller design.

Decomposition of modulated inputs: The spectrum of the square wave \( v_s(t) \) in Fig. 1 contains a fundamental component as well as higher-order harmonics. If the switching frequency \( f_s \) is close to the resonant frequency of the resonant tank \( f_0 \) and the tank has high Q-factor, the sinusoidal approximation can be applied [4]. Then the input to the resonant tank behaves similar to an AM or FM sinusoidal input. If the magnitude of the modulating signal is very small compared to the carrier signal, the AM and FM inputs can be decomposed into three individual inputs [5]. For example, if the bus voltage is \( V_s \) and modulating frequency is \( f_m \), then for a half-bridge switching network, the FM input can be approximately expressed as

\[
\begin{align*}
V_{s,FM}(t) &= \frac{2V_s}{\pi} \left( \cos(o_p t) + \frac{c_m}{2f_m} \cos(\omega_m t) \right) \\
&\quad - \frac{c_m}{2f_m} \cos(\omega_m t)
\end{align*}
\]

(1)

where \( \omega_m = 2\pi f_m \), \( \omega_m = 2\pi f_m \) and \( c_m \) is the modulation coefficient. Similar result can also be obtained for AM input.

Derivation of small-signal model: Based on the input decomposition, the output of the inverter (a linear network) is the summation of the network responses to the three inputs. In the following, the FM input is used as an example to derive the small-signal transfer function from frequency to output envelope. From (1), the system response in the frequency domain can be easily written as

\[
X_{out}(j\omega) = V_{s,FM}(j\omega)G(j\omega)
\]

(2)

where \( X_{out}(j\omega) \) represents the frequency response of any output, \( V_{s,FM}(j\omega) \) is Fourier transformation of \( V_{s,FM}(t) \) with \( c_m = 1 \) and \( G(j\omega) \) is the well-defined input-to-output transfer function of the resonant tank [4]. The reverse transformation of \( X_{out}(j\omega) \) leads to

\[
x_{out}(t) = \frac{2V_s}{\pi} |\omega| \cos(o_p t + \angle A)
\]

where

\[
\begin{align*}
A &= A_0 + A_1 e^{-j\omega_m t} + A_2 e^{j\omega_m t}, A_0 = G(j\omega_o) \\
A_1 &= -\frac{1}{2f_m} G(j(\omega_o - \omega_m)), A_2 = \frac{1}{2f_m} G(j(\omega_o + \omega_m))
\end{align*}
\]

(4)

Hence, the output envelope equals to \( 2V_s/|\omega| \) \( A \).

Next the small-signal portion of \( |A| \) is to be extracted. One notes that

\[
|A|^2 = A^2 + \frac{2\pi}{\omega_m} \left| A_1e^{j\omega_m t} + A_2e^{-j\omega_m t} \right| \cos(\omega_m t + \angle A_1e^{j\omega_m t} + A_2e^{-j\omega_m t})
\]

(5)

where ‘*’ represents complex conjugate. In (5), the small-signal assumptions \( |A_1|, |A_2| \ll |A_0| \) have been applied. By finding the square root of \( |A|^2 \) and linearising \( |A| \), one obtains

\[
|A| \simeq |A_0| + \frac{|A_1e^{j\omega_m t} + A_2e^{-j\omega_m t}|}{|A_0|} \cos(\omega_m t + \angle (A_1e^{j\omega_m t} + A_2e^{-j\omega_m t}))
\]

(6)

Equation (6) explicitly expresses \( |A| \) as a DC component \( |A_0| \) and an AC component. The DC component represents the steady-state response of \( x_{out}(t) \) at the switching frequency, while the AC component models small variations in the envelope caused by frequency modulation.

From (6), the transfer function of frequency-to-output envelope is obtained as

\[
G_{x_{out}}(j\omega_o) = \frac{2V_s}{\pi} \left( A_0 + A_1 e^{-j\omega_m t} + A_2 e^{j\omega_m t} \right)
\]

(7)

Following similar steps, the AM input can be solved to derive the transfer function from line to output envelope as

\[
G_{x_{out}}(j\omega_o) = \frac{2A_0e^{j\omega_m t} + A_1 e^{-j\omega_m t}}{|A_0|}
\]

(8)

with \( A_0, A_1, A_2 \) defined as

\[
A_0 = V_s G(j\omega_o), A_1 = \frac{1}{2} G(j(\omega_o - \omega_m)), A_2 = \frac{1}{2} G(j(\omega_o + \omega_m))
\]

(9)

Equations (7) and (8) explicitly relate the envelope transfer functions to the transfer function of the original tank and reveal the inherent relationship between these two transfer functions.

Discussion: In the following discussion, it is assumed that \( G(j\omega) \) is of the form:

\[
G(j\omega) = \frac{N(j\omega)}{D(j\omega)}
\]

(10)

By substituting (10) into the transfer function defined in (7) and (8), the denominator of the envelope transfer functions can be generally expressed as

\[
D_{x_{out}}(j\omega_o) = D(j\omega_o) + j\omega_o N(j\omega_o - \omega_o)
\]

(11)

From (11), it is clear that poles of (7) and (8) can be obtained by shifting the poles of the resonant tank up and down by \( j\omega_o \).
relationship between the poles of the envelope transfer functions and the poles of the resonant tank for a typical LCC inverter is shown in Fig. 2. For most applications, there is a pair of low-frequency poles as
\[ \omega_1 = \sqrt{\omega_0^2 - 2\omega_0\omega_2\sqrt{1 - 1/(4Q^2)}} + \omega_0 \geq 2\omega_0 \text{ if } Q > 1/2 \]

where \( \omega_0 \) and \( Q \) are resonant frequency and quality factor of the resonant tank. This result is the same as the one obtained in [1] and also in agreement with the approximation made in [2].

Model verification: The LCC resonant inverter as shown in Fig. 1 is studied with \( L = 539 \) \( \mu \)H, \( C_s = 4.3 \) nF, \( C_p = 3.8 \) nF, \( R = 300 \) \( \Omega \) and \( V_g = 155 \) V, and the theoretical results are verified by examining the envelope transfer function of output current \( i_{\text{out}} \). Fig. 3 compares the transfer functions from switching frequency to output current envelope. The agreement between theoretical and experimental results is quite good.

![Fig. 3 Frequency-to-output current envelope transfer function](image)

Conclusion: In this Letter, a direct approach to model the dynamics of envelope signals in resonant converters is provided to facilitate controller design. The theoretical results for the small signal transfer functions of envelope signals are derived, which reveal the inherent relationship between the dynamics of the envelopes and those of the resonant tanks. The theoretical results are verified by simulations and experiments.

Acknowledgments: This work is sponsored by General Electric Co. Global Research, through the Colorado Power Electronics Center and is cofunded by the Department of Energy’s National Energy Technology Laboratory under Cooperative Agreement DE-FC26-02NT41252.

© IEE 2004
Electronics Letters online no: 20040556
doi: 10.1049/el:20040556
Y. Yin, R. Zane and R. Erickson (Colorado Power Electronics Center, Department of Electrical and Computer Engineering, UCB 425, University of Colorado, Boulder, CO 80309-0425, USA)
E-mail: zane@colorado.edu
J. Glaser (General Electric Global Research Center, Building K1-4C22, One Research Circle, Niskayuna, NY 12309, USA)

References