ECEN 3300
Linear Systems
Class Meeting 34

Review of Filters and responses
Today’s Topics

• Time Domain Properties of Ideal Continuous and Discrete Filters (Section 6.3)
  – Impulse responses of ideal filters
  – Variable phase ideal filters
  – Step responses of ideal filters
Fourier Transform

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \exp(j\omega t) \, d\omega \]

\[ X(j\omega) = \int_{-\infty}^{\infty} x(t) \exp(-j\omega t) \, dt \]

- There is no basic period, the limit where k over the basic period becomes continuous.
Discrete Fourier Transform

\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \exp(j\omega n) \, d\omega \]

\[ X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \exp(-j\omega n) \]

- The complex exponential is periodic in omega
- Even if the signal \( x[n] \) is aperiodic, the transform is periodic with period 2 pi and inverse requires an integration range of 2 pi
Ideal Low Pass Filters (Section 6.3)

- We can find these transforms
Ideal Low Pass Filter IR’s

- Replace t by n for the DFT
Ideal Low Pass with Phase Function

- What does a phase function do?
Continuous Linear Phase Function

\[ e^{j\omega_s t} h(t) \leftrightarrow H(j(\omega - \omega_s)) \]

\[ h(t - t_0) \leftrightarrow e^{j\omega t_0} H(j\omega) \]

• What does this look like?
Continuous Linear Phase Function

- A similar result occurs in the discrete case?
Discrete Linear Phase Function

\[ e^{j\omega_s n} h[n] \leftrightarrow H \left( e^{j(\omega - \omega_s)} \right) \]

\[ h[n - n_0] \leftrightarrow e^{j\omega n_0} H \left( e^{j\omega} \right) \]

• Why does phase shift cause delay?
Phase and Group Velocity

\[ y(t) = x(t) * h(t) \]

\[ Y(j\omega) = X(j\omega)H(j\omega) \]

\[ Y(j\omega) = |X(j\omega)||H(j\omega)|e^{j\psi}e^{\phi(\omega)} \]

\[ \tau(\omega) = \frac{d\phi(\omega)}{d\omega} \]

- Phi is the phase delay
- Tau is the group delay time
Phase and Group Velocity

- When phase is constant, there is no delay
- When phase is linear, all frequency components arrive at the same delayed time
- When phase is nonlinear, the initial signal disperses – that is, breaks up
Step Response (see also Chapter 2)

\[ y(t) = x(t) \ast h(t) \]

\[ Y(j\omega) = X(j\omega)H(j\omega) \]

\[ y[n] = x[n] \ast h[n] \]

\[ Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \]

- Step response S refers to response when input function is a step function
A Transform Pair

\[
\frac{d}{dt} \int_{-\infty}^{t} x(t') dt' = x(t)
\]

\[
\int_{-\infty}^{t} x(t') dt' \leftrightarrow \frac{X(j\omega)}{j\omega}
\]

- Step response S refers to response when input function is a step function
Resulting Step Response

\[ s(t) = \int_{-\infty}^{t} h(t') dt' \]

\[ s[n] = \sum_{m=-\infty}^{t} h[m] \]

• These were derived from generalized functions in chapter 2
• What do these look like?
Step Response

- Do partial fraction expansions yield analytical results?