Fourier Series of Discrete Signals
Today’s Topics: Properties of FS

• Fourier Series in General
• Properties of Discrete Fourier Series
  – Linearity through Parseval (15 properties)
• Example 3.13 through 3.15
• Fourier Series and LTI Systems
Continuous Fourier Series

\[ x(t) = \sum_{k=-\infty}^{\infty} a_k \exp(\jmath k\omega_0 t) \]

\[ a_k = \frac{1}{T} \int_{0}^{T} x(t) \exp(-\jmath k\omega_0 t) \, dt \]

\[ \omega_0 = \frac{2\pi}{T} \]

• Definition of Fourier Series Coefficients
Discrete Fourier Series

\[ x[n] = \sum_{k=\langle N \rangle} a_k \exp(jk\omega_0 n) \]

\[ a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \exp(-jk\omega_0 n) \]

\[ \omega_0 = \frac{2\pi}{N} \]

- Definition of Discrete Fourier Series
Properties of Discrete Fourier Series

- Linearity
- Time Shifting
- Frequency Shifting
- Conjugation
- Time Reversal
- Time Scaling
- Convolution
- Multiplication
- First Difference
- Running Sum
- Conjugate Symmetry
- Real and Even
- Real and Odd
- Even Odd
- Decomposition
- Parseval
Example 3.13

- Find the Fourier Series for

- How to approach?
Example 3.13 • Can write

- Recall examples 3.12 and 3.8
Example 3.12

• How to find the a’s of this expansion of x[n]?
Example 3.12

- How to find the a’s of this expansion of x[n]?

\[ a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \exp(-jk\omega_0 n) \]
Example 3.12

\[ a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} \exp(-jk\omega_0 n) \]

• Can we simplify?
Example 3.12

\[ a_k = \begin{cases} 
\frac{1}{N} \frac{\sin[2\pi k(N_1 + 1/2)/N]}{\sin \pi k/N} \\
\frac{2N_1 + 1}{N} 
\end{cases} \]

\(k \neq 0, \pm N, \ldots\)

\(k = 0, \pm N, \ldots\)
Example 3.8

\[ x(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT) \]

- The a are given by

\[ a_k = \frac{1}{T} \int_{0}^{T} x(t) \exp(-jk\omega_0 t) \, dt = \frac{1}{T} \]
Example 3.8

\[ x[n] = \sum_{m=-\infty}^{\infty} \delta[n - m] = \sum_{m=\langle N \rangle} \delta[n - m] \]

- Find the \( a \) in

\[ a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \exp(-jk\omega_0 n) \]

\[ a_0 = 1, \quad k = 0, \pm N, \pm 2N, \ldots \]

\[ a_k = 0, \quad k \neq 0, \pm N, \pm 2N, \ldots \]
Example 3.13

- The expansion for

- is then
Example 3.13

- The expansion for

\[ a_k = \begin{cases} 
\frac{1}{5} \frac{\sin(3\pi k/5)}{\sin(\pi k/5)}, & \text{for } k \neq 0, \pm N, \pm 2N, \ldots \\
8/5, & \text{for } k = 0, \pm N, \pm 2N, \ldots 
\end{cases} \]
Example 3.14

1. \( x[n], \, N = 6 \)

2. \[ \sum_{n=0}^{5} x[n] = 2 \]

3. \[ \sum_{n=2}^{7} (-)^n x[n] = 1 \]

4. \( x[n] \) is minimum power
Example 3.14

\[ \sum_{n=0}^{5} x[n] = 2 \rightarrow a_0 = \frac{1}{3} \]

\[ \sum_{n=2}^{7} (-)^{n} x[n] = 1 \rightarrow a_3 = \frac{1}{6} \]

- The first result is from \( k=0 \)
- The second is from \((-)^{n} = \exp(j\pi n)\)
Example 3.14

\[ x[n] = \frac{1}{3} + (-)^n \frac{1}{6} \]

• Has the proper properties