ECEN 3300
Linear Systems
Class Meeting 9

Linear Time Invariant Systems
Today’s Topics: LTI Systems

• Deriving the Picture at Beginning of Chapter
• Discrete LTI
  – Representation as Impulses
  – Impulse Response and Convolution
• Examples
  – 2.1 through 2.4
Chapter 2: Linear Time Invariant Systems

• What is this picture at the beginning of the chapter? What system might this be?
An RC Circuit

\[ v_s(t) = \frac{V_s}{T} u(t - T/2) \]

- What is the output?
Linear Time Invariant Systems

- What properties of linear systems make them useful as abstractions of real systems?
- What are the 2 properties of linear systems?
An RC Circuit

\[ v_s(t) = \frac{V_s}{T} \sum_{n=0}^{\infty} u(t - (2n + 1)T/2) \]

- What is the output?
Properties of LTI Systems

- Linearity yields superposition
- Time Invariance allows superposition of different signals at different times
- Superposition of the same signal at different times allows representation of a system by a transfer function

\[ y(t) = \int_{-\infty}^{\infty} h(t - t') x(t') \, dt' \]
We want to represent this as a sum of values times values of the impulse function at the points of evaluation.
A function $x[n]$ can be expanded as the values of the function at the evaluation points times impulses at the times of the evaluations.
Representation by Impulses

\[ x[k] \delta[n - k] = \begin{cases} 
  x[k] & n = n_0 \\
  0 & n \neq n_0 
\end{cases} \]

\[ x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k] \]

- The function can be expanded as the values times impulses at the times of the evaluations
- Is called the sifting property of the impulse
Example: Step Function

\[ u[n] = \sum_{k=-\infty}^{n} u[k] \delta[k] \]

\[ = \sum_{k=-\infty}^{n} \delta[k] \]

• Have seen this before as a definition
• Is this invertible?
System Impulse Response

\[ x[n] = \delta[n - k] \rightarrow y[n] = h_k[n] \]

- When one impulse at one time is input to the system, the (measured) response is the impulse response

\[ h_k[n] \]
Response as Sum of Impulse Responses

\[ x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k] \]

- Then it follows that:

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k] h_k[n] \]
Example: Sum of Impulse Responses I

That there are only three non-zero values for $x$ says that there are only 3 terms in the sum

$$y[n] = \sum_{k=-1}^{1} x[k] h_k[n]$$
Example: Sum of Impulse Responses II

Then follows from

\[ y[n] = \sum_{k=-1}^{1} x[k]h_k[n] \]
Example: Sum of Impulse Responses

$$h_k[n] = h_0[n - k] = h[n - k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

- Use the response at time $k=0$ as the impulse response
- The form is now that of a discrete convolution
Example 2.1

- Find the function $y[n]$ from the above and

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n - k]$$
Example 2.1

The function $y[n]$ is a sum of two functions

$$y[n] = \sum_{k=0}^{1} x[k] h[n - k]$$

$$= x[0] h[n] + x[1] h[n - 1]$$

• The function $y[n]$ is a sum of two functions
Example 2.1

- Find $y[n]$ from the above and

$$y[n] = \sum_{k=0}^{1} x[k]h_k[n]$$
Example 2.2

• Find the values of $y[n]$ from the above

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \]
Example 2.2

• Find the values $y[n]$ 

$y[0] = x[0]h[0]$  