ECEN 3300
Linear Systems
Class Meeting 8
Linear Time Invariant Systems
Today’s Topics

• Properties of Systems
• Linear Time Invariant Systems
  – Convolution in
  • Discrete Time Systems
  • Continuous Time Systems
Properties of Systems

- Memory
- Invertibility
- Causality
- Stability
- Time Invariance
- Linearity
Properties of Systems

• Memory – output depends on input at other than the present time
A Continuous System with Memory

- Capacitors has storage and, thereby, RC filters delay response
Properties of Systems I

- Invertibility – can express (find) input as a function of the output (monotonic and one to one)
Invertibility

- RC circuit has inverse
- Simple delay circuits have inverses
- Sine wave has no memory but is multi-valued and does not have unique inverse
Properties of Systems II

- Causality – output depends only on input at earlier time – can be tricky when system has memory – memory also affects linearity
Causality in Discrete Systems

\[ y[n] = \sum_{n' = 0}^{n} A_{n-n'} x[n - n'] \]

• System output only depends on inputs at earlier times
Causality in Discrete Systems II

\[ y[n] = \sum_{n' = 0}^{n} A_{n-n'} x[n - n'] + \sum_{n' = 0}^{n} B_{n-n'} y[n - n'] \]

- System output only depends on inputs and state of the system at earlier times – the systems could have memory
- If the system has memory, then there was an input at a still earlier time
Properties of Systems III

• Stability – output remains bounded when input is bounded – oscillators are unstable, that is, there is output without input
Stability in Discrete Systems

\[ y[n] = \sum_{n'=0}^{n} T_{n-n'} x[n - n'] \]

If \( A < x[n] < B \) \( \forall n \)

Then \( C < y[n] < D \) \( \forall n \)

- System input is bounded, system output is bounded
- Stability depends on strongly on the coefficients
Stability in Continuous Systems

\[ y(t) = \int_{t'=0}^{t} h(t - t') x(t') dt' \]

If \( A < x(t) < B \quad \forall t \)

Then \( C < y(t) < D \quad \forall t \)

- System input is bounded, system output is bounded
- Stability depends on strongly on the impulse response
Oscillators

- An LC circuit can have output without input
Properties of Systems IV

• Time Invariance
  – Discrete Systems
    • Example
  – Continuous Systems
    • Examples
Time Invariance in Discrete Systems

If \( x[n] \rightarrow y[n] \) for all \( n \)

Then \( x[n - n_0] \rightarrow y[n - n_0] \) for all \( n \)

- System output does not depend on time at which input is applied
- Equivalent to saying the system does not change with time although the inputs could
Discrete System Examples of TI

\[ y[n] = nx[n] \]

\[ y[n] = ax[n] + bx^2[n - 1] + cx^3[n - 2] \]

• Are the above time invariant
Time Invariance in Continuous Systems

If \( x(t) \rightarrow y(t) \quad \forall t \)

Then \( x(t - t_0) \rightarrow y(t - t_0) \quad \forall t \)

- System output does not depend on time at which input is applied
- Equivalent to saying the system does not change with time although the inputs could
Continuous System Examples of TI

\[ y(t) = \sin(x(t)) \]

\[ y(t) = x(2t) \]

• Are the above time invariant?
Continuous System Example of TI II

\[ y(t) = x(2t) \]

- Are the above time invariant?
Continuous System Example II

\[ y(t) = x(2t) \]

- Not invariant
Linearity in Discrete Systems

If
Then

\( x_i[n] \rightarrow y_i[n] \quad \forall n \)

\( x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n] \quad \forall n \)

and

If
Then

\( x_i[n] \rightarrow y_i[n] \quad \forall n \)

\( ax_i[n] \rightarrow ay_i[n] \quad \forall n \)

• System output is a sum of system outputs for given inputs (superposition) and the system output scales with system input (homogeneity)
Example for Linearity Discrete Systems

\[ y[n] = \Re\{x[n]\} \]

\[ y[n] = 2x[n] + 3 \]

- Are the above systems linear?
Incrementally Linear

- Adding a constant to a linear output is not a linear operation
- This is linear about an operating point (o. p.)
- Are other systems linear about an o. p.?
Linearity in Continuous Systems

If \( x_i(t) \rightarrow y_i(t) \quad \forall t \)
Then \( x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \quad \forall t \)

and

If \( x_i(t) \rightarrow y_i(t) \quad \forall t \)
Then \( ax_i(t) \rightarrow ay_i(t) \quad \forall t \)

• System output is a sum of outputs for given system inputs (superposition) and the system output scales with system input (homogeneity)
Examples of Linearity in Continuous Systems

\[ y(t) = x^2(t) \]

\[ y(t) = ax(t) + b \]

- Are the above linear? Incrementally linear?
Chapter 2: Linear Time Invariant Systems

• What is this picture at the beginning of the chapter? What system might this be?
Linear Time Invariant Systems

- What properties of linear systems make them useful as abstractions of real systems?
- What are the 2 properties of linear systems?
Linear Time Invariant Systems

- What properties of linear time invariant systems make them useful abstractions of real systems?
- What do these properties allow? Why?