Surveying Game Theoretic Approaches for Wind Farm Optimization

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This paper surveys recent results in game theory and cooperative control and highlights their implications for the problem of optimizing energy production in wind farms. One such result is a simple payoff-based learning rule that is completely decentralized and leads to an efficient configuration of actions in virtually any distributed system. We demonstrate that this learning rule can be used to provably maximize energy production in wind farms without explicitly modeling the aerodynamic interaction amongst the turbines.

I. Introduction

Wind energy is widely becoming recognized as one of the most cost-efficient sources of renewable energy. Accordingly, expectations for wind energy are at unprecedented levels as the overarching goal is for wind energy to become a dominant source for global electricity needs. In the US, a goal has been set for wind energy capacity to meet 20% of the country’s electrical energy demands by 2030.1 One of the keys to realizing this goal in a cost-efficient manner is to utilize existing wind farms in a more efficient manner through improved control algorithms.

Most of the existing research on the control of wind turbines focuses on the single turbine setting.2,3 The control of an array of turbines in a wind farm is fundamentally more challenging than controlling a single turbine because of the aerodynamic interactions amongst the turbines which render most of these single turbine control algorithms highly inefficient for optimizing power capture in wind farms.4–6 The potential for improving performance, both in terms of increasing power capture as well as mitigating loads across the wind farm, has led to several new research efforts in coordinating the control of arrays of wind turbines.7–12 One approach for dealing with these aerodynamic interactions is to develop wake models for use in the distributed control algorithms.13–15 However, the variable and chaotic nature of wind makes such a task incredibly challenging. An alternative approach, and the goal of this paper, is to develop an online control algorithm where each turbine adjusts its own axial induction factor in response to local information, such as the individual turbine’s power generation, local wind conditions, or minimal information regarding neighboring turbines. The goal is to develop such an algorithm that permits the set of turbines to reach a desirable set of axial induction factors that lead to good system level behavior, e.g., power maximization or load minimization, without the need for explicitly modeling the wind.

In this paper we survey recent results in game theory and cooperative control and highlight their implications in the problem of optimizing energy production in wind farms.16–18 The field of game theory provides an analytical framework for analyzing systems comprised of enmeshed decision-makers. In terms of wind farms, the decision makers represent the individual turbines and the enmeshment follows from the fact that the decision of one turbine impacts the wind conditions and potential power generation by another turbine. The game theoretic framework is broad enough to model several phenomena that are relevant to distributed control of wind farms, including multiple and heterogeneous decision makers (e.g., turbines with variations in blade size), limited information in decision making (e.g., each turbine has limited information regarding the environment), robustness to environmental uncertainties (e.g., variability in wind conditions), and more.

The control and design of multi-agent systems parallels the theme of distributed optimization. One of the core differences between these two domains is the fact that multi-agent systems frequently place restrictions on the set of admissible controllers. In terms of distributed optimization, this places a restriction on the set of admissible distributed
algorithms. For example, in wind farms the control strategies for the individual turbines are limited by the following informational constraints:

(i) Each turbine does not have access to the choices of other turbines. This is because of the lack of a suitable communication system.

(ii) Each turbine does not have access to the functional form of the power generated by the wind farm. This is because the aerodynamic interaction between the turbines is poorly understood.

Accordingly, the applicability of some of the common approaches to distributed optimization, e.g., subgradient methods, are inapplicable because of these informational constraints. Accordingly, recent research focuses on the use of genetic algorithms for wind farm optimization. However, genetic algorithms do not provide any guarantees on convergence times or the quality of the solution.

The focus of this exposition is to illustrate developments in game theoretic control for multi-agent systems on the problem of wind farm optimization under the aforementioned informational constraints. We start by defining the wind farm model in Section II and a tractable example in Section III. Next, in Section IV we present a game theoretic distributed learning algorithm, termed Safe Experimentation Dynamics, which can be utilized by a distributed control algorithm to provide convergence to the collection of axial induction factors that optimize the power production in the wind farm. This algorithm is model-free, which means that it does not require a characterization of the aerodynamic interaction between the turbines to provide the desired convergence. However, the algorithm does require each of the individual turbines to have knowledge of the total power produced in the wind farm. Recent work focuses on an alternative distributed learning algorithm which provides the same performance guarantees under less demanding communication requirements.

Lastly, in Section V we present several illustrations. First, we start with a simple three-turbine wind farm where the turbines are positioned in a row, as defined in Section III. In this setting, the inefficiency associated with locally optimal controllers when compared to the globally optimally controllers exceeds 7% as shown in Section III. Here, locally optimal controllers are the well-studied single turbine controllers which seek to achieve an axial induction factor of 1/3. In Section V.A, we provide simulation results on this three-turbine wind farm example using the Safe Experimentation Dynamics algorithm. The simulation results demonstrate that the performance of the wind farm rapidly approaches the optimal system performance with a running time of less than 0.1 seconds. Next, in Section V.B we provide simulation results on a more complex 80-turbine wind farm which replicates the layout of Horns Rev. In this setting, the simulation results show that the Safe Experimentation Dynamics algorithm also ensures that the performance of the wind farm efficiently approaches the optimal system performance with a running time on the order of 17 seconds. It is important to highlight that the algorithm does not rely on any information regarding the wind farm model. Therefore, one would see similar results when using an alternative wind farm model or with measurements obtained through implementation in an existing wind farm.

II. Wind Farm Model

This section presents the wind farm model utilized in this paper. The wind farm model consists of the following three components:

(i) **Wake model**: When a wind turbine extracts energy out of the wind it creates a wake downstream where the wind speed is reduced. A wake model seeks to characterize the wake resulting from a single turbine.

(ii) **Wake interaction model**: A wake interaction model seeks to characterize how wakes from multiple turbines interact with one another.

(iii) **Power model**: Power models seek to characterize the amount of power generated by a wind turbine given the local wind conditions and the control of the wind turbine. We represent the control parameters of a wind turbine by the turbine’s axial induction factor which represents the fractional decrease in wind velocity between the free stream conditions and those seen at the rotor plane. We parameterize our wind model by the axial induction factors as opposed to more traditional control parameters, e.g., tip-speed ratio and pitch angle, to provide a more compact representation of the wind farm model studied in this paper.
A. Preliminaries

We consider a wind farm consisting of \( n \) wind turbines denoted by the set \( N := \{1, 2, \ldots, n\} \). For simplicity in initially developing and exploring our approach, we assume uniform wind of a constant speed \( U_{\infty} \) and a constant direction. Furthermore, we assume throughout that all wind turbines are oriented so that the turbine’s axis is parallel to the wind direction. Each turbine \( i \in N \) is characterized by the diameter of the disk generated by the turbine blades, denoted by \( D_i \), and a two-dimensional location \((x_i, r_i)\) relative to a common vertex, where \( x_i \) represents the distance from this vertex in the wind direction and \( r_i \) represent the distance from this vertex in the orthogonal direction. We represent joint axial induction factors by the tuple \( a := (a_1, a_2, \ldots, a_n) \) where \( a_i \) denotes the axial induction factor of turbine \( i \). The set of admissible axial induction factors for turbine \( i \) is given by the set \( {\mathcal A}_i : = \{a : 0 \leq a_i \leq 0.5\} \) and \( {\mathcal A} := {\mathcal A}_1 \times \ldots \times {\mathcal A}_n \) represents the set of admissible joint axial induction factors. We will frequently express a joint axial induction factor \( a \in {\mathcal A} \) by \( a = (a, a_{-i}) \) where \( a_{-i} := (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n) \) represents the collection of axial induction factors of all wind turbines other than turbine \( i \).

B. Wake Model

We start by reviewing the Park model\( ^a \) which is one of the most prevalent wake models studied in the existing literature.\( ^{25,31,33-35} \) Consider the situation highlighted in Figure 1 where turbine \( i \in N \) represents the only turbine and let \( V_i(x, r; a_i) \) represent the velocity profile of the wake generated by turbine \( i \) relative to the vertex \((x_i = 0, r_i = 0)\). According to the Park model, the velocity profile takes on the form

\[
V_i(x, r; a_i) = U_{\infty} \left( 1 - \delta V_i(x, r; a_i) \right)
\]

where \( \delta V_i(x, r; a_i) \) represents the (fractional) deficit of the velocity at the point \((x, r)\) downstream of turbine \( i \). This velocity deficit is defined by

\[
\delta V_i(x, r; a_i) = \begin{cases} 
2a_i \left( \frac{D_i}{D_i + 2kr} \right)^2 & \text{for any } r \leq \frac{D_i + 2kr}{2}, \\
0 & \text{for any } r > \frac{D_i + 2kr}{2},
\end{cases}
\]

where \( k \) is a roughness coefficient.\( ^b \) The two dominant traits of this model are (i) the velocity profile is constant along the radial direction of a wake, i.e., \( V_i(x, r; a_i) = V_i(x, r'; a_i') \) for any \( r, r' \leq (D_i + 2kr)/2 \), and (ii) the velocity approaches \( U_{\infty} \) at large distances from the turbine. According to (2), the diameter of the wake of turbine \( i \) at a distance \( x \) downstream is given by

\[
D_i^w(x) = D_i + 2kx.
\]

\( ^a \)The Park model is utilized in popular software for wind farm simulations including Openwind,\( ^{28} \) WindPRO,\( ^{29} \) WindFarmer,\( ^{30} \) and SimWindFarm\( ^{31} \) that have come out of the Aeolus Project.\( ^{32} \)

\( ^b \)The roughness coefficient defines the slope at which the wake expands out from the turbine. Roughness coefficients have been found empirically for many different environments, e.g., \( k = 0.075 \) for farmlands and \( k = 0.04 \) for offshore locations.\( ^{34} \)
Figure 2. Consider the two turbine example depicted above where the wind seen at turbine 2 is nonuniform. Using (5), the aggregate velocity deficit at turbine 2 is \( \delta V_2(a) = 2a_1 \left( \frac{D_1}{D_1 + 2k(x_2 - x_1)} \right)^2 \frac{A_{1 \rightarrow 2}^{\text{overlap}}}{A_2} \). Note that if the wake of turbine 1 completely encompasses turbine 2, i.e., \( A_{1 \rightarrow 2}^{\text{overlap}} = A_2 \), then we recover (2) as expected.

C. Wake Interaction Model

A core modeling challenge associated with the multiple turbine setting is characterizing how overlapping wakes interact with one another. A common approach in the existing literature is that of momentum balance. Rather than deriving an entire velocity profile, here we concentrate on deriving an aggregate wind velocity seen by each turbine \( i \in N \) which we will represent by \( V_i(a) \), respectively. For any wind turbine \( i \in N \), the aggregate wind velocity is given by

\[
V_i(a) = U_\infty(1 - \delta V_i(a))
\]

where the aggregate velocity deficit seen by turbine \( i \) is given by

\[
\delta V_i(a) = 2 \sum_{j \in N: x_j < x_i} \left( a_j \left( \frac{D_j}{D_j + 2k(x_i - x_j)} \right)^2 \frac{A_{j \rightarrow i}^{\text{overlap}}}{A_i} \right)^2
\]

\[A_i\] is the area of the disk generated by the blades of turbine \( i \), and \( A_{j \rightarrow i}^{\text{overlap}} \) is the area of the overlap between the wake generated by turbine \( j \) and the disk generated by the blades of turbine \( i \). See Figure 2 for a simple illustration with two turbines. Accordingly, (4) takes on the form

\[
V_i(a) = U_\infty \left( 1 - 2 \sqrt{\sum_{j \in N: x_j < x_i} (a_j c_{ji})^2} \right)
\]

where

\[
c_{ji} = \left( \frac{D_j}{D_j + 2k(x_i - x_j)} \right)^2 \frac{A_{j \rightarrow i}^{\text{overlap}}}{A_i}.
\]

D. Power Model

The power generated by turbine \( i \) is characterized by \[\rho A_i C_P(a_i) V_i(a)^3\]

\[P_i(a) = \frac{1}{2} \rho A_i C_P(a_i) V_i(a)^3\]

where \( \rho \) is the density of air and \( C_P(a_i) \) is the power efficiency coefficient which takes on the form

\[C_P(a_i) = 4a_i(1 - a_i)^2\]
Figure 3. Simple 3-turbine wind farm considered in Section III. We denote the turbine on the far left as turbine 1, the turbine in the middle as turbine 2, and the turbine on the far right as turbine 3. Each turbine has a diameter of 80 meters, and the turbines are spaced 400 meters apart from one another. The wind direction is in the direction of the $x$-axis with a fixed upwind velocity $U_\infty$. It is important to note that according to (8) and (10) the wind velocity does not factor into the optimal profile of axial induction factors.

The total power generated in the wind farm is simply

$$ P(a) = \sum_{i \in N} P_i(a). \quad (10) $$

III. A Motivating Example

This paper focuses on developing wind turbine control strategies for maximizing power capture in a wind farm. More specifically, we focus on the attainment of the optimal joint axial induction factor

$$ a_{opt} \in \arg \max_{a \in A} P(a). $$

It is important to note that the bulk of the existing control strategies for wind turbines are geared at stabilizing an axial induction factor of $1/3$. The reason for this stems from (8) where we know that for any $a_{-i} \in A_{-i} := \prod_{j \neq i} A_j$ we have

$$ 1/3 = \arg \max_{a_i \in A_i} P_i(a_i, a_{-i}). $$

We refer to this locally optimal control strategy as greedy and we let $a^{\text{greedy}} := \{1/3, \ldots, 1/3\}$. Does this greedy control policy efficiently extend to the wind farm setting? More formally, how does the power production associated with $a^{\text{greedy}}$ compare to the power production associated with $a_{opt}$?

To shed light on this question, we focus on the three-turbine wind farm illustrated in Figure 3. In this setting, the power produced by the wind farm takes on the form

$$ P(a) = \frac{1}{2} \rho A \left( C_p(a_1)U_\infty^3 + C_p(a_2)V_2(a)^3 + C_p(a_3)V_3(a)^3 \right) \quad (11) $$

where $U_\infty$ is the upwind velocity and $A = A_1 = A_2 = A_3$. Solving for $V_2(a)$ and $V_3(a)$ using (6) we obtain

$$ V_2(a) = U_\infty (1 - 2a_1 c_{12}), \quad (12) $$

$$ V_3(a) = U_\infty \left( 1 - 2\sqrt{(a_1 c_{13})^2 + (a_2 c_{23})^2} \right). \quad (13) $$

Optimizing (11) over the set of admissible joint axial induction factors $A$ gives us

$$ \{a_1^{opt}, a_2^{opt}, a_3^{opt}\} = \{0.232, 0.208, 0.333\}. \quad (14) $$

It is important to note that there are alternative objectives in wind farm control, e.g., mitigating loads. While we will not explicitly formulate such objectives, the control techniques developed in this paper are applicable for these settings as well.

There are currently no well-known alternative control strategies for stabilizing axial induction factors that are not equal to $1/3$. The study presented in this paper establishes that developing such controllers is definitely warranted to improve the operational efficiency of existing wind farms.
It is straightforward to verify that $a_{3}^{\text{opt}} = 1/3$ as there are no further turbines downstream. For this simple setting, we can attain $a_{1}^{\text{opt}}$ and $a_{2}^{\text{opt}}$ through an exhaustive search over the set $A_{1} \times A_{2}$. The efficiency of the locally optimal axial induction factors is

$$\frac{P(a^{\text{greedy}})}{P(a^{\text{opt}})} = 0.9265.$$  \hspace{1cm} (15)

meaning that the efficiency loss is greater than 7%. The reason for this degradation is clear as the locally optimal axial induction factors do not account for the aerodynamic interactions between the turbines.

## IV. Wind Farm Control

The previous section demonstrates that control algorithms for wind turbines in the single-turbine setting do not efficiently extend to the multiple-turbine setting. This section explores approaches for developing control algorithms for this multiple turbine setting. Here, we focus purely on the objective of maximizing total power production in a wind farm given fixed exogenous wind conditions. The challenge with such an objective is dealing with the fact that (i) the aerodynamic interaction between the turbines is not well characterized and (ii) the information available to each of the wind turbines may be limited. Nonetheless, in this section we demonstrate that these limitations can be overcome to meet our desired objectives.

### A. Preliminaries: Cooperative Control

In this section we formulate the problem of wind farm optimization as a cooperative control problem. The forthcoming control designs establish an interaction framework which produces a sequence of joint axial induction factors $a(0)$, $a(1)$, ..., where at each iteration $t \in \{0, 1, 2, \ldots \}$ the decision of each turbine $i \in N$ is chosen independently according to a local control law of the form

$$a_{i}(t) = \Pi_{i}(\text{Information available to agent } i \text{ at time } t).$$ \hspace{1cm} (16)

The control policy of turbine $i$, $\Pi_{i}(\cdot)$, designates how each turbine processes available information to formulate a decision at each iteration. We will refer to a turbine’s axial induction factor as the turbine’s action or decision. The goal is to design the local control policies $\{\Pi_{i}(\cdot)\}_{i \in N}$ within the desired informational constraints such that the collective behavior converges to a collection of axial induction factors $a^{*}$ that optimizes the total power production in the wind farm, i.e., $a^{*} \in \arg \max_{a \in A} P(a)$. Here, we focus on the design of turbine control policies that are model-free. That is, control policies that do not rely on a characterization of the aerodynamic interaction between the turbines.

### B. Model-Free with Communication

We will now introduce the Safe Experimentation Dynamics which requires that the set of axial induction factors for each turbine be a discretized set.\textsuperscript{26} In the Safe Experimentation Dynamics, each turbine $i \in N$ possesses a local state variable which impacts the agent’s control policy. We represent an agent’s state by the tuple $[\bar{a}_{i}, \bar{p}_{i}]$ where

- The benchmark action is $\bar{a}_{i} \in A_{i}$.
- The benchmark power is $\bar{p}_{i}$, which is in the range of $P(\cdot)$.

The Safe Experimentation Dynamics takes on the following form:

1. **Initialization:** At time $t = 0$, each turbine $i \in N$ randomly selects an axial induction factor $a_{i}(0) \in A_{i}$. This will be initially set as the turbine’s baseline action at time $t = 1$ and is denoted by $\bar{a}_{i}(1) = a_{i}(0)$. The turbine’s baseline power at time $t = 1$ is given by $\bar{p}_{i}(1) = P(a(0))$.

2. **Action Selection:** At each subsequent time step, each turbine selects the baseline action with probability $(1 - \epsilon)$ or experiments with a new random action with probability $\epsilon$, i.e.,

$$a_{i}(t) = \begin{cases} \bar{a}_{i}(t) \text{ with probability } (1 - \epsilon), \\
\text{RAND} \text{ with probability } \epsilon, \end{cases}$$ \hspace{1cm} (17)

where $\epsilon > 0$ will be referred to as the agent’s exploration rate and RAND represents that $a_{i}(t)$ is chosen randomly accordingly to a uniform distribution over the set $A_{i}$.

\textsuperscript{26}In general, uniformity is not necessary to provide the asymptotic guarantees given in Theorem 1.\textsuperscript{26}
3. **Baseline Strategy Update:** Each turbine \( i \in N \) updates the baseline action as

\[
\bar{a}_i(t + 1) = \begin{cases} 
    a_i(t), & P(a(t)) > P^{\text{max}}(t), \\
    \bar{a}_i(t), & P(a(t)) \leq P^{\text{max}}(t),
\end{cases}
\]

and the baseline power as

\[
\bar{p}_i(t + 1) = \max \{ P(a(t)), \bar{p}_i(t) \}.
\]

This step is performed whether or not Step 2 involved exploration.

4. Return to Step 2 and repeat.

This learning algorithm is called Safe Experimentation Dynamics since \( P(\bar{a}(t)) \) is non-decreasing with respect to time, i.e., the power generated by the wind farm when using the baseline action is non-decreasing. We now state the following characterization on the limiting behavior associated with the Safe Experimentation Dynamics.\(^{26}\)

**Theorem 1** Suppose all turbines use the Safe Experimentation dynamics as highlighted above. Given any probability \( p < 1 \), if the exploration rate \( \epsilon > 0 \) is sufficiently small, then for all sufficiently large times \( t \), \( a(t) \in \arg \max_{a \in A} P(a) \) with at least probability \( p \).

There are several important properties regarding the applicability of Theorem 1 to the wind farm optimization. First, note that there is no underlying dependence on a given wind model and wake interaction model. Accordingly, the above characterization holds for any setting which is of fundamental importance since developing accurate wind models and wake interaction models is an active research area.\(^{7-12}\) Second, it is important to highlight that the characterization provided in Theorem 1 refers to probabilistic convergence as opposed to almost sure convergence. This means that the joint axial induction factors will not converge to the optimal axial induction factors. Rather, the individual wind turbines will spend the majority of the time using the optimal axial induction factors. The reason for this is that the individual turbines do not have access to the structural form of \( P(a) \). Therefore, the turbines perpetually probe the system, albeit with small probability, to gain information. Lastly, there are extensions of the presented algorithm for scenarios with non-deterministic power production functions \( P(\cdot) \).\(^{26}\) This non-deterministic case may yield better performance in the wind farm setting as wind conditions are not static.

**V. Illustrations**

We now present several illustrations of the distributed control algorithm developed in the previous section on the problem of wind farm optimization. We first focus on the simple three-turbine row farm example from Section III and will then proceed to a more involved 80-turbine wind farm example which replicates the layout of Horns Rev.

**A. A Three-Turbine Example**

Recall the three-turbine wind farm illustrated in Figure 3. As shown previously in (15), the efficiency loss associated with the locally optimal controllers was over 7% when compared with the globally optimal controllers. While this 7% was evaluated focusing on a specific wind model and wake interaction model, i.e., Park model with momentum balance, it seems very realistic that alternative models, in addition to reality, would see similar deficiencies. Ultimately, it is imperative that the underlying control strategy accounts for the aerodynamic interaction between the turbines.

We simulated the learning algorithm Safe Experimentation Dynamics on this three-turbine wind farm example depicted in Section III. The model parameters are \( k = 0.075, \rho = 1.225 \text{ (kg/m}\text{)}^3\), and \( U_\infty = 16 \text{ (m/s)} \). The results are presented in Figure 4 for the algorithm parameter \( \epsilon = 0.05 \), discretized action sets of the form \( A_i = [0.1 : 0.01 : 0.33] \), and an initial profile of axial induction factors \( a(0) = \{0.33, 0.33, 0.33\} \). The simulation results demonstrate that the power produced by the wind farm rapidly approaches the optimal power that could be produced by the wind farm for the given wind conditions. Furthermore, the axial induction factors quickly approach the optimal axial induction factors identified in (14). It is important to emphasize that this algorithm was effectively able to optimize system performance without exploiting any information regarding the underlying wind model or wake interaction model. The running time of the algorithm was approximately 0.08 seconds on a typical laptop computer for the entire simulation shown in Figure 4.
Figure 4. Simulation results of the Safe Experimentation Dynamics on the three-turbine wind farm with $\epsilon = 0.05$, discretized action sets of the form $A_i = [0.1 : 0.01 : 0.33]$, and an initial profile of axial induction factors $a(0) = [0.33, 0.33, 0.33]$. Note that while the system perpetually experiments, as expected, the average performance is not impacted by these experimentations. Subfigure (a) presents the evolution of the power produced in the wind farm. Subfigure (b) shows the evolution of the axial induction factors.

Figure 5(b) demonstrates that if the wind direction is due east then the efficiency loss associated with the locally optimal control policy could be over 8%. This means the potential loss could exceed 10 MW which is quite staggering. Figure 6 characterizes the energy loss associated with all wind directions between ±90 degrees about the east direction. It is important to point out that the optimal power produced by the wind farm is calculated using well established convex optimization programs. If the power produced by the wind farm, i.e., $P(a)$, is convex over $a \in A$, then the highlighted efficiency gap is in fact accurate. If $P(a)$ is not convex over $a \in A$, then the highlighted efficiency gap would serve as a lower bound for the efficiency gap. This means that the efficiency gap could exceed 10% for some settings. This figure demonstrates that there is a strong correlation between underlying wind direction and the resulting efficiency loss. Furthermore, much of the wind distribution seen by Horns Rev this year lies in the range where there is a significant efficiency gap.

VI. Summary

This paper surveys recent results in game theory and cooperative control and highlights their implications for the problem of optimizing energy production in wind farms. In particular, we have demonstrated that the distributed learning algorithm Safe Experimentation Dynamics can be used to provably maximize energy production in wind farms without explicitly modeling the aerodynamic interaction amongst the turbines. This distributed control algorithm provides increases in energy production of almost 10%. While these efficiency results are derived using the Park model, it seems likely that similar gains in energy production could be realized in actual wind farms as well. Throughout this paper we defined the control strategies of each wind turbine as the choice of axial induction factor. It is worth noting that there are currently no well-known control strategies for stabilizing axial induction factors that are not equal to 1/3. Hence, the study presented in this paper establishes that developing such control strategies is definitely warranted to improve the operational efficiency of existing wind farms.
Figure 5. Subfigure (a) illustrates the layout of a wind farm consisting of 80 wind turbines considered in this example. This layout seeks to replicate the Horns Rev wind farm located off the shore of Denmark. Horns Rev is composed of Vestas V80 2MW turbines each with a diameter of 80 meters. The turbines are laid out in an oblique rectangle with a turbine spacing of 7 turbine diameters, i.e., 560 meters, in both the $x$ and $y$ directions. The $x$-direction represents due east while the $y$-direction represents due north. Subfigure (b) illustrates the evolution of the power produced by the 80-turbine wind farm using the Safe Experimentation Dynamics with $\epsilon = 0.03$, discretized action sets of the form $A_i = [0 : 0.001 : 0.333]$, and initial axial induction factors $a_i(0) = 0.333$ for each turbine $i \in N$. Note that while the system perpetually experiments, as expected, the average performance is not impacted by these experimentations. The greedy policy produces around 91.5% of the potential wind power. When using the Safe Experimentation Dynamics, after 2000 iterations, the average power produced is over 98% of the potential wind power.

**References**

Figure 6. Subfigure (a) illustrates the relationship between wind direction and the resulting efficiency loss in power production when comparing $\alpha^{\text{greedy}}$ versus $\alpha^{\text{opt}}$ for the 80-turbine wind farm illustrated in Figure 5(a). Subfigure (b) illustrates the density of the wind conditions (direction and speed) seen by Denmark’s Horns Rev 1 wind farm this year. The radial direction is wind speed ranging from 0 to 12 knots.

28AWSTtruewind, OPENWIND: Theoretical Basis and Validation, 2010.
32Aeolus European research project, Distributed Control of Large-Scale Offshore Wind Farms, http://www.ict-aeolus.eu.