Today’s Topics

• Notation for Signals
• Notation for Power
• Transformations of Independent Variable
  – Time Shifts
  – Reflections
  – Examples 1.1 – 1.3
  – Periodic Signals
  – Example 1.4
A one-dimensional continuous signal is always in this course to be represented by $x(t)$, $t$ real, $x$ real or complex.
A one-dimensional discrete signal is in this course to be represented by \( x[n] \), \( n \) positive or negative integer, \( x \) real or complex.
Mathematical Descriptions of Power and Energy

- Power and energy contained in a signal are derived from the signal representation

\[ p(t) = |x(t)|^2 \]

\[ E(t) = \int_{t_1}^{t_2} |x(t')|^2 \, dt' \]
Mathematical Descriptions of Discrete Signal Power and Energy

- Power and energy contained in a signal are derived from the signal representation

\[ p[n] = |x[n]|^2 \]

\[ E(t) = \sum_{n_1}^{n_2} |x[n]|^2 \]
Mathematical Descriptions of Power and Energy

• The total energy in a signal can be defined as

\[ E_\infty(t) = \lim_{{T \to \infty}} \int_{-T}^{T} |x(t')|^2 \, dt' \]

If a signal is not limited to a finite time interval (not time limited), then often

\[ E_\infty(t) \to \infty \]
Average Power

- Average power over an interval of $2T$ and over all time (called average power) can be defined as

$$p_{2T}(t) = \frac{1}{2T} \int_{t-T}^{t+T} |x(t')|^2 dt'$$

- If average power is finite but signal has power over infinite range, energy diverges

$$P_\infty(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{t-T}^{t+T} |x(t')|^2 dt'$$
Discrete Average Power

- Discrete average power can be defined as

\[
p_{2n}[N] = \frac{1}{2n} \sum_{N-n}^{N+n} |x[n]|^2
\]

- \[\lim_{n \to \infty} \frac{1}{2n} \sum_{-n}^{n} |x[n]|^2\]

- What kinds of signals are infinite in range?
Signal Transformations

• We will treat continuous and discrete signals separately in the following
Time Shift of Continuous Signal

- Used when one needs to shift the origin of time. For example, necessary for synchronizing multiple signals.
Time Shift of Discrete Signal

- Is a necessary operation for preprocessing “experimental” data. Why would one leave zeros between origin and first data point?
Reflecting a Continuous Signal

- What mathematical operation requires reflecting one signal before multiplying with another and integrating?
Reflecting a Discrete Signal

Is there a DSP operation that requires signal reflection? Where does the need for reflection come about during processing?
Scaling a Continuous Signal

Why would we want to scale a signal before sampling? Why do we not have an example of scaling a discrete signal here after the continuous one? What is different about scaling a discrete signal?
General Problem:

\[ x(t) \rightarrow x(\alpha t + \beta) \]

Specific Problem

\[ x(t) \rightarrow x \left( \frac{3}{2} t + 1 \right) \]
Example 1.1

\[ x(t) \rightarrow x \left( \frac{3}{2}t + 1 \right) \]

What is the first step? What does it look like?
Example 1.1 - 2

\[ x(t) \rightarrow x \left( \frac{3}{2}t + 1 \right) \]

What is the next step? What does it look like?
Example 1.1 - 3

\[ x(t) \rightarrow x \left( \frac{3}{2}t + 1 \right) \]

What if the scale factor had been negative?
Example 1.1 – 3+

\[ x(t) \rightarrow x(-t+1) \]

A good general approach is to delay, invert (if necessary), and then scale
What is the energy in a periodic continuous signal? What is the average power?

\[ x(t) = x(t + T) \]
What other kinds of signals will contain infinite energy?
An Even Signal

Definition: \[ x(t) = x(-t) \quad \forall t \]
An Odd Signal

Definition: \[ x(t) = -x(-t) \quad \forall \ t \]
Example: A Signal Decomposition

How to decompose into even and odd parts?
A Signal Decomposition

\[ \mathcal{E}\{x(0)\} = x(t) + x(-t) \]

\[ \mathcal{O}\{x(0)\} = x(t) - x(-t) \]