1. Consider a pin junction. The difference from the regular pn junction is that the pin junction has an intrinsic region between the p-type and n-type regions.

(a) Write down Poisson’s equation for the depletion region on the p-side \((-x_p \leq x \leq -x_{pi})\), the intrinsic region \((-x_{pi} \leq x \leq x_{ni})\), and the depletion region on the n-side \((x_{ni} \leq x \leq x_n)\), where \(x_{ni} = 1.0\mu m\), \(x_{pi} = 0.5\mu m\) are the \(n-i\) and \(p-i\) boundaries and \(x_n, x_p\) are the depletion region edges for n- and p-sides, respectively. Solve Poisson’s equation in each region and match the boundary conditions to find the electric field and potential as a function of \(x\).

Solution Gauss equation \(\nabla \cdot \mathbf{D} = \rho\) can be written in the full depletion approximation for the one dimensional doped semiconductor as

\[
\frac{d\mathcal{E}_x}{dx} = q \frac{N_d^+ - N_a^-}{\epsilon_s}.
\]

For the pin, we can write region by region

\[
\frac{\epsilon_s}{q} \frac{d\mathcal{E}_x}{dx} = \begin{cases} 
-N_a & -x_p \leq x \leq -x_{pi} \\
0 & -x_{pi} \leq x \leq x_{ni} \\
N_d & x_{ni} \leq x \leq x_n
\end{cases}.
\]

Using boundary conditions that \(\mathcal{E}_x(x_p) = \mathcal{E}_x(x_n) = 0\) and that \(\mathcal{E}_x\) is continuous at \(x_{pi}\) and \(x_{ni}\), we find that

\[
\frac{\epsilon_s}{q} \mathcal{E}_x = \begin{cases} 
-N_a(x + x_p) & -x_p \leq x \leq -x_{pi} \\
-N_a(x_p - x_{pi}) = -N_d(x_n - x_{ni}) & -x_{pi} \leq x \leq x_{ni} \\
-N_d(x_n - x) & x_{ni} \leq x \leq x_n
\end{cases}.
\]

Here we see that we can identify

\[
\mathcal{E}_{max} = -N_a(x_p - x_{pi}) = -N_d(x_n - x_{ni})
\]

\[
x_d = x_p - x_{pi} + x_n - x_{ni}
\]

\[
x_p - x_{pi} = \frac{N_d}{N_a + N_d} x_d
\]

\[
x_n - x_{ni} = \frac{N_a}{N_a + N_d} x_d.
\]
Integrating the expression for $\mathcal{E}_x$, we find the $\varphi_i$ to be

$$\frac{2\epsilon}{q} \varphi_i = N_a(x_p - x_{pi})^2 + N_d(x_n - x_{ni})^2 + 2N_a(x_p - x_{pi})x_{pi} + 2N_d(x_n - x_{ni})x_{ni}$$

Writing $\varphi_i$ in terms of $x_d$, we obtain

$$\frac{2\epsilon}{q} \varphi_i = \frac{N_a \left( \frac{N_d}{N_a + N_d} \right)^2 x_d^2 + N_d \left( \frac{N_a}{N_a + N_d} \right)^2 x_d^2 + 2 \frac{N_a N_d}{N_a + N_d} x_i x_d}{N_a + N_d}$$

where $x_i = x_{pi} + x_{ni}$ has been used. This leads to a quadratic

$$x_d^2 + 2x_i - \frac{2\epsilon N_a + N_d}{q N_a N_d} \varphi_i = 0$$

with solution for $x_d$

$$x_d = -x_i + \sqrt{x_i^2 + \frac{2\epsilon N_a + N_d}{q N_a N_d} \varphi_i}$$

We are now ready to write $\varphi(x)$ in the form

$$\frac{2\epsilon}{q} \varphi(x) = \begin{cases} \frac{N_a(x + x_p)^2}{N_a + N_d} & -x_p \leq x \leq -x_{pi} \\ -N_a(x_p - x_{pi})^2 \frac{x-x_{pi}}{x_{pi}+x_{ni}} + \left( \frac{2\epsilon}{q} \varphi_i - N_d(x_n - x_{ni})^2 \right) \frac{x+x_{pi}}{x_{pi}+x_{ni}} & -x_{pi} \leq x \leq x_{ni} \\ \frac{2\epsilon}{q} \varphi_i - N_d(x_n - x_{ni})^2 & x_{ni} \leq x \leq x_n \end{cases}$$

(b) Compare the maximum electric field in this pin junction with that in the regular pn junction with the same doping densities but without the intrinsic region.

**Solution:** The maximum field with the intrinsic region is $N_a(x_p - x_{pi}) = \frac{N_a N_d}{N_a + N_d} x_d$ whereas in the pn junction, it is $N_a x_p = \frac{N_a N_d}{N_a + N_d} x_d$. The intrinsic region does not make a difference to the expression. However, the width of the depletion region $x_d$ can be approximated when the $x_i \gg x_d$ by

$$x_d = -x_i + \sqrt{x_i^2 + \frac{2\epsilon N_a + N_d}{q N_a N_d} \varphi_i}$$

that is always smaller than in the pn junction limit and may be very much smaller when $x_i$ is comparatively large.

(c) Derive an expression for the junction capacitance under a reverse bias $V_R$. Plot $1/C^2$ vs. $V_R$ for the pin junction and the regular pn junction with the same doping densities but without the intrinsic region.
Solution: Here we need to use

$$C = \frac{dQ}{dV}$$

where we need to remember that $C$ and $Q$ are actually per unit area values. We can write that

$$\frac{Q}{A} = qN_d(x_n - x_{ni})$$

where the $A$ is the cross-sectional area and

$$x_n - x_{ni} = \frac{N_a}{N_a + N_d} x_d.$$ 

We have that

$$\frac{C}{A} = \frac{dQ}{dV} = q \frac{N_aN_d}{N_a + N_d} \frac{dx_d}{dV_a}.$$ 

Noting that

$$\frac{dx_d}{dV_a} = \frac{d}{dV_a} \left[ -x_i + \sqrt{x_i^2 + \frac{2\varepsilon N_a + N_d}{q N_a N_d} (\phi_i - V)} \right]$$

$$= \frac{2\varepsilon N_a + N_d}{q N_a N_d} \frac{1}{\sqrt{x_i^2 + \frac{2\varepsilon N_a + N_d}{q N_a N_d} (\phi_i - V)}}$$

we can write $C/A$ in the form

$$C = \frac{2\varepsilon A}{x_d + x_i}.$$ 

Here we see the most important result that, when $x_i \gg x_d$ then $C$ is independent of voltage. The relation for $1/C^2$ then is

$$\frac{4\varepsilon^2 A^2}{C^2} = x_i^2 + \frac{2\varepsilon N_a + N_d}{q N_a N_d} (\phi_i - V_a).$$

Forward bias will have to be large to remove this depletion region.

2. Consider a step pn junction made of GaAs at $T = 300^\circ$K. The junction capacitance at zero bias is $C(0)$ and the capacitance with a 10 V reverse bias is $C(10)$. The ratio of the capacitances was measured to be

$$\frac{C(0)}{C(10)} = 3.13$$

(a) Calculate the built-in potential, $\phi_i$. 

3
Solution: The capacitance is given by

\[ C = \frac{\epsilon_s A}{x_d} \]

where

\[ x_d = \sqrt{\frac{2\epsilon_s N_a N_d}{q} \left( \phi_i - V_a \right)} \]

so that we can write that

\[ \frac{C(0)}{C(10)} = \sqrt{\frac{\phi_i - V_a}{\phi_i}} = \sqrt{\frac{\phi_i - 10}{\phi_i}} = 3.13 \]

that yields a \( \phi_i \) roughly 1.1 V.

(b) Given that the depletion region width on the p-side, \( x_p \), was found to be 20% of the entire depletion region width, \( x_d \), find the doping densities, \( N_d \) and \( N_a \), on the n- and p-sides, respectively.

Solution: we have that

\[ x_p = \frac{N_d}{N_a + N_d} x_d \]
\[ x_n = \frac{N_a}{N_a + N_d} x_d \]

that follows from

\[ N_a x_p = N_d x_n \]

and \( x_p + x_n = x_d \). Using

\[ 0.2 = \frac{N_d}{N_a + N_d} \]

we find that \( N_d = N_a / 4 \). Using

\[ N_a N_d = n_i^2 \exp \left( \frac{q\phi_i}{kT} \right) \]

we find

\[ N_a = 2n_i \exp \left( \frac{q\phi_i}{2kT} \right) = 1.3 \times 10^{16} \text{ cm}^{-3} \]

so that \( N_d \approx 3.3 \times 10^{15} \text{ cm}^{-3} \).

3. Consider a one-sided step pn junction made of silicon in which the doping density in the n-side is much greater than that in the p-side, \( N_d \gg N_a \). This junction is under a reverse bias of 10 V. If the doping density, \( N_a \), in the p-side is doubled, what would be the percent change in
(a) junction capacitance, $C$, and

**Solution:** We have that the junction capacitance (which is capacitance/area)

$$C = \frac{\epsilon_s}{x_d}$$

where

$$x_d = \sqrt{\frac{2\epsilon_s N_a N_d}{q N_a + N_d (\phi_i - V_a)}}$$

so that we can write that

$$C \approx \sqrt{\frac{q\epsilon_s N_a}{2 \phi_i - V_a}}.$$  

If $N_a \to 2N_a$, then $C \to \sqrt{2}C$.

(b) the built-in potential, $\phi_i$.

**Solution:** As we have that

$$\phi_i = \frac{kT}{q} \ln \left( \frac{N_a N_d}{n_i^2} \right)$$

then when $N_a \to 2N_a$, $\phi_i \to \phi_i + \frac{kT}{q} \ln(2)$. 