3.3.7. Solution to Poisson's Equation

To assess the error made when using the full depletion approximation we now derive the correct solution by solving Poisson's equation analytically[2]. The actual solution for the potential is then obtained by numerically integrating the expression for the electric field. We start from the charge density, r, in a semiconductor for the general case where electrons, holes, ionized acceptors and ionized donors are present:

$$r(f) = q(p + N_d^+ - n - N_a^-) \tag{3.3.19}$$

where f is the potential in the semiconductor. The potential is chosen to equal zero deep into the semiconductor. For an *n*-type semiconductor without acceptors or free holes this can be further reduced to:

$$r(f) = qN_d(1 - \exp(\frac{qf}{kT})) \tag{3.3.20}$$

assuming the semiconductor to be non-degenerate and fully ionized. A similar expression can be obtained for *p*-type material. Poisson's law can then be rewritten as:

$$\frac{d^2f}{dx^2} = -\frac{r(f)}{e_s} = -qN_d(1 - \exp(\frac{qf}{kT})) \tag{3.3.21}$$

Multiplying both sides with df/dx, this equation can be integrated between an arbitrary point x and infinity. The electric field at infinity (deep in the semiconductor) is taken to be zero. The electric field for a given potential is then:

$$E(f) = \sqrt{2\int_0^f \frac{-qN_d}{e_s}(1 - \exp(\frac{qf}{kT}))\,df} = sign(f)\frac{V_t}{L_D}\sqrt{2[\exp(\frac{f}{V_t}) - (\frac{f}{V_t}) - 1]} \tag{3.3.22}$$

Where the sign function equals +1 or -1 depending on the sign of f and L_D is the Debye length given by, $L_D = \sqrt{\frac{e_s kT}{q^2 N_d}}$. Equation (3.3.22) is plotted in Figure 3.3.4 using normalized parameters. Depletion occurs for negative potentials while accumulation occurs for positive potentials.

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[2]This derivation follows that of Goodman and Perkins, J. Appl. Phys. **35**, p 3351, 1964.

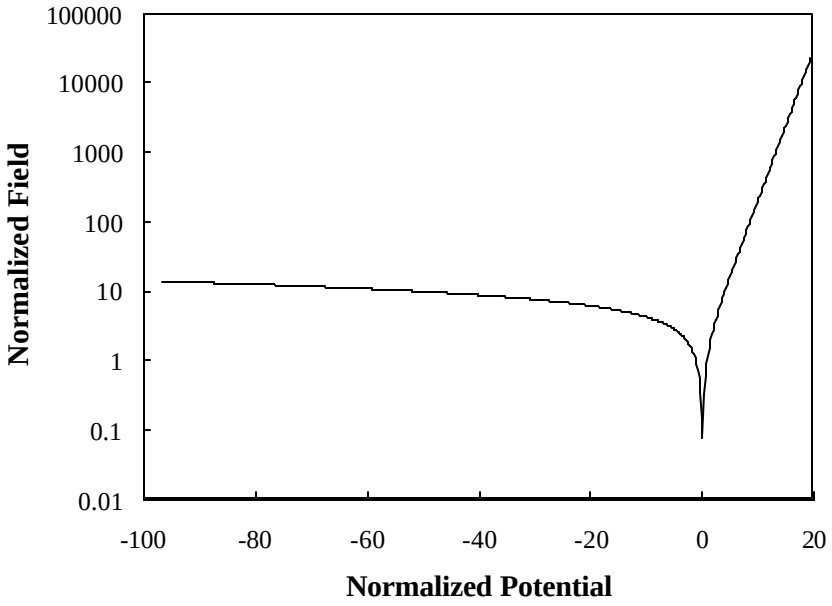

Figure 3.3.4 Absolute value of the normalized electric field, $|E|\ L_D/V_t$, versus normalized potential, f/V_t

Applying Gauss's law ($Q = e_s E$), we then find the relation between the total charge in the semiconductor region and the total potential across the semiconductor. The capacitance can also be obtained from:

$$C = \left|\frac{dQ}{dV_a}\right| = e_s \left|\frac{dE}{dV_a}\right| = \frac{e_s}{L_D}\left|\frac{\exp(\frac{f}{V_t}) - 1}{\sqrt{2[\exp(\frac{f}{V_t}) - (\frac{f}{V_t}) - 1]}}\right| \tag{3.3.23}$$

where f_s is the potential across the semiconductor and equals $-f_i + V_a$. This expression can be approximated for $f_s < 0$ and $|f_s| >> V_t$, yielding:

$$C = \frac{e_s}{L_D}\frac{1}{\sqrt{2\frac{f_i - V_a - V_t}{V_t}}} = \sqrt{\frac{e_s q N_d}{2(f_i - V_a - V_t)}} \tag{3.3.24}$$

This expression equals (3.3.10) as derived using the full depletion approximation, except for the added term, V_t, in the denominator. This expression yields the capacitance value with a relative accuracy better than 0.3 % for $V_a < f_i - 6V_t$.

3.3.7.1.Numeric solution

A numeric solution can be obtained by integrating equation (3.3.21). The solution to the energy band diagram, the charge density, the electric field and the potential are shown in the figures below: Integration was started four Debye lengths to the right of the edge of the depletion region as obtained using the full depletion approximation. Initial conditions were obtained by assuming the potential at the starting point to be adequately expressed by a solution to the homogenous equation:

$$f = V_t \exp\left(-\frac{(x - x_d)}{L_D}\right) \tag{3.3.25}$$

Shown are solutions for a gold-silicon M-S junction with Φ_M = 4.75V, c = 4.05V, N_d = 10^{16} cm^{-3} and e_s/e_0 = 11.9.

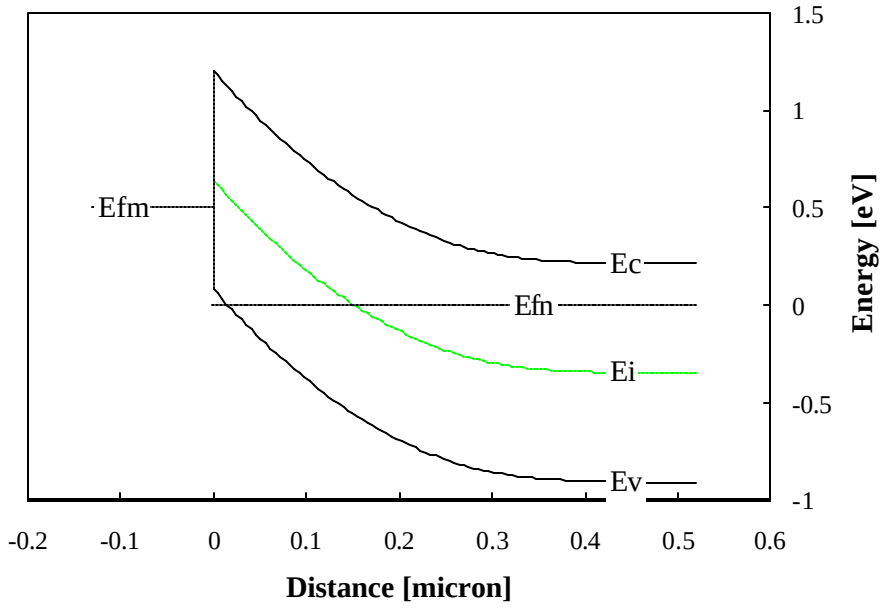

Figure 3.3.5 Energy band diagram of an M-S junction

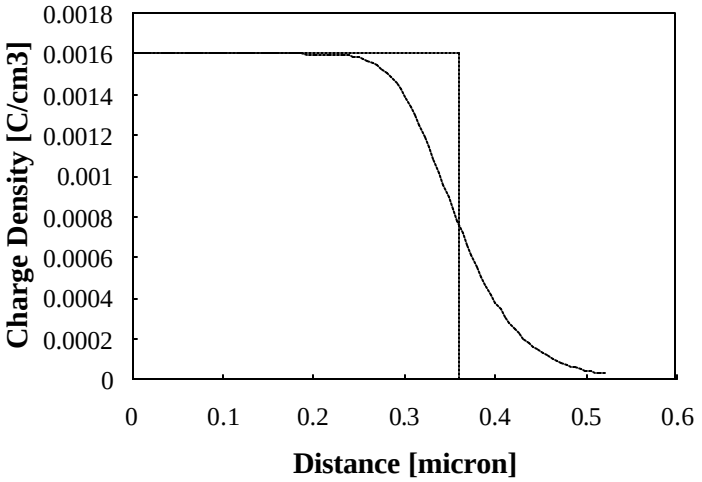

Figure 3.3.6 Charge density versus position in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

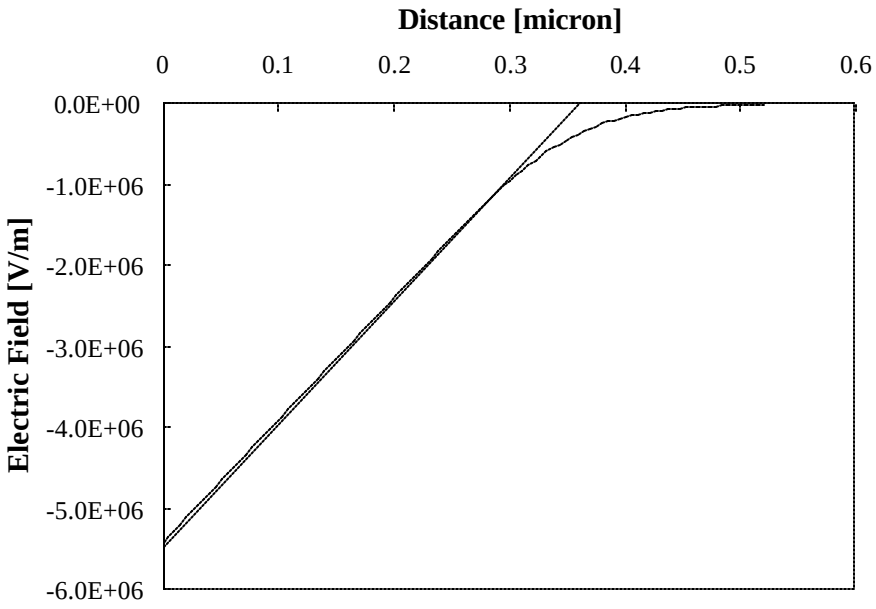

Figure 3.3.7 Electric field versus distance in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

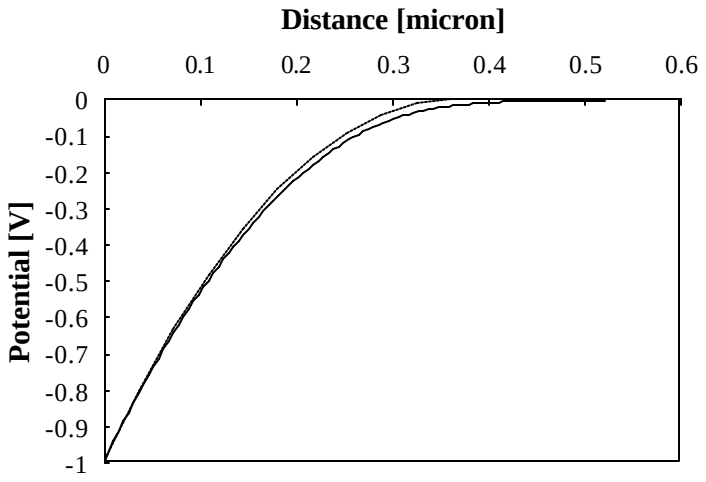

Figure 3.3.8 Potential versus distance of an M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.