ECEN 3300
Linear Systems
Class Meeting 37

Notes Before Exam 3
Today’s Topics

• Some Transforms
  – Causal
  – Centered
  – Periodic

• Some Properties

• Problem Set 9 Problem 4

• Questions of Sample Exam
Some Transforms: Causal

\[ \alpha^n u[n] \leftrightarrow \frac{1}{1 - \alpha e^{-j\omega}} \]

\[ u[n] - u[n - M] \leftrightarrow \frac{1 - e^{-j\omega M}}{1 - e^{-j\omega}} \]

• delta[n] is included as a case
Some Transforms: Centered

\[\frac{\sin(\omega_c n)}{\pi n} \leftrightarrow \text{rect} \left[ \frac{\omega}{2\omega_c} \right]\]

\[u[n + M] - u[n - M - 1] \leftrightarrow \frac{\sin(\omega M)}{\sin \omega}\]

\[\text{rect} \left[ \frac{n}{2M + 1} \right] \leftrightarrow \frac{\sin(\omega M)}{\sin \omega}\]

- \text{delta}[n] \text{ is included as a case } M=0
Some Transforms: Periodic

$$\sum_{k=-\infty}^{\infty} \delta[n - kN] \leftrightarrow \sum_{k=0}^{\infty} \delta \left( \omega - \frac{2\pi k}{N} \right)$$

$$\sum_{k=-\infty}^{\infty} a_k e^{(jk2\pi/N)} \leftrightarrow \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2\pi a_k \delta \left( \omega - \frac{2\pi k}{N} - 2\pi l \right)$$

- The $-2\pi l$ is missing in the book.
Some Properties

\[ x[n - n_0] \leftrightarrow e^{j\omega n_0} X(e^{j\omega}) \]

\[ e^{j\omega_s n} x[n] \leftrightarrow X(e^{j(\omega - \omega_s)}) \]

- These are the most important
4. Consider the function

\[ x[n] = \alpha^n \cos(\omega_0 n) \quad N_s/2 \leq n \leq N_s/2 - 1 \]

Here we want to numerically, using MatLab, find the DFT of these assumed aperiodic signal \( x[n] \) for \( N_s \) values of \( \omega \) such that \( -\pi \leq \omega \leq \pi \). Assume \( N_s = 200 \) and consider the 6 cases,

(a) \( \alpha = 1 \) and \( \omega_0 = 2\pi \),
(b) \( \alpha = 1 \) and \( \omega_0 = \pi/5 \),
(c) \( \alpha = 1 \) and \( \omega_0 = \pi/50 \),
(d) \( \alpha = 0.95 \) and \( \omega_0 = 2\pi \),
(e) \( \alpha = 0.95 \) and \( \omega_0 = \pi/5 \), and
(f) \( \alpha = 0.95 \) and \( \omega_0 = \pi/50 \).

• The transforms should appear clean
PS9P4 for $\alpha=1.$
PS9P4 for alpha=0.95:
Sample Midterm Problems: 1

- Some Transforms

\[
\cos \pi n \\

u[n - n_0] - u[n - n_0 - 1] \\

\sum_{k=-\infty}^{\infty} \delta[n - kN] \\

e^{j\omega (n - n_0)} \alpha^{(n - n_0)} u[n - n_0]
\]
Sample Midterm Problems: 2

- Some Inverse Transforms

\[
\frac{1 - ae^{j(\omega - \omega_s)M}}{1 - ae^{-j(\omega - \omega_s)}}
\]

\[
\sum_{k=0}^{\infty} e^{j\omega k}
\]

\[
\sum_{k=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi k}{N} \right)
\]

\[
\sum_{k=-\infty}^{\infty} \text{rect} \left( \frac{\omega - \omega_c - \frac{2\pi k}{N}}{\omega_c} \right)
\]
Sample Midterm Problems: 3

- Some Systems Responses

\[ h[n] = \alpha^n u[n] \]

\[ x[n] = \delta[n] \]

\[ x[n] = u[n - n_0] - u[n - n_0 - M] \]

\[ x[n] = \beta^n u[n] \]
Sample Midterm Problems: 4

- A Difference Equations with Some Limits

\[ y[n] - b_1 y[n - 1] + b_2 y[n - 2] = a_0 x[n] + a_1 x[n - 1] \]

\[ H(e^{j\omega}) = ? \]

\[ y[n] = ? \]
More Questions?