Large-Domain MOM Solution of Complex Electromagnetic Problems

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Abstract— **A numerical method is presented for the analysis and design of a wide variety of electromagnetic structures consisting of dielectric and conducting parts of arbitrary shapes. The method is based on the integral-equation formulation in frequency domain, and represents a large-domain (high-order expansion) Galerkin-type version of the method of moments (MOM). The method is formulated in two versions concerning the type of the equivalence (volume and surface) utilized in the treatment of the dielectric parts of the structure. It is demonstrated on two unconventional examples that a well designed and carefully optimized moment-method can be a highly efficient and reliable tool for numerical solutions of real-world problems.**

I. Introduction

Most of the existing frequency-domain integral-equation methods for analysis of arbitrary 3D electromagnetic structures are subdomain (small-domain) type methods of moments. More precisely, the structure is approximated by many electrically small geometrical elements (on the order of $\lambda/10$ in each dimension), with low-order expansion functions for currents (e.g., [1,2]). In the authors' opinion, an entire-domain approach can greatly extend the versatility, accuracy, reliability, and efficiency of moment-methods. More precisely, the entire-domain (large-domain) technique utilizes high-order expansion functions defined in electrically large geometrical elements (on the order of 2λ in each dimension).

This paper outlines a large-domain method of moments (MOM) for analysis of structures composed of arbitrarily excited and loaded dielectric and conducting bodies of arbitrary shapes. It is based on a frequency-domain integral-equation formulation. The method has two versions. The first version utilizes the volume equivalence principle to compensate the dielectric and conducting materials by the actual induced electric currents in a vacuum. These currents represent the unknown quantities to be determined in the solution procedure. The other version of the method invokes the surface equivalence principle (generalized Huygens' principle), and utilizes as unknown quantities equivalent (artificial) electric and magnetic surface currents over boundary surfaces between the homogeneous regions of the structure. For metallic structures in a vacuum, the two versions of the method are identical. For structures which include dielectrics, each version has advantages in solving specific problems. For example, the volume-equivalence approach is a better choice when analyzing highly inhomogeneous dielectric scatterers. In both versions, the geometry is modelled by flexible parametric elements with high-order polynomial expansions for the approximation of currents, enabling electrically large elements to be used. In this manner the number of unknowns is greatly reduced (for an order of magnitude) when compared with subdomain solutions.

The paper is aimed at demonstrating that the method of moments, if well designed and carefully optimized, can be regarded a highly efficient and reliable tool for the analysis and design of a wide class of complex 3D electromagnetic structures. MOM is often thought to be the method for solving "canonical" problems of subwavelength size. The large-domain (high-order current approximation) approach, used surprisingly rarely, promotes the applicability of the MOM to the solutions of real-world problems.

II. Integral Equations for Currents

Consider a structure consisting of arbitrarily shaped metallic and dielectric parts. Let the structure be situated in a time-harmonic incident (impressed) field of complex electric field intensity $\boldsymbol{E}_\mathrm{i}$ and angular frequency ω. The integral-equation numerical analysis of such a structure can be performed by employing either the volume or the surface equivalence principle.

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A. Volume Equivalence Principle

The incident field induces conduction and polarization currents, of density $\boldsymbol{J}$, in the structure volume. According to the volume equivalence principle, these currents can be considered to be in a vacuum, so that the scattered electric field, $\boldsymbol{E}$, can be expressed in terms of these currents through integrals which involve the free-space Green's function [3]. For the perfectly conducting parts of the structure, the volume currents can be replaced by surface currents, of density $\boldsymbol{J}_s$, over the perfectly conducting surfaces (PEC surfaces).

Generalized local Ohm's law and boundary condition on PEC give the following relations:

$$\frac{\boldsymbol{J}}{\sigma_e} - \boldsymbol{E}(\boldsymbol{J}, \boldsymbol{J}_s) = \boldsymbol{E}_i \quad \text{(inside dielectrics)} , \qquad (1)$$

$$-[\boldsymbol{E}(\boldsymbol{J}, \boldsymbol{J}_s)]_{\text{tang}} = (\boldsymbol{E}_i)_{\text{tang}} \quad \text{(over PEC surfaces)} , \qquad (2)$$

where $\sigma_e = \sigma + j\omega(\epsilon - \epsilon_0)$ is the dielectric equivalent complex conductivity. Note that the dielectrics can be both inhomogeneous and lossy. The integral equations (1) and (2), which include the integral expressions for the scattered field, $\boldsymbol{E}(\boldsymbol{J}, \boldsymbol{J}_s)$, represent a system of coupled, simultaneous electric-field integral equations, with $\boldsymbol{J}$ and $\boldsymbol{J}_s$ as unknowns.

B. Surface Equivalence Principle

Suppose now that the system under considerations consists of N homogeneous dielectric regions (domains), which generally include PEC surfaces. One of the domains is the external space (most often air, but can also be water, real ground, etc.) surrounding the structure. We can use the surface equivalence principle (generalized Huygens' principle) to break the system into N subsystems, each of them representing one of the dielectric regions, together with the belonging PEC surfaces, with the remaining space being filled with the same medium. To achieve this, a layer of equivalent (artificial) surface electric currents, of density $\boldsymbol{J}_s$, and a layer of equivalent (artificial) surface magnetic currents, of density $\boldsymbol{M}_s$, must be placed on the boundary surface of each dielectric region, with the objective to produce a zero total field in the surrounding space. These current densities are given by

$$\boldsymbol{J}_s = \boldsymbol{n} \times \boldsymbol{H}_{\text{tot}} , \qquad \boldsymbol{M}_s = -\boldsymbol{n} \times \boldsymbol{E}_{\text{tot}} , \qquad (3)$$

where $\boldsymbol{n}$ is the inward normal on the dielectric surface, and $\boldsymbol{E}_{\text{tot}}$ and $\boldsymbol{H}_{\text{tot}}$ are the total electric and magnetic field at the surface. On the PEC surfaces, only the (actual) surface electric currents ($\boldsymbol{J}_s$) exist. The scattered electric and magnetic fields in the region of complex permittivity ϵ and complex permeability μ can be expressed in terms of these currents as follows:

$$\boldsymbol{E} = \boldsymbol{E}(\boldsymbol{J}_s, \boldsymbol{M}_s, \epsilon, \mu) = -j\omega\boldsymbol{A} - \text{grad}\Phi - \frac{1}{\epsilon}\,\text{curl}\boldsymbol{F} , \qquad (4)$$

$$\boldsymbol{H} = \boldsymbol{H}(\boldsymbol{J}_s, \boldsymbol{M}_s, \epsilon, \mu) = -j\omega\boldsymbol{F} - \text{grad}U + \frac{1}{\mu}\,\text{curl}\boldsymbol{A} , \qquad (5)$$

$$\boldsymbol{A} = \mu \int_S \boldsymbol{J}_s\, g\, dS , \qquad \boldsymbol{F} = \epsilon \int_S \boldsymbol{M}_s\, g\, dS , \qquad (6)$$

$$\Phi = \frac{j}{\omega\epsilon} \int_S \text{div}_s \boldsymbol{J}_s\, g\, dS , \qquad U = \frac{j}{\omega\mu} \int_S \text{div}_s \boldsymbol{M}_s\, g\, dS . \qquad (7)$$

Here, $\boldsymbol{A}$ and $\boldsymbol{F}$ are the magnetic and electric vector potential, while Φ and U are the electric and magnetic scalar potential, respectively. S is the boundary surface of the region considered, and g the Green's function for the unbounded homogeneous medium of parameters ϵ and μ,

$$g = \frac{e^{-\gamma R}}{4\pi R} , \qquad \gamma = j\omega\sqrt{\epsilon\mu} , \qquad (8)$$

γ being the propagation coefficient in the medium and R the distance of the field point from the source point.

The boundary conditions for the tangential components of the total electric and magnetic field vectors on the boundary surface between dielectric domains 1 and 2 yield

$$[\boldsymbol{E}(\boldsymbol{J}_s, \boldsymbol{M}_s, \epsilon_1, \mu_1)]_{\text{tang}} + (\boldsymbol{E}_i)_{\text{tang}}$$
$$= [\boldsymbol{E}(-\boldsymbol{J}_s, -\boldsymbol{M}_s, \epsilon_2, \mu_2)]_{\text{tang}} \quad \text{(on surface 1-2)} , \qquad (9)$$

$$[\boldsymbol{H}(\boldsymbol{J}_s, \boldsymbol{M}_s, \epsilon_1, \mu_1)]_{\text{tang}} + (\boldsymbol{H}_i)_{\text{tang}}$$
$$= [\boldsymbol{H}(-\boldsymbol{J}_s, -\boldsymbol{M}_s, \epsilon_2, \mu_2)]_{\text{tang}} \quad \text{(on surface 1-2)} , \qquad (10)$$

where we assume that the incident/impressed field is present only in domain 1. Note the negative sign on the equivalent currents in the expressions of the fields in domain 2, which is due to the opposite directions of the normal $\boldsymbol{n}$ in Eq.(3) when applying the equivalence principle for the two adjacent domains. On the PEC surfaces, we reduce the boundary conditions (9) and (10) to $(\boldsymbol{E}_{\text{tot}})_{\text{tang}} = 0$ [as in Eq.(2)] only. Having in mind the integral expressions for the fields in Eqs.(4)–(8), the equations (9) and (10) represent a set of coupled electric/magnetic field integral equations for $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$ as unknowns.

III. Geometrical Modelling

We approximate all the surfaces (PEC and dielectric surfaces) by a system of bilinear quadrilaterals. A bilinear quadrilateral (Fig.1) is defined uniquely by its four vertices, that can be positioned arbitrarily in space. The parametric equation of the quadrilateral in a local

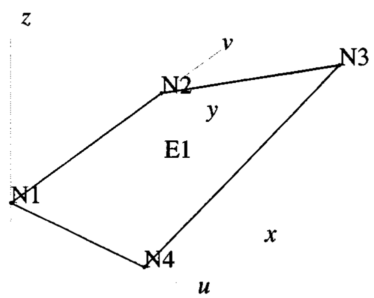

Fig. 1. Bilinear quadrilateral.

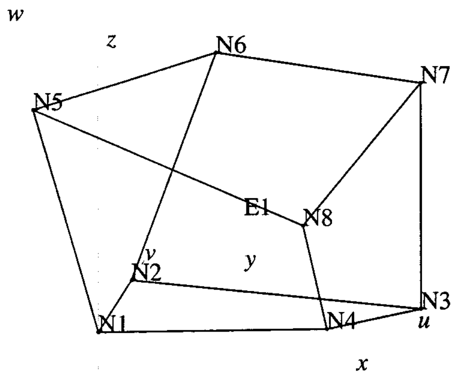

Fig. 2. Trilinear hexahedron.

(generally non-orthogonal) u-v coordinate system in the figure reads

$$\boldsymbol{r}(u,v) = \boldsymbol{r}_{\mathrm{c}} + \boldsymbol{r}_u u + \boldsymbol{r}_v v + \boldsymbol{r}_{uv} uv\ , \quad 0 \le u, v \le 1\ , \tag{11}$$

where $\boldsymbol{r}(u,v)$ is the position vector of a quadrilateral point, and $\boldsymbol{r}_{\mathrm{c}}$, $\boldsymbol{r}_u$, $\boldsymbol{r}_v$, and $\boldsymbol{r}_{uv}$ are constant vectors that can be expressed in terms of the position vectors of the quadrilateral vertices. The quadrilateral edges and all parametric coordinate lines are straight, but its surface is generally curved (inflexed).

As the basic volume element for the approximation of (inhomogeneous) dielectric bodies we adopt a trilinear hexahedron, sketched in Fig.2 [3]. This is a body determined solely by eight arbitrary points in space, which represent its vertices. The hexahedron edges and all coordinate lines are straight, while its sides (bilinear quadrilaterals) in the general case are curved.

Finally, if the EM structure contains wire-like PEC surfaces, we model them by straight wire segments, and approximate the actual current distribution $\boldsymbol{J}_{\mathrm{s}}$ over the wire surface by a line current, of intensity I, along a generatrix of the wire (the reduced-kernel approximation for thin wires).

IV. High-Order Expansion Functions for Currents

For the approximation of the u-, v-, and w-components of $\boldsymbol{J}$ inside bodies, the u- and v-components of $\boldsymbol{J}_{\mathrm{s}}$ and $\boldsymbol{M}_{\mathrm{s}}$ over surfaces, and the current intensity I along wires we use the polynomials in u, v, and w coordinates which satisfy automatically the corresponding current-continuity boundary conditions at the junctions of elements in a geometrical model. We adopt the testing (weighting) functions to be the same as the basis functions (Galerkin method).

The method includes models of generators, as well as of lumped and distributed loadings.

V. Results

A. Reflector Antenna Modelled After a Bat's Ear

Fig.3 shows the physical (experimental) and geometrical (simulation) model of a reflector antenna designed after the external ear geometry of a bat species (*Plecotus auritus*). The measured dimensions of the ear [4], are normalized with respect to the acoustic wavelength at 50 kHz (navigation acoustic frequency of the bat), and the antenna (physical model) is fabricated from the copper with dimensions normalized to the electromagnetic wavelength at 10 GHz, $\lambda_0 = 3$ cm. The motivation is to benefit from the accuracy and sensitivity of bat biosonar systems in the design of direction-finding radar. The antenna model consists of the primary (larger) and secondary (smaller) reflector, with a monopole feed, as described in Fig.3.

This antenna is complex in shape and is about $5.5\lambda_0 \times 2\lambda_0 \times 1\lambda_0$ large. The number of unknowns for the approximation of currents in the MOM simulation amounts to 986, and the CPU time required for the analysis, including the postprocessing, to 8 minutes on a PC Pentium 166 MHz (16 MB RAM memory).

The antenna exhibits a perfect match to 50 Ω at 10 GHz. The measured return loss using an HP8510B network analyzer is 16.7 dB. The MOM analysis is in excellent agreement giving 15.3 dB.

Fig.4 shows the simulation results for the gain of the antenna in the plane $\phi = 0°$ against the angle θ (using standard definition of spherical coordinates).

B. Microstrip-fed slot antenna

Shown in Fig.5 is a microstrip-fed slot antenna. It consists of a slot aperture in a finite-size square plate, a finite-size perfect-dielectric substrate that covers the plate, a strip situated on top of the substrate, and a wire, with a generator at its base, embedded in the dielectric from the strip beginning to the plate. The end of the strip is left open. The antenna is designed by the

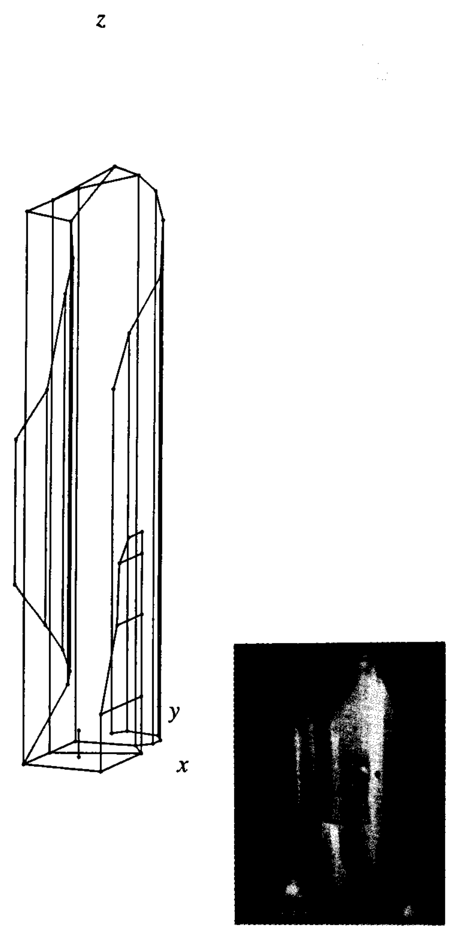

Fig. 3. Geometrical (simulation) model and photograph of bat-ear antenna. The primary reflector is 165 mm high in the z-direction and 60 mm wide. The distance between the secondary reflector (*tragus*), which is 62 mm high and 21.5 mm wide in its base, and the back of the primary reflector is 23.5 mm (x-direction). The antenna feed is a vertical 7.5 mm wire monopole. The distance of the wire axis from the *tragus* is 12.5 mm, and the wire diameter is 0.6 mm.

novel MOM code to be perfectly matched to 50 Ω at 7 GHz ($|s_{11}| = -27$ dB), near its second resonance. The code is utilized in the surface-equivalence version, which appears to be more suitable in cases when the metallic plates and the dielectric surfaces overlap exactly. The simulation is in excellent agreement with the experiment, which gives $f_{\text{match}} = 7.05$ GHz ($|s_{11}| = -16$ dB).

VI. Conclusions

The paper presents a large-domain Galerkin-type method of moments for the analysis of electromagnetic structures composed of arbitrarily excited and loaded dielectric and conducting bodies of arbitrary shape. It

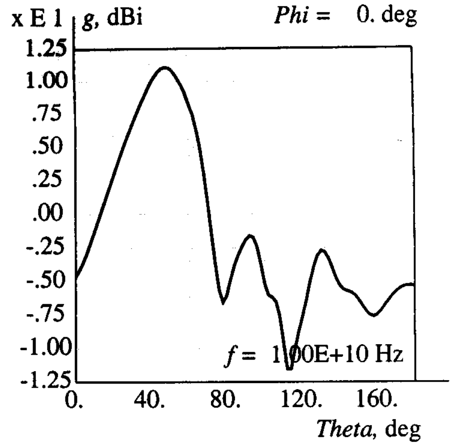

Fig. 4. Gain of antenna in Fig.3 in plane $\phi = 0°$.

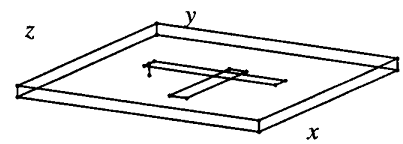

Fig. 5. Microstrip-fed slot antenna. The square-plate edge length is 70 mm, the thickness of the dielectric substrate 0.5 mm, and the substrate relative permittivity 2.2. The slot is 30.6 mm long and 1.66 mm wide. The length and width of the strip are 28 mm and 1.57 mm, respectively. The distance of the open-circuited end of the line from the center of the slot is 11 mm.

is founded on the integral-equation formulation in the frequency domain, and is built in two versions concerning the type of the equivalence invoked in the treatment of the dielectric parts of the structure, and consequently the type of unknown quantities in the dielectrics. Numerical results obtained by the novel MOM code, some of which are presented in the paper, have demonstrated that a well designed and carefully optimized moment-method can provide accurate and reliable analysis and design of complex and electrically quite large EM structures on even standard personal computers.

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