4.3.7. Solution to Poisson's Equation for an Abrupt p-n Junction

Applying Gauss's law one finds that the total charge in the n-type depletion region equals minus the charge in the p-type depletion region:

$$Q_n = e_s \left| E(x=0) \right| = -Q_p \tag{4.3.45}$$

Poisson's equation can be solved separately in the n and p-type region as was done in section 3.1.1 yielding an expression for $E(x = 0)$ which is almost identical to equation [3.1.4]:

$$\left| E(x=0) \right| = \frac{V_t}{L_{D,n}} \sqrt{2(\exp \frac{f_n}{V_t} - \frac{f_n}{V_t} - 1)} = \frac{V_t}{L_{D,n}} \sqrt{2(\exp \frac{f_p}{V_t} - \frac{f_p}{V_t} - 1)} \tag{4.3.46}$$

where f_n and f_p are assumed negative if the semiconductor is depleted. Their relation to the applied voltage is given by:

$$f_n + f_p = V_a - f_i \tag{4.3.47}$$

Solving the transcendental equations one finds f_n and f_p as a function of the applied voltage. In the special case of a symmetric doping profile, or $N_d = N_a$ these equations can easily be solved yielding:

$$f_n = f_p = \frac{V_a - f_i}{2} \tag{4.3.48}$$

The depletion layer widths also equal each other and are given by:

$$x_n = x_p = \left| \frac{Q_n}{qN_d} \right| = \left| \frac{Q_p}{qN_a} \right| = \left| \frac{e_s E(x=0)}{qN_d} \right| \tag{4.3.49}$$

Using the above expression for the electric field at the origin we find:

$$x_n = x_p = L_D \sqrt{2 \exp \frac{V_a - f_i}{2V_t} + \frac{f_i - V_a - V_t}{V_t}} \tag{4.3.50}$$

where L_D is the extrinsic Debye length. The relative error of the depletion layer width as obtained using the full depletion approximation equals:

$$\frac{\Delta x_n}{x_n} = \frac{\Delta x_p}{x_p} = \frac{1-\sqrt{1-\frac{V_t}{f_i - V_a}+\frac{V_t}{2(f_i - V_a)}\exp(\frac{V_a - f_i}{2V_t})}}{\sqrt{1-\frac{V_t}{f_i - V_a}+\frac{V_t}{2(f_i - V_a)}\exp(\frac{V_a - f_i}{2V_t})}} \tag{4.3.51}$$

for $\frac{f_i - V_a}{V_t}$ = 1, 2, 5, 10, 20 and 40 one finds the relative error to be 45, 23, 10, 5.1, 2.5 and 1.26 %.