ECEN 3300
Linear Systems
Class Meeting 42

Aliasing and DSP
Today’s Topics: Sampling Theorem

• Quick Go-Over
  – Sampling Theorem
  – A Zero Order Hold
  – Example: P7.31

• Aliasing

• Discrete Processing of Continuous Signals
  – Time and point representations
• Pulse train multiplication is an approximation but we will see it is alright
Sampling: Frequency Domain

- The above is for perfect sampling pulses
- Does the sampling theorem hold for finite pulses?
Sample Recovery

• Filtering can exactly reproduce any sampled signal whose sampling satisfies the sampling theorem.

• Problems – 1) samplers are not perfect and 2) filters are not perfect
Sampling with a Hold

- Not possible to sample at infinite with infinite frequencies so sample with almost none
• The samples are not centered on the sample point – required by causality
• (Not easy to design a sampling head)
• We do not want to wait until infinity to reconstruct
Sampling Spectrum

\[ X_o(j\omega) = e^{-j\omega T/2} \frac{2 \sin (\omega T/2)}{\omega} \sum_{n=-\infty}^{\infty} x(nT)e^{jn\omega T} \]

- How to recover here?
The Sampling Theorem’

If $X(j\omega)$ is bandlimited to $\omega_m$

Then $x(t)$ can be recovered from a distorted sampled version $x'_p(t)$

if $\omega_s > 2\omega_m$ (Why $>$?)

- Distorted: 1) finite sampling peak, 2) hold filtering
Recovery Filter

\[
H_r(j\omega) = \frac{e^{j\omega T/2}H(j\omega)}{2 \sin(\omega T/2)}
\]

- Is often called a whitening filter because of the effect on a black and white photo
Example: P7.31

7.31. Shown in Figure P7.31 is a system that processes continuous-time signals using a digital filter $h[n]$ that is linear and causal with difference equation

$$y[n] = \frac{1}{2} y[n-1] + x[n].$$

For input signals that are band limited such that $X_c(j\omega) = 0$ for $|\omega| > \pi/T$, the system in the figure is equivalent to a continuous-time LTI system.

Determine the frequency response $H_c(j\omega)$ of the equivalent overall system with input $x_c(t)$ and output $y_c(t)$.

- A primary reason for sampling is to use the superior characteristics of digital filters – How?
Aliasing

- What happens when the sampling theorem is not satisfied?
Sampling a Sinusoid

- Here the theorem is satisfied?
Sampling a Sinusoid II

- Still satisfied?
Sampling a Sinusoid III

- Oops! The wrong peaks are being sampled (with the wrong sign) in the last two. What do these last ones look like?
Sampling in Time

- These both look alright - the sample is restored
Sampling in Time II

- These are not alright! Phase is reversed.