

#### 4.4.6. High Injection Effects

High injection of carriers<sup>1</sup> causes to violate one of the approximations made in the derivation of the ideal diode characteristics, namely that the majority carrier density equals the thermal equilibrium value. Excess carriers will dominate the electron and hole concentration and can be expressed in the following way:

$$n_p p_p = n_i^2 \exp\left(\frac{V_a}{V_t}\right) = n_p (p_{p0} + n_p) \quad (4.4.50)$$

$$n_n p_n = n_i^2 \exp\left(\frac{V_a}{V_t}\right) = p_n (n_{n0} + p_n) \quad (4.4.51)$$

where all carrier densities with subscript n are taken at  $x = x_n$  and those with subscript p at  $x = x_p$ . Solving the resulting quadratic equation yields:

$$n_p = \frac{N_a}{2} \left( \sqrt{1 + \frac{4n_i^2 \exp\left(\frac{V_a}{V_t}\right)}{N_a^2}} - 1 \right) \cong n_i^2 \exp\left(\frac{V_a}{2V_t}\right) \quad (4.4.52)$$

$$p_n = \frac{N_d}{2} \left( \sqrt{1 + \frac{4n_i^2 \exp\left(\frac{V_a}{V_t}\right)}{N_d^2}} - 1 \right) \cong n_i^2 \exp\left(\frac{V_a}{2V_t}\right) \quad (4.4.53)$$

where the second terms are approximations for large  $V_a$ . From these expressions one can calculate the minority carrier diffusion current assuming a "long" diode. We also ignore carrier recombination in the depletion region.

$$J_n + J_p = q \left( \frac{D_n}{L_n} + \frac{D_p}{L_p} \right) n_i \exp\left(\frac{V_a}{2V_t}\right) \quad (4.4.54)$$

This means that high injection in a p-n diode will reduce the slope on the current-voltage characteristic on a semi-logarithmic scale to 119mV/decade.

High injection also causes a voltage drop across the quasi-neutral region. This voltage can be calculated from the carrier densities. Let's assume that high injection only occurs in the (lower doped) p-type region. The hole density at the edge of the depletion region ( $x = x_p$ ) equals:

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<sup>1</sup>A more complete derivation can be found in R.S. Muller and T.I. Kamins, "Device Electronics for Integrated Circuits", Wiley and sons, second edition p. 323-324.

$$p_p(x = x_p) = N_a \exp\left(\frac{V_1}{V_t}\right) = N_a + n_p(x = x_p) = N_a + n_{p0} \exp\left(\frac{V_a - V_1}{V_t}\right) \quad (4.4.55)$$

where  $V_1$  is the voltage drop across the p-type quasi-neutral region. This equation can then be solved for  $V_1$  yielding

$$V_1 = V_t \ln\left[\frac{\sqrt{1 + \frac{4n_i^2 \exp\left(\frac{V_a}{V_t}\right)}{N_a^2}} - 1}{2}\right] \quad (4.4.56)$$

High injection occurs (by definition) when the excess minority carrier density exceeds the doping density in the material. It is under such conditions that also the majority carrier density increases since, for charge neutrality to exist, the excess electron density has to equal the excess hole density: If there exists a net charge, the resulting electric field causes the carriers to move so that charge neutrality is restored.

Up till now we have assumed that the minority carrier densities in the n-type and p-type region are small compared to the doping density. However as the forward bias voltage is increased the minority carrier density exceeds the doping density.

The analysis starts by assuming a certain excess carrier density so that the total density can be written as the sum of the thermal equilibrium density plus the excess carrier density. For an n-type region this yields the following equations:

$$n_n = n_{n0} + \mathbf{d} n_n \quad (4.4.57)$$

$$p_n = p_{p0} + \mathbf{d} p_n \quad (4.4.58)$$

The product of the carrier densities can be expressed as a function of the intrinsic density in the following way:

$$n_n p_n = n_i^2 \exp\left(\frac{V_a}{V_t}\right) = (n_{n0} + \mathbf{d} n_n)(p_{p0} + \mathbf{d} p_n) \quad (4.4.59)$$

where it was assumed that the semiconductor is non-degenerate and that the difference between the electron and hole quasi Fermi energies in electron volt equals the applied voltage in volt. Quasi-neutrality implies that the excess densities are the same, which yields a quadratic equation for the minority carrier density  $p_n$ :

$$(\mathbf{d} p_n)^2 + N_a \mathbf{d} p_n - n_i^2 \left(\exp\left(\frac{V_a}{V_t}\right) - 1\right) = 0 \quad (4.4.60)$$

which yields a value for the minority carrier density at the edge of the depletion region.

$$\mathbf{d} p_n = \frac{N_d}{2} \left( \sqrt{1 + \frac{4n_i^2 [\exp(\frac{V_a}{V_t}) - 1]}{N_d^2}} - 1 \right) \quad (4.4.61)$$

The associated current is a diffusion current and using a procedure similar to that for calculating the ideal diode current in a "long" diode one obtains the following hole current<sup>2</sup>.

$$I_p = qA \frac{D_p \mathbf{d} p_n}{L_p} = qA \frac{D_p N_d}{2L_p} \left( \sqrt{1 + \frac{4n_i^2 [\exp(\frac{V_a}{V_t}) - 1]}{N_d^2}} - 1 \right) \quad (4.4.62)$$

The electron current due to diffusion of electrons in the p-type region is given by a similar expression:

$$I_n = qA \frac{D_n \mathbf{d} n_p}{L_n} = qA \frac{D_n N_a}{2L_n} \left( \sqrt{1 + \frac{4n_i^2 [\exp(\frac{V_a}{V_t}) - 1]}{N_a^2}} - 1 \right) \quad (4.4.63)$$

These expressions can be reduced to the ideal diode expressions provided that the excess minority density is much smaller than one quarter of the doping density, or:

$$I_p = qA \frac{D_p n_{p0}}{L_p} [\exp(\frac{V_a}{V_t}) - 1] \quad , \text{ for } n_{p0} [\exp(\frac{V_a}{V_t}) - 1] \ll \frac{N_d}{4} \quad (4.4.64)$$

while if the excess minority carrier density is much larger than one quarter of the doping density and expression is obtained which is only valid under high injection conditions:

$$I_p = qA \frac{D_p n_i}{L_p} \exp(\frac{V_a}{2V_t}) \quad , \text{ for } n_{p0} [\exp(\frac{V_a}{V_t}) - 1] \gg \frac{N_d}{4} \quad (4.4.65)$$

A closer examination of the problem prompts the question whether the full depletion approximation is still valid since the sign of potential across the semiconductor reverses. However the increase of the majority carrier density beyond the doping density causes a potential variation across the "quasi-neutral" region. This voltage in the n-type region is given by:

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<sup>2</sup> The reader should note that the potential across the "quasi-neutral" regions goes hand-in-hand with an electric field. The region is therefore not neutral so that the diffusion equation is no longer valid. Instead one has to calculate the current based on the diffusion and drift of the carriers. The equations above were derived using the diffusion equation and should therefore be used with caution.

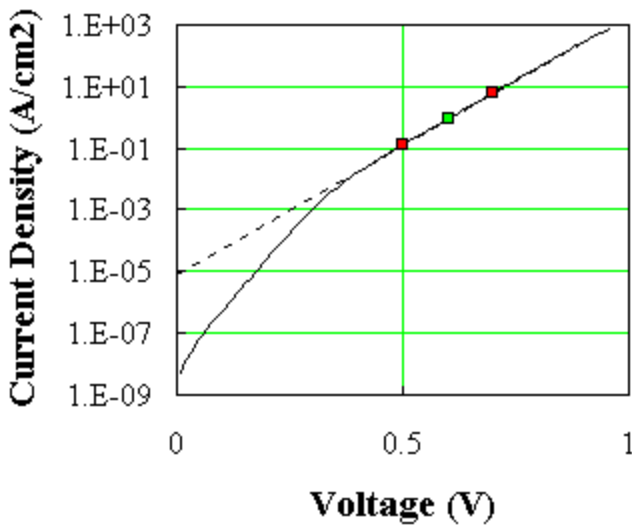
$$V_n = V_t = \ln \frac{n_{n0} + \mathbf{d}n_n}{n_{n0}} = V_t \ln \left[ \frac{\sqrt{1 + \frac{4n_i^2 \exp(\frac{V_a}{V_t})}{N_d^2}} - 1}{2} \right] \quad (4.4.66)$$

and similarly for the p-type region:

$$V_p = V_t = \ln \frac{p_{p0} + \mathbf{d}p_p}{p_{p0}} = V_t \ln \left[ \frac{\sqrt{1 + \frac{4n_i^2 \exp(\frac{V_a}{V_t})}{N_a^2}} - 1}{2} \right] \quad (4.4.67)$$

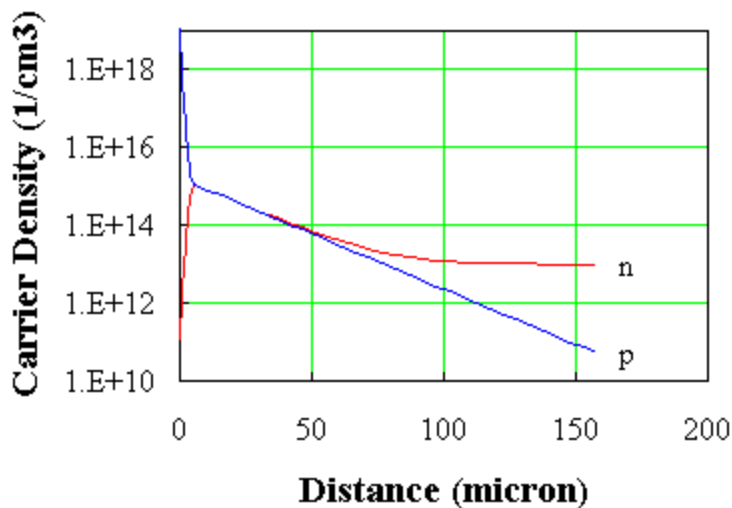
The potentials across the "quasi-neutral" region causes a larger potential across the depletion layer, since they have opposite sign, so that the depletion layer width as calculated using the modified potential  $\mathbf{f} = \mathbf{f}_i - V_a + V_n + V_p$  does not become zero.

As an example we now consider an abrupt one-sided p-n diode. The current is shown as function of the voltage in the figure below. It is calculated for an n-type doping of  $10^{13} \text{ cm}^{-3}$ .

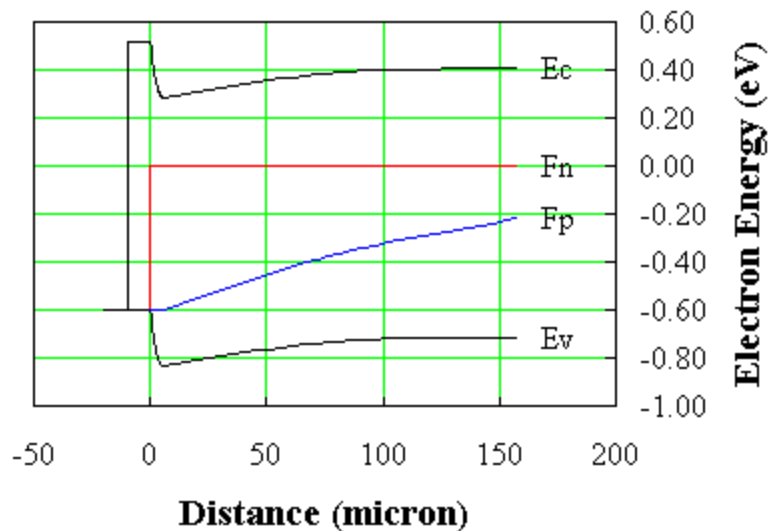


**Figure 4.4.7** Current-Voltage characteristics of a  $p^+ - n$  diode including the effects of high injection and a linear series resistance.

The dotted line on the figure fits the current at high forward bias. The slope is 1 decade/120 mV which corresponds to an ideality factor of 2.



**Figure 4.4.8** Electron and hole density under high injection conditions.



**Figure 4.4.9** Energy band diagram under high injection conditions.

#### Influence of the Series Resistance

The influence of a series resistance in a p-n diode can be calculated using Ohm's law. The total series resistance is the sum of the contact resistances and the resistances of the neutral semiconductor regions. For each semiconductor region one can find the resistivity if the doping density is known. The only complication arises when the exact length of the semiconductor regions is concerned, since it equals the average length the majority carriers travel through these

regions.

Once the total resistance is determined, one finds the external diode voltage by adding the voltage drop across the resistor to the internal diode voltage.

### Numerical Analysis and Comparison

The numerical analysis is performed for an abrupt one-sided silicon p-n diode. The diode consists of a thin highly doped p-type region on top of a lower doped n-type substrate. Only the hole current is calculated since it is dominant in such a diode. The current due to recombination in the depletion region was not included in this analysis. Included are the diffusion of holes even if their density exceeds the doping of the n-type substrate and the series resistance of the low-doped n-type substrate. The contact resistances to the n and p-type semiconductor as well as the resistance of the thin highly doped p-type layer have been ignored.

### Related Exercises:

Find the inverse slope of the current voltage characteristic between 0.5 and 0.7 Volt in units of Volt/decade. What is the corresponding ideality factor? Write down a relation between the ideality factor and the inverse slope.

"Measure" the ideality factor at high forward bias and compare it to the ideality factor at low bias. Verify that the ideality factor  $n = 2$  when high injection occurs

Increase the doping concentration of the n-type substrate and observe the changes. Why does the high injection region as characterized with an  $n = 2$  region disappear for high substrate doping?