Electronics Design Laboratory
Lecture #4
Experiment 2 – Robot DC Motor

Part A

• Measure DC motor characteristics
• Develop a Spice circuit model for the DC motor and determine model parameters based on experiments
• Validate the model: compare experimental and simulation results

Part B

• Design a speed sensor circuit (tachometer) that outputs voltage proportional to wheel speed
• Use LTspice simulations to verify and debug the design
• Build, test and demo the speed sensor circuit
DC Motor System

- DC voltages move the motor at some angular frequency.
- This angular frequency is translated into a frequency by a shaft encoder.

**Inputs:** differential voltage $V_{DC}$ and motor current $I_{DC}$

- $-10 < V_{DC} < +10$ V

**Outputs:** Encoder signals with 50% duty cycle, $f_{enc} \propto \omega$
Encoder output pulses, frequency $f_{enc}$ [Hz] is proportional to speed.

It is hard to measure frequency, but easy to measure voltage so we want to translate $f_{enc}$ to a proportional voltage.

**Goal:** Convert frequency to voltage
Converting Frequency to Voltage

Encoder Pulses

\[ V_{ENC} \]

“One-Shot” Output

\[ V_{OS} \]

Pulse of width \( T_{on} \) generated at each rising edge of the encoder. **Same frequency as encoder signal, but different average.**

Average Output

\[ V_{speed} \]

\[ T_{enc} = \frac{1}{f_{enc}} = \frac{1}{(K_e \omega)} \]

\[ T_{ON} \text{ (Independent of } f_{enc}!) \]

\[ \langle V_{OS} \rangle = V_{cc} \frac{T_{ON}}{T_{ENC}} = (V_{cc}T_{ON} K_e) \omega \]

\[ = K_{OS} \omega \]

\[ V_{speed} = \text{average } \langle V_{OS} \rangle = K_{OS} \omega \]
Converting Frequency to Voltage

Encoder Pulses $V_{ENC}$

"One-Shot" Output $V_{OS}$

Lower frequency means one-shot pulses further apart... **average value is lower.**

Average Output $V_{speed}$

$$V_{enc} = 1/f_{enc} = 1/(K_e \omega)$$

$$T_{ON} \text{ (Independent of } f_{enc}!)$$

$$\langle V_{OS} \rangle = V_{cc} \frac{T_{ON}}{T_{ENC}} = (V_{cc}T_{ON}K_e)\omega$$

$$= K_{OS} \omega$$

$$V_{speed} = \text{average } \langle V_{OS} \rangle = K_{OS} \omega$$
Tachometer Block Diagram

\[
v_{\text{speed}} \propto f_{\text{enc}} = K_e \omega
\]
Tachometer Circuit

Motor with Shaft Encoder

One-shot Circuit

Average Circuit

Encoder

One-shot

Average
How to approach the analysis (or design, or debugging) of a complex circuit

1. **DON’T**: dive right in and start writing a lot of loop and node equations
   - You will make an algebraic mess and get nowhere
   - *When debugging*: don’t just build the whole thing and turn it on, expecting it to work first time

2. **DO**: Break the circuit down into smaller functional blocks that can be separately understood
   - First try to explain in words how each block works
     - Isolate sections that you don’t understand. Explain the ones you do understand first.
   - Get the first block to work before moving on to the next
     - Don’t try and solve it all at once!
3. **DO**: For each block, decide what you need to know, and what analysis will be feasible

   - **Identify the input and output signals**

   - **Write simple equations**
     * Develop additional constraints based on your understanding of how the circuit is supposed to work

   - **Solve the equations** for the element values; often there is more than one valid answer
     * Chose impedance levels so that currents and power consumption are reasonable, e.g. mA not A
Tachometer Circuit Blocks

- **Trigger**
  - 555 One-shot
  - Low-pass filter

- **Motor** – Solved in Part A
- **Encoder**
- **One-Shot Circuit**
- **Average Circuit**

Diagram with components and connections labeled:
- VCC
- C7, 47μF
- C5, 1μF
- C6, .1μF
- 1N4148
- R1, R2
- LM555
- C1, C2, C3, C4
- Resistors
- Capacitors
- Diode

Solved in Part A.
555 One-Shot: Inside the “555 Timer”

**Comparator**
- Output depends on relative value of both inputs.
- Commonly used to detect signal level

\[
V_{out} = \begin{cases} 
V^+ & V_{in} > V^- \\
V^- & V_{in} \leq V^- 
\end{cases} \quad V_{CC}, \quad 0V
\]

**SR-Latch**
- Output Q dependent on “set input” S, and “reset input” R.
- Output changes on rising edge of input signal.
- Logic level 1 corresponds to a voltage of \(V_{cc}\).

<table>
<thead>
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<th>S</th>
<th>R</th>
<th>Q</th>
<th>(\bar{Q})</th>
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<td>1</td>
<td>1</td>
<td>X</td>
<td>X</td>
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**Buffer**
- Output voltage equals the input voltage.
- Buffers are used to ‘strengthen’ signals.
  The buffer is able to drive large currents.
**Inputs:** Encoder pulses with 50% duty cycle

**Outputs:** Fixed on-time pulse

Things we want to know:

- How does this circuit generate the \( t_{on} \) pulse?
- **How long is** \( t_{on} \), and how should we choose \( R_2 \) and \( C_2 \)?
One-shot timing

- $t_{on}$: must be shorter than shortest $T_{enc}$
- Design output pulse $t_{on}$ to set duty cycle at maximum frequency
- Want $t_{on}$ as long as possible, try to achieve $t_{on} = (0.9)\text{MIN}[T_{enc}]$
Tachometer Circuit Blocks

Motor – Solved in Part A

Solved

555 One-shot

Solved

Low-pass filter
The set pulse needs to terminate before it is time to reset the latch. But the datasheet for the 555 timer specifies a minimum pulse width of 1 µsec.

**Inputs:** $v_{enc}(t)$, square wave from encoder

**Outputs:** Set pulse going to latch

Things we want to know:
- How is $f$ related to the ground speed of the wheels? (left as exercise for students)
- How does this circuit generate the set pulse?
- How long is $t_{trig}$, and how should we choose $R_1$ and $C_1$?
Trigger Circuit Analysis: Preliminaries

\[ V^+ = \frac{V_{CC}}{15k\Omega} = \frac{V_{CC}}{3} \]

- \( D_1 \) acts as a switch, creating two equivalent circuits depending on \( V_{ENC} + V_{C1} \):
  - \( V_{ENC} + V_{C1} < 5V \)
  - \( V_{ENC} + V_{C1} \geq 5V \)

- Equivalent circuit ‘B’ is solved visually. All nodes have known voltages!
- Equivalent circuit A is unknown.
  - **Inputs**: \( V_{ENC} \)
  - **Outputs**: \( V_{TRIG} \)

Characteristics of silicon \( p-n \) diode
Trigger Circuit: Waveforms

\[ v_{enc}(t) \]

\[ V_{CC} \]

\[ f = 1/T \]

\[ t_{trig} \]

\[ V_{CC}/3 \]

\[ V_{CC} \]

\[ 0 \]

\[ v_{C1}(t) \]

\[ V_{CC}/3 \]

\[ V_{CC} \]

\[ 0 \]

\[ D_1 \text{ does not allow } TRIG > V_{CC} \]

so \[ v_{C1} = 0 \]
Trigger Circuit: Solution

• Assume $t = 0$ at the falling edge of $V_{enc}$
• Redraw the equivalent circuit in the time domain...

• Or with the Laplace transform...

• Then solve for capacitor voltage $V_{c1}$
• $t_{trig}$ is the time at which $V_{c1} = V^+$

• Using the Laplace transform...

$$v_{c1}(s) = \frac{V_{CC} \left(1 / sC_1\right)}{R + 1 / sC_1} = \frac{V_{CC} 1}{s \left(1 + sR_1C_1\right)}$$

• Now use partial fraction expansion to take inverse Laplace transform; the result is:

$$v_{c1}(t) = V_{CC} \left(1 - e^{-t_{trig}/R_1C_1}\right)$$

• The trigger circuit comparator causes the set pulse to end when $v_{c1} = V_{CC}/3$, at time $t = t_{trig}$. Hence:

$$v_{c1}(t_{trig}) = \frac{V_{CC}}{3} = V_{CC} \left(1 - e^{-t_{trig}/R_1C_1}\right)$$

• Solving for $t_{trig}$

$$V_{CC} \left(1 - e^{-t_{trig}/R_1C_1}\right) = \frac{V_{CC}}{3} \quad t_{trig} = R_1C_1 \ln\left(\frac{3}{2}\right)$$
Tachometer Circuit Blocks

Motor – Solved in Part A

555 One-shot

Low-pass filter

Trigger

Solved

Encoder

One-Shot Circuit

Average Circuit

Solved

Electronics Design Laboratory
**Low-Pass Filter Circuit**

**Input:** Pulse-width-modulated signal $OUT$

**Output:** “speed” signal having a DC value proportional to $f$

Things we want to know:
- How does this circuit operate on the pulse-width-modulated OUT signal to produce the speed signal?
- How should we choose $R_3$ and $C_3$?
Low-Pass Filter Circuit: Analysis

The pulse-width-modulated signal \( OUT(t) \) can be represented by Fourier analysis as a DC component \( V_0 \) plus a sum of sinusoids called harmonics:

\[
OUT(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n2\pi ft + \theta_n)
\]

The amplitude spectrum is a plot of the harmonic amplitudes vs. frequency:

The harmonics have frequencies that are integral multiples of the fundamental frequency \( f \). The DC component is given by the average value:

\[
V_0 = \frac{1}{T} \int_0^T OUT(t) \, dt
\]

We want to attenuate the harmonics (frequencies above DC) and leave the DC component untouched.
Filter design

The effect of the $R_3-C_3$ filter on each individual harmonic can be found by phasor analysis of the circuit: use phasors to solve the circuit and find how the amplitude of a sinusoid is changed by the circuit, as a function of frequency.

We want to choose $R_3$ and $C_3$ so that the filter passes the DC component and any very low-frequency variations that occur as a result of the changing speed of the robot. But we want the filter to reject the components of $\text{OUT}$ at the fundamental frequency $f$ and its harmonics. So the filter should have a transfer function (i.e., the ratio of its output voltage amplitude to its input voltage amplitude, vs. frequency) that looks like this:

![Filter transfer function diagram](image)

Use phasor analysis to solve for the transfer function of the $R_3-C_3$ filter. Select appropriate values for $R_3$ and $C_3$. 

Phasor Analysis of LPF

Voltage divider with impedances –
- Replacing capacitor by its impedance, 1/(jωC)
Solve for the ratio of phasors Vout/Vin

\[
\frac{V_{out}}{V_{in}} = \frac{1}{j\omega C} \frac{1}{(R + \frac{1}{j\omega C})}
\]

Multiplying by jωC/jωC leads to

\[
\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}
\]

- As frequency ω increases, jωRC increases
- As denominator becomes greater, Vout/Vin becomes smaller
- Therefore, higher frequency signal voltage components are attenuated
- Another way to look at this—reactance \(X_c = \frac{1}{j\omega C}\) approaches zero with higher frequencies which appears as a direct short with all voltage across R and none at Vout
Frequency Response of 1KHz LPF

Bode plot: Magnitude of phasor ratio $\frac{V_{out}}{V_{in}}$
Plot log of frequency on x axis
Plot log of magnitude in decibels “dB” on y axis

$$\left| \frac{V_{out}}{V_{in}} \right| = 20 \log \left| \frac{V_{out}}{V_{in}} \right|$$

-3 dB down point
@ $f_c = 1$kHz
corner frequency
Filter Design with $f_c$

Corner frequency, $f_c$ -
- 3dB (1/2 power point) occurs
  Frequencies equal to and greater than corner frequency are attenuated

Corner frequency is defined as:

$$f_c = \frac{1}{(2\pi RC)}$$

Design low pass filter for Lab 2 -
- Decide what frequencies to preserve and set corner frequency to just above the limit.
- Remember, the lower the corner frequency, the slower the system response time
- Pick an available capacitor value for $C_3$ and calculate the necessary $R_3$ from the corner frequency equation above.
- Pick the closest available resistor value to calculated value.
Summary of Time Constants in the Tachometer circuit

- **R1, C1**
  - $1\mu s < t_{\text{trig}} << t_{\text{on}}$
  - $C_1 >>$ capacitance at node TRIG
  - $R_1 >> R_{\text{encoder}}$

- **R2, C2**
  - Set $t_{\text{on}}$ to determine output voltage of speed sensor at maximum speed (based on VCC and duty cycle)

- **R3, C3**
  - Low-pass filter PWM output; determines voltage ripple on the speed sensor output voltage
  - $R3 \times C3 >>$ lowest expected PWM period
  - $R3 \times C3 <$ desired response time of the speed sensor (e.g. $<< 1$ sec)