Bandpass filter analysis

The circuit given in Exp. 5:

(see also Thomas and Rosa section on bandpass filter design)

Analysis

1. The $R_4 - R_5 - C_4 - V_{cc}$ circuitry provides a dc bias that keeps the op amp input terminals within their common mode range of zero to $V_{cc}$. The voltage at the op amp positive input is purely dc, $V_{cc}$, with no ac variation.

2. There are two sources, $v_1$ and $V_{cc}$, and one output, $v_2$. So $v_2$ is a superposition of terms coming from $v_1$ and $V_{cc}$. We want to find the transfer function.
from \( v_1 \) to \( v_2 \)

\[ G(s) = \frac{v_2(s)}{v_1(s)} \]

Set \( v_{cc} \) to zero to analyze, and solve for \( G(s) \). This makes the voltage at the op amp + terminal equal to zero, and the circuit becomes

\[ \frac{v_1(s)}{R_1} - \frac{1}{sC_1} - R_3 = 0 \]

Now solve:

Thevenin equivalent:

\[ \frac{v_1(s)}{R_1} \quad \Rightarrow \quad \frac{R_2}{R_1 + R_2} v_1(s) \]

Circuit becomes:

\[ \frac{R_2}{R_1 + R_2} \quad \pm \quad \frac{1}{sC_1} \quad \pm \quad R_3 \]
- The experiment chooses \( C_1 = C_2 = C \)

- Virtual ground: \( V^- = 0 \)

- Use node analysis to eliminate \( V_a(s) \) and find \( V_2(s) \)

\[
\frac{R_2}{R_1 + R_2} \cdot V_1 \quad \text{at} \quad \text{node}\]

\[
i_1 = \frac{R_2}{R_1 + R_2} \cdot \frac{V_1}{R_1 || R_2} = \frac{V_a}{R_1 || R_2}
\]

\[
i_3 = \frac{V_a}{1/SC}
\]

\[
i_2 = \frac{V_a - V_2}{1/SC}
\]

\[
V_2 = -i_3R_3 = -V_a < R_3C
\]

**Algebra**

\[
i_1 = \frac{1}{R_1 || R_2} \left( \frac{R_2}{R_1 + R_2} \cdot V_1 - V_a \right) = i_2 + i_3 = SC \left( V_a + sR_3C \right) + SC \frac{V_a}{1/SC}
\]

\[
\Rightarrow \frac{1}{R_1 || R_2} \cdot \frac{R_2}{R_1 + R_2} \cdot V_1 = V_a \left[ \frac{1}{R_1 || R_2} + 2SC + s^2C^2R_3 \right]
\]
implies\[ \alpha = \frac{R_2}{R_1 + R_2} \alpha, \quad \frac{1}{(1 + 2sc \cdot R_1 \| R_2 + s^2 c^2 R_3 \cdot R_1 \| R_2)} \]

and\[ v_2 = -v_\alpha s R_3 c \]

\[ = -\alpha \cdot \frac{R_2}{R_1 + R_2} \cdot \frac{s R_3 c}{(1 + 2sc \cdot R_1 \| R_2 + s^2 c^2 R_3 \cdot R_1 \| R_2)} \]

which leads to\[ \frac{v_2(s)}{v_1(s)} = G(s) = -\frac{R_2}{R_1 + R_2} \frac{s R_3 c}{(1 + 2sc \cdot R_1 \| R_2 + s^2 c^2 R_3 \cdot R_1 \| R_2)} \]

This transfer function is of the form

\[ G(s) = -G_0 \frac{\frac{s}{\omega_0^2}}{\left(1 + \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)} \]

with\[ G_0 = \frac{R_2}{R_1 + R_2} \]

\[ \frac{1}{\omega_0^2} = c^2 R_3 \cdot R_1 \| R_2 = \omega_0^2 = \frac{1}{c \sqrt{R_3 \cdot R_1 \| R_2}} \]

\[ \frac{1}{\omega_0} = 2c \cdot R_1 \| R_2 \Rightarrow \omega_0 = \frac{1}{2 \sqrt{R_3 \cdot R_1 \| R_2}} \]
Bode plot - magnitude

\[ |G(j\omega)| = \left| \frac{R_2}{R_1 + R_2} \right| \]

constant

\[ \left\| \frac{s}{\omega s^2 + (\omega_0)^2} \right\| = j\omega \] (possibly complex) poles

\[ \left\| \frac{s}{\omega s + (\omega_0)^2} \right\| = j\omega \] zeros at origin

\[ \left\| \frac{s}{\omega s + (\omega_0)^2} \right\| = 1 \] 

\[ \omega_0 \]

\[ +20 \text{dB/decade} \]

\[ \frac{R_2}{R_1 + R_2} \]

\[ -40 \text{dB/dec} \]

\[ \omega \]

\[ \omega_0 \]

\[ -20 \text{dB/dec} \]

\[ \omega \]

\[ \omega_0 \]

\[ -20 \text{dB/dec} \]

\[ \omega_0 \]
The low frequency asymptote \((\omega \ll \omega_0)\) is
\[
\left| -\frac{R_2}{R_1+R_2} \right| \cdot \left| \frac{\frac{i\omega}{w_2}}{1 + \frac{s}{\omega_0} + \left(\frac{s}{w_2}\right)^2} \right| \\
\leq 1
\]
\[
= \frac{R_2}{R_1+R_2} \cdot \frac{\omega}{\omega_2}
\]

The value of \(\|G\|\) at \(\omega = \omega_0\) is
\[
G_{pk} = Q \cdot (\text{value of low frequency asymptote at } \omega = \omega_0)
\]
\[
= Q \cdot \frac{R_2}{R_1+R_2} \cdot \frac{\omega_0}{\omega_2}
\]

Plug in for \(Q, \omega_0, \omega_2\) to find
\[
G_{pk} = \frac{1}{2} \cdot \frac{R_3}{R_1}
\]

We will choose \(R_1\) to make \(G_{pk} = 1\)