ECEN 3300
Linear Systems
Class Meeting 17

First Midterm
Today’s Topics: Singularity Functions

- Singularity Functions
- Representations of Delta Functions
- Driving with Finite Impulses
- Delta Identities
- Derivatives of Delta Functions
Singularity Functions

\[ \delta(t) = \delta(t) \ast \delta(t) \]

\[ f(t)\delta(t) = f(0) \ast \delta(t) \]

\[ \frac{d^2y(t)}{dt^2} = y(t) \ast u_1(t) \ast u_1(t) \]

• Singularity functions arise from using idealizations such as steps and, especially, impulses
Sifting for Identities

\[ x(t) = x(t) * \delta(t) \]

- **When**

\[ x(t) = \delta(t) \]

\[ \delta(t) = \delta(t) * \delta(t) \]

- **Can this be right?**
Representations for Delta

\[ \delta(t) = \lim_{\Delta \to 0} \delta_{\Delta}(t) \]

\[ \delta_{\Delta}(t) = \frac{1}{\Delta} (u(t) - u(t - \tau)) \]

\[ r_{\Delta}(t) = \delta_{\Delta}(t) \ast \delta_{\Delta}(t) \]

- What is the limit of this?
- What does it look like?
Representations for Delta II

• What is the area under the curve?
• What is the height and where?
Representations for Delta III

\[ \delta(t) = \sum_{k=0}^{\infty} a_k \delta(t) \ast_k \delta(t) \]

\[ = a_0 \delta(t) + a_1 \delta(t) \ast \delta(t) + a_2 \delta(t) \ast \delta(t) \ast \delta(t) + \ldots \]

- Can you think of some functions that reduce to delta(t)?
- Why would it be useful to know some?
Pulsed Drivers: Example 2.16

\[
\frac{dy(t)}{dt} + 2y(t) = x(t)
\]

\[
x(t) = \begin{cases} 
\delta_\Delta(t) \\
r_\Delta(t) \\
\delta_\Delta(t) * r_\Delta(t) \\
r_\Delta(t) * r_\Delta(t)
\end{cases}
\]

How to find these? What do they look like?
Example 2.16 Pictures

Do these look familiar?
Example 2.16
more Pictures

Do these look familiar?
Defining Delta without Infinity

\[ x(t) = x(t) \ast \delta(t) \]

\[ \text{If } \quad x(t) = 1 \]

\[ \text{Then } \quad \int_{-\infty}^{\infty} \delta(\tau) d\tau = 1 \]

Other properties of generalized functions follow from the sifting property
Using Delta to Sample

\[ x(t) = x(t) \ast \delta(t) \]

If \[ x(t) = g(-t) \]

Then \[ \int_{-\infty}^{\infty} g(\tau) \delta(\tau) d\tau = g(0) \]

Other properties of generalized functions follow from the sifting property
Using Delta to Fix Argument

From

\[ x(0) = \int_{-\infty}^{\infty} x(\tau) \delta(\tau) d\tau \]

Can show

\[ f(t) \delta(t) = f(0) \delta(t) \]

How?
Derivatives of Delta Functions

What happens then when

\[ y(t) = x(t) \ast h(t) \]

What circuit realizes this?
Derivatives of Delta Functions II

\[ \frac{dx(t)}{dt} = x(t) \ast h(t) \]

with solution

\[ h(t) = u_1(t) \]

• Can we measure this \( h(t) \)?
• What does it look like?
A Picture of $u_1$

- Are there more?
Higher Derivatives

\[ y(t) = x(t) \ast u_1(t) \ast u_1(t) \]

with alternate representation

\[ h(t) = u_1(t) \ast u_1(t) = u_2(t) \]

- Can we measure this \( h(t) \)?
- What does it look like?
Still Higher Derivatives

What are

\[ y(t) = x(t) \ast u_n(t) \]

\[ y(t) = \sum_{k=0}^{\infty} a_k x(t) \ast u_k(t) \]

Can we generate anything we want?
Negative Derivatives

\[ y(t) = x(t) \ast h(t) \]

What happens then when

\[ y(t) = \int_{0}^{t} x(\tau) \, d\tau \]

What circuit realizes this?
Negative Derivatives II

\[
\int_{0}^{t} x(\tau) d\tau = x(t) * h(t)
\]

with solution

\[ h(t) = u(t) = u_{-1}(t) \]

- Can we measure this \( h(t) \)?
- What does it look like?
More Negative

\[ y(t) = x(t) \ast u_{-1}(t) \ast u_{-1}(t) \]

with alternate representation

\[ h(t) = u_{-1}(t) \ast u_{-1}(t) = u_{-2}(t) \]

• What does this look like?
Picturing an Integral Operator

- Can we define $u_{-n}(t)$?
- Can we draw $u_{-n}(t)$?
Still More Negative

What are?

\[ y(t) = x(t) \ast u_{-n}(t) \]

\[ y(t) = u_k(t) \ast u_r(t) \]