Chapter 2: Semiconductor Fundamentals

2.2. Crystals and crystal structures

$$\vec{r} = k\,\vec{a}_1 + l\,\vec{a}_2 + m\,\vec{a}_3 \tag{2.2.1}$$

2.3. Energy bands

$$E_g(T) = E_g(0) - \frac{aT^2}{T+b} \tag{2.3.1}$$

$$J_{vb} = \frac{1}{V} \sum_{\substack{\text{filled}\\ \text{states}}} (\text{-}q) v_i \tag{2.3.2}$$

$$J_{vb} = \frac{1}{V}\Big(\sum_{\substack{\text{all}\\ \text{states}}} (-q) v_i - \sum_{\substack{\text{empty}\\ \text{states}}} (-q) v_i \Big) \tag{2.3.3}$$

$$J_{vb} = \frac{1}{V} \sum_{\substack{\text{empty}\\ \text{states}}} (+q) v_i \tag{2.3.4}$$

2.4. Density of states

$$\Psi = A \sin(k_x x) + B \cos(k_x x) \tag{2.4.1}$$

$$k_x = \frac{np}{L}, \text{ n} = 1,2,3... \tag{2.4.2}$$

$$N = 2 \times \frac{1}{8} \times \left(\frac{L}{p}\right)^3 \times \frac{4}{3} \times p \times k^3 \tag{2.4.3}$$

$$\frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE} = \left(\frac{\text{L}}{p}\right)^3 p\, k^2 \frac{dk}{dE} \tag{2.4.4}$$

$$E(k) = \frac{\hbar^2 k^2}{2m^*} \tag{2.4.5}$$

$$g(E) = \frac{1}{L^3}\frac{dN}{dE} = \frac{8p\sqrt{2}}{h^3} m^{*3/2} \sqrt{E}, \text{ for } E \geq 0 \tag{2.4.6}$$

$$g_c(E) = \frac{1}{L^3}\frac{dN}{dE} = \frac{8p\sqrt{2}}{h^3} m^{*3/2} \sqrt{E - E_c}, \text{ for } E \geq E_c \tag{2.4.7}$$

$$g_c(E) = 0, \text{ for } E < E_c$$

2.5. Carrier distribution functions

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}} \tag{2.5.1}$$

$$f_{donor}(E_d) = \frac{1}{1+\frac{1}{2}e^{(E_d-E_F)/kT}} \tag{2.5.2}$$

$$f_{acceptor}(E_a) = \frac{1}{1+4e^{(E_a-E_F)/kT}} \tag{2.5.3}$$

$$f_{BE}(E) = \frac{1}{e^{(E-E_F)/kT}-1} \tag{2.5.4}$$

$$f_{MB}(E) = \frac{1}{e^{(E-E_F)/kT}} \tag{2.5.5}$$

2.6. Carrier densities

$$n(E) = g_c(E) f(E) \tag{2.6.1}$$

$$p(E) = g_v(E)[1 - f(E)] \tag{2.6.2}$$

$$g_c(E) = \frac{8p\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E-E_c}, \text{ for } E \geq E_c \tag{2.6.3}$$

$$g_v(E) = \frac{8p\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E_v-E}, \text{ for } E \leq E_v \tag{2.6.4}$$

$$n = \int_{E_c}^{\text{top of the conduction band}} n(E)dE = \int_{E_c}^{\text{top of the conduction band}} g_c(E) f(E) dE \tag{2.6.5}$$

$$n_o = \int_{E_c}^{\infty} g_c(E) f(E) dE \tag{2.6.6}$$

$$n_o = \int_{E_c}^{\infty} \frac{8p\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E-E_c} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE \tag{2.6.7}$$

$$p_o = \int_{-\infty}^{E_v} g_v(E)[1-f(E)]dE \tag{2.6.8}$$

$$p_o = \int_{-\infty}^{E_v} \frac{8p\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_v-E} \frac{1}{1+e^{\frac{E_F-E}{kT}}} dE \tag{2.6.9}$$

$$n_o = \int_{E_c}^{E_F} g_c(E)dE \text{ at } T = 0\text{ K} \tag{2.6.10}$$

$$n_o = \frac{2}{3}\frac{\sqrt{2}}{p^2}\left(\frac{qm^*}{\hbar^2}\right)^{3/2}(E_F - E_c)^{3/2}, \text{ for } E_F \geq E_c \tag{2.6.11}$$

$$n_o \cong \int_{E_c}^{\infty} \frac{8p\sqrt{2}}{h^3} m_e^{*3/2}\sqrt{E - E_c}\, e^{\frac{E_F - E}{kT}} dE = N_c e^{\frac{E_F - E_c}{kT}} \tag{2.6.12}$$

$$N_c = 2[\frac{2p\, m_e^* kT}{h^2}]^{3/2} \tag{2.6.13}$$

$$p_o \cong \int_{-\infty}^{E_v} \frac{8p\sqrt{2}}{h^3} m_h^{*3/2}\sqrt{E_v - E}\, e^{\frac{E - E_F}{kT}} dE = N_v e^{\frac{E_v - E_F}{kT}} \tag{2.6.14}$$

$$N_v = 2[\frac{2p\, m_h^* kT}{h^2}]^{3/2} \tag{2.6.15}$$

$$\frac{E_F - E_c}{kT} \cong \ln\frac{n_o}{N_c} + \frac{1}{\sqrt{8}}\frac{n_o}{N_c} - (\frac{3}{16} - \frac{\sqrt{3}}{9})(\frac{n_o}{N_c})^2 + \ldots \tag{2.6.16}$$

$$\frac{E_v - E_F}{kT} \cong \ln\frac{p_o}{N_v} + \frac{1}{\sqrt{8}}\frac{p_o}{N_v} - (\frac{3}{16} - \frac{\sqrt{3}}{9})(\frac{p_o}{N_v})^2 + \ldots \tag{2.6.17}$$

$$n_i = n_o\big|_{(E_F = E_i)} = N_c e^{(E_i - E_c)/kT} \tag{2.6.18}$$

$$n_i = p_o\big|_{(E_F = E_i)} = N_v e^{(E_v - E_i)/kT}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \tag{2.6.19}$$

$$n_o \cdot p_o = N_c N_v e^{(E_v - E_c)/kT} = n_i^2 \tag{2.6.20}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{1}{2}kT\ln(\frac{N_v}{N_c}) \tag{2.6.21}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4}kT\ln(\frac{m_h^*}{m_e^*}) \tag{2.6.22}$$

$$n_o = n_i e^{(E_F - E_i)/kT} \tag{2.6.23}$$

$$p_o = n_i e^{(E_i - E_F)/kT} \quad (2.6.24)$$

$$E_F = E_i + kT \ln \frac{n_o}{n_i} \quad (2.6.25)$$

$$E_F = E_i - kT \ln \frac{p_o}{n_i} \quad (2.6.26)$$

$$N_d^+ \cong N_d \quad (2.6.27)$$

$$N_a^- \cong N_a \quad (2.6.28)$$

$$n_o \cong N_d^+ - N_a^-, \quad \text{if} \quad N_d^+ - N_a^- >> n_i \quad (2.6.29)$$

$$p_o \cong N_a^- - N_d^+, \quad \text{if} \quad N_a^- - N_d^+ >> n_i \quad (2.6.30)$$

$$E_c - E_d = 13.6 \frac{m_{cond}^*}{m_0 e_r^2} \text{ eV} \quad (2.6.31)$$

$$r = q(p_o - n_o + N_d^+ - N_a^-) = 0 \quad (2.6.32)$$

$$n_o = \frac{n_i^2}{n} + N_d^+ - N_a^- \quad (2.6.33)$$

$$n_o = \frac{N_d^+ - N_a^-}{2} + \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2 + n_i^2} \quad (2.6.34)$$

$$p_o = \frac{N_a^- - N_d^+}{2} + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2 + n_i^2} \quad (2.6.35)$$

$$p_o + N_d^+ = n_o + N_a^- \quad (2.6.36)$$

$$n = n_o + d\,n = n_i \exp\left(\frac{F_n - E_i}{kT}\right) \quad (2.6.37)$$

$$p = p_o + d\,p = n_i \exp\left(\frac{E_i - F_p}{kT}\right) \quad (2.6.38)$$

2.7. Carrier Transport

$$I = \frac{Q}{t_r} = \frac{Q}{L/v} \quad (2.7.1)$$

$$\vec{J} = \frac{Q}{AL}\vec{v} = r\vec{v} = qn\vec{v} \tag{2.7.2}$$

$$\vec{F} = m\vec{a} = m\frac{d < \vec{v} >}{dt} \tag{2.7.3}$$

$$\vec{F} = q\vec{E} - \frac{m < \vec{v} >}{t} \tag{2.7.4}$$

$$q\vec{E} = m\frac{d < \vec{v} >}{dt} + \frac{m < \vec{v} >}{t} \tag{2.7.5}$$

$$m \stackrel{\Delta}{=} \frac{|\vec{v}|}{|\vec{E}|} = \frac{qt}{m} \tag{2.7.6}$$

$$\vec{J} = qnm_n\vec{E} \tag{2.7.7}$$

$$m = \frac{qt}{m^*} \tag{2.7.8}$$

$$m = m_{\min} + \frac{m_{\max} - m_{\min}}{1 + (\frac{N}{N_r})^a} \tag{2.7.9}$$

$$J = qnv_e + qpv_h = q(nm_n + pm_p)E \tag{2.7.10}$$

$$s \stackrel{\Delta}{=} \frac{J}{E} = q(nm_n + pm_p) \tag{2.7.11}$$

$$r = \frac{1}{s} = \frac{1}{q(m_n n + m_p p)} \tag{2.7.12}$$

$$R_s = \frac{r}{t} \tag{2.7.13}$$

$$R = R_s \frac{L}{W} \tag{2.7.14}$$

$$v(E) = \frac{mE}{1 + \frac{mE}{v_{sat}}} \tag{2.7.15}$$

$$v_{th} = \frac{l}{t} \tag{2.7.16}$$

$$\Phi_{n,left \rightarrow right} = \tfrac{1}{2} v_{th} n(x = -l) \tag{2.7.17}$$

$$\Phi_{n,right \to left} = \tfrac{1}{2} v_{th} n(x = l) \tag{2.7.18}$$

$$\Phi_n = \Phi_{n,left \to right} - \Phi_{n,right \to left} = \tfrac{1}{2} v_{th} [n(x = -l) - n(x = l)] \tag{2.7.19}$$

$$\Phi_n = -l v_{th} \frac{n(x = l) - n(x = -l)}{2l} = -l v_{th} \frac{dn}{dx} \tag{2.7.20}$$

$$J_n = -q\Phi_n = q l v_{th} \frac{dn}{dx} \tag{2.7.21}$$

$$J_n = q D_n \frac{dn}{dx} \tag{2.7.22}$$

$$J_p = -q D_p \frac{dp}{dx} \tag{2.7.23}$$

$$v_{th} = \frac{l}{t} \tag{2.7.24}$$

$$\frac{kT}{2} = \frac{m^* v_{th}^2}{2} \tag{2.7.25}$$

$$l v_{th} = \frac{m^* v_{th}^2}{q} \frac{qt}{m^*} = \frac{kT}{q} m \tag{2.7.26}$$

$$D_n = m_n \frac{kT}{q} = m_n V_t \tag{2.7.27}$$

$$D_p = m_p \frac{kT}{q} = m_p V_t \tag{2.7.28}$$

$$J_n = q n m_n E + q D_n \frac{dn}{dx} \tag{2.7.29}$$

$$J_p = q p m_p E - q D_p \frac{dp}{dx} \tag{2.7.30}$$

$$I_{total} = A(J_n + J_p) \tag{2.7.31}$$

2.8. Carrier recombination and generation

$$U_n = R_n - G_n = \frac{n_p - n_{p0}}{t_n} \tag{2.8.1}$$

$$U_p = R_p - G_p = \frac{p_n - p_{n0}}{t_p} \tag{2.8.2}$$

$$U_{b-b} = b(np - n_i^2) \tag{2.8.3}$$

$$U_{SHR} = \frac{pn - n_i^2}{p + n + 2n_i \cosh(\frac{E_i - E_t}{kT})} N_t v_{th} \sigma \tag{2.8.4}$$

$$U_n = R_n - G_n = \frac{n_p - n_{p0}}{\tau_n} \tag{2.8.5}$$

$$U_p = R_p - G_p = \frac{p_n - p_{n0}}{\tau_p} \tag{2.8.6}$$

$$\tau_n = \tau_p = \frac{1}{N_t v_{th} \sigma} \tag{2.8.7}$$

$$U_{s,SHR} = \frac{pn - n_i^2}{p + n + 2n_i \cosh(\frac{E_i - E_{st}}{kT})} N_{st} v_{th} \sigma_s \tag{2.8.8}$$

$$U_{s,n} = R_{s,n} - G_{s,n} = v_s (n_p - n_{p0}) \tag{2.8.9}$$

$$v_s = N_{st} v_{th} \sigma_s \tag{2.8.10}$$

$$U_{Auger} = \Gamma_n n(np - n_i^2) + \Gamma_p p(np - n_i^2) \tag{2.8.11}$$

$$G_{p,light} = G_{n,light} = \alpha \frac{P_{opt}(x)}{E_{ph} A} \tag{2.8.12}$$

$$\frac{dP_{opt}(x)}{dx} = -\alpha P_{opt}(x) \tag{2.8.13}$$

2.9. Continuity equation

$$\frac{\partial n(x,t)}{\partial t} A\,dx = (\frac{J_n(x)}{-q} - \frac{J_n(x+dx)}{-q})A + (G_n(x,t) - R_n(x,t))\,A\,dx \tag{2.9.1}$$

$$J_n(x+dx) = J_n(x) + \frac{d\,J_n(x)}{dx} dx \tag{2.9.2}$$

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q} \frac{\partial J_n(x,t)}{\partial x} + G_n(x,t) - R_n(x,t) \tag{2.9.3}$$

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q} \frac{\partial J_p(x,t)}{\partial x} + G_p(x,t) - R_p(x,t) \tag{2.9.4}$$

$$\frac{\partial n(x,t)}{\partial t} = \mu_n n \frac{\partial E(x,t)}{\partial x} + \mu_n E \frac{\partial n(x,t)}{\partial x} + D_n \frac{\partial^2 n(x,t)}{\partial x^2} + G_n(x,t) - R_n(x,t) \tag{2.9.5}$$

$$\frac{\partial p(x,t)}{\partial t} = -\mu_p p \frac{\partial E(x,t)}{\partial x} - \mu_p E \frac{\partial p(x,t)}{\partial x} + D_p \frac{\partial^2 p(x,t)}{\partial x^2} + G_p(x,t) - R_p(x,t) \tag{2.9.6}$$

$$\frac{\partial n(x,y,z,t)}{\partial t} = \tfrac{1}{q} \vec{\nabla} \vec{J}_n(x,y,z,t) + G_n(x,y,z,t) - R_n(x,y,z,t) \tag{2.9.7}$$

$$\frac{\partial p(x,y,z,t)}{\partial t} = -\tfrac{1}{q} \vec{\nabla} \vec{J}_p(x,y,z,t) + G_p(x,y,z,t) - R_p(x,y,z,t) \tag{2.9.8}$$

$$\frac{\partial n(x,t)}{\partial t} = D_n \frac{\partial^2 n_p(x,t)}{\partial x^2} - \frac{n_p(x,t) - n_{p0}}{\tau_n} \tag{2.9.9}$$

$$\frac{\partial p(x,t)}{\partial t} = D_p \frac{\partial^2 p_n(x,t)}{\partial x^2} - \frac{p_n(x,t) - p_{n0}}{\tau_p} \tag{2.9.10}$$

$$0 = D_n \frac{d^2 n_p(x)}{dx^2} - \frac{n_p(x) - n_{p0}}{\tau_n} \tag{2.9.11}$$

$$0 = D_p \frac{d^2 p_n(x)}{dx^2} - \frac{p_n(x) - p_{n0}}{\tau_p} \tag{2.9.12}$$

$$p_n(x \ge x_n) = p_{n0} + A e^{-(x-x_n)/L_p} + B e^{(x-x_n)/L_p} \tag{2.9.13}$$

$$n_p(x \le -x_p) = n_{p0} + C e^{-(x+x_p)/L_p} + D e^{(x+x_p)/L_p} \tag{2.9.14}$$

$$L_n = \sqrt{D_n \tau_n} \tag{2.9.15}$$

$$L_p = \sqrt{D_p \tau_p} \tag{2.9.16}$$

$$n = n_0 + \delta n \tag{2.9.17}$$

$$p = p_0 + \delta p \tag{2.9.18}$$

$$0 = \frac{d^2(dn_p)}{dx^2} - \frac{dn_p}{L_n^2} \qquad (2.9.19)$$

$$0 = \frac{d^2(d\,p_n)}{dx^2} - \frac{d\,p_n}{L_p^2} \qquad (2.9.20)$$

2.10. The drift-diffusion model

$$r = q(p - n + N_d^+ - N_a^-) \qquad (2.10.1)$$

$$\frac{dE}{dx} = \frac{r}{e} \qquad (2.10.2)$$

$$\frac{df}{dx} = -E \qquad (2.10.3)$$

$$\frac{dE_i}{dx} = qE \qquad (2.10.4)$$

$$n = n_i e^{(F_n - E_i)/kT} \qquad (2.10.5)$$

$$p = n_i e^{(E_i - F_p)/kT} \qquad (2.10.6)$$

$$J_n = qnm_n E + qD_n \frac{dn}{dx} \qquad (2.10.7)$$

$$J_p = qpm_p E - qD_p \frac{dp}{dx} \qquad (2.10.8)$$

$$0 = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh(\frac{E_t - E_i}{kT})}\frac{1}{t} \qquad (2.10.9)$$

$$0 = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh(\frac{E_t - E_i}{kT})}\frac{1}{t} \qquad (2.10.10)$$