ECEN 3300
Linear Systems
Class Meeting 18

Introduction to Fourier Series
Today’s Topics: Fourier Series

- Some History
- Response of LTI System to Complex Exponentials
- Fourier Series Representations of Continuous Time Signals
• What does the above have to do with Fourier Series?
• What is the function is depicted above?
Modes of a Vibrating String

- What is the wave equation?
- How do we solve the wave equation?
- How to apply boundary conditions?
Jean Baptiste Joseph Fourier

- Followed Bernoulli (1753), Euler, Lagrange (1759)
- Applied “Fourier” series to heat conduction (1807)
An oversimplified picture of the phenomena of rogue waves

Waves greater than 100 feet do exist in deep water

https://www.youtube.com/watch?v=sCxr_XzyGO8
Basis Functions: Chapter 2 - Delta

\[ x[n] = \sum_{-\infty}^{\infty} x[k] \delta[n - k] \]

\[ x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) \, d\tau \]

- Can represent functions in a delta basis
- Functions in delta basis are a series of points
Basis Functions: Chapter 3 – z and s

\[ x[n] = \sum_{-\infty}^{\infty} a_k z_k^n \]

\[ x(t) = \sum_{-\infty}^{\infty} a_k \exp(s_k t) \]

- Can represent functions in other than delta
- Functions in other bases have coefficients and rules how to calculate the coefficients
Response of LTI Systems to Sinusoids and Powers

When \( x(t) \rightarrow \exp(st) \)
Then \( y(t) \rightarrow H(s)\exp(st) \)

When \( x[n] \rightarrow z^n \)
Then \( y[n] \rightarrow H(z)z^n \)

- Eigenfunctions elicit simple responses
- Complex exponentials are eigenfunctions of LTI systems
Continuous Time $H(s)$

\[ y(t) = h(t) \ast x(t) \]

\[ y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau \]

- What happens when $x(t)$ is a complex exponential?

\[ x(t) = \exp(st) \]
Finding $H(s)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \exp(s(t - \tau))d\tau$$

$$y(t) = \exp(st) \int_{-\infty}^{\infty} h(\tau) \exp(-s\tau)d\tau$$

• $H(s)$ is then

$$H(s) = \int_{-\infty}^{\infty} h(\tau) \exp(-s\tau)d\tau$$
An Example

\[ x(t) = \sum_{k=1}^{3} a_k \exp(s_k t) \]

\[ = a_1 \exp(s_1 t) + a_2 \exp(s_2 t) + a_3 \exp(s_3 t) \]

• Complex exponentials are eigenfunctions of LTI systems
• Eigenfunctions elicit simple systems responses – what is the response here?
An Example: Result

\[ y(t) = \sum_{k=1}^{3} a_k H(s_k) \exp(s_k t) \]

\[ = a_1 H(s_1) \exp(s_1 t) + a_2 H(s_2) \exp(s_2 t) + a_3 H(s_3) \exp(s_3 t) \]

• Can simply write down the system response in terms of the response function \( H(s) \)