ECEN 3300
Linear Systems
Class Meeting 14
Causal Linear Time Invariant Systems II
Today’s Topics: Causal LTI Systems

• Step Response of Causal LTI’s
• Causal differential equations
• Example 2.14
Unit Step Response of a Discrete LTI?

When \( x[n] = u[n] \)

Then \( y[n] = s[n] = u[n] \ast h[n] \)

- Find \( s[n] \)?
- Can \( s[n] \) be inverted to find \( h[n] \)?
Step Response of a Continuous LTI?

When \( x(t) = u(t) \)
Then \( y(t) = s(t) = u(t) \ast h(t) \)

- What is \( s(t) \)?
- Can \( s(t) \) be inverted to find \( h(t) \)?
Signals and Noise

\[ y(t) = s(t) + n(t) \]

\[ s(t) = s_0 \cos \omega_s t \]

\[ n(t) = n_0 \cos(\omega_n t + \phi(t)) \]

\[ n_0 \ll s_0 \quad \omega_s \ll \omega_n \]

• What does this look like?
• What happens if we differentiate?
Causality in Discrete LTI

If \( x[n] = 0 \) \( n < 0 \)
Then \( y[n] = 0 \) \( n < 0 \)

• What condition does this pose on \( h[n] \)?
Causality in Continuous LTI

If \( x(t) = 0 \) \( \quad t < 0 \)

Then \( y(t) = 0 \) \( \quad t < 0 \)

- What condition does this pose on \( h(t) \)?
LTI Differential Equations

\[
\frac{dy(t)}{dt} + 2y(t) = x(t)
\]

• What makes it TI?
• Does the equation look familiar?
• Can this differential equation be inverted?
• How do we solve a differential equation in general? When is solution causal.
Example 2.14

\[
\frac{dy(t)}{dt} + 2y(t) = K \exp(3t)u(t)
\]

- Find the homogeneous solution?
- Find a particular solution?
- Construct the general solution?
Causal LTI Continuous DE: General

\[ x(t) \overset{\text{Continuous-time system}}{\rightarrow} y(t) \]

\[ \sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \]

\[ y(t_0) = \frac{dy(t_0)}{dt} = \ldots \frac{d^{N-1}y(t_0)}{dt^{N-1}} = 0 \]

- Why is this causal? What is the general solution?