Example 2.12  

A 1 cm long piece of undoped silicon with a lifetime of 1 ms is illuminated with light, generating \( G_{\text{opt}} = 2 \times 10^{19} \text{cm}^{-2} \text{s}^{-1} \) electron-hole pairs in the middle of the silicon.

This bar silicon has ideal Ohmic contacts on both sides. Find the excess electron density throughout the material using the simple recombination model and assuming that \( \mu_n = \mu_p = 1000 \text{ cm}^2/\text{V} \cdot \text{s} \).

Also find the resulting electron current density throughout the material.

Solution  

Because of the symmetry one can treat each half separately with half the number of electron-hole pairs generated on both sides.

Then we solve the diffusion equation:

\[
0 = D_n \frac{d^2 n}{dx^2} - \frac{n - n_o}{\tau_n}
\]

The general solution to the diffusion equation equals:

\[
n - n_o = A \exp(x / L_n) + B \exp(-x / L_n)
\]

where \( A \) and \( B \) need to be determined by applying the boundary conditions.

Since an ideal contact implies that the material is in thermal equilibrium at the contact,

\[
n(x=0) = n_o
\]

so that \( A = -B \) and, linking the current density due to the carrier generation to the carrier gradient.

\[
J_n(x = L/2) = qD_n \frac{dn}{dx} \bigg|_{x=L/2} = q \frac{G_{\text{opt}}}{2}
\]

one finds that:

\[
\frac{dn}{dx} \bigg|_{x=L/2} = \frac{A}{L_n} \exp(-L / 2L_n) - \frac{B}{L_n} \exp(-L / 2L_n) = \frac{G_{\text{opt}}}{2D_n}
\]

or

\[
A = \frac{G_{\text{opt}}}{4D_n} \frac{L_n}{\cosh(L / 2L_n)}
\]

And the solution for the electron density becomes:
\[ n - n_o = \frac{G_{opt} L_n}{2D_n} \frac{L_n}{\cosh(\frac{L}{2L_n})} \sinh\left(\frac{x}{L_n}\right) \]

The corresponding electron current density is then:

\[ J_n(x) = \frac{G_{opt}}{2} \frac{\cosh\left(\frac{x}{L_n}\right)}{\cosh\left(\frac{L}{2L_n}\right)} \]

Both the excess electron density and the electron current density are plotted versus position on the graph below.