Chapter 4: p-n Junctions

4.2. Structure and principle of operation

$$f_i = V_t \ln \frac{N_d N_a}{n_i^2} \tag{4.2.1}$$

$$f = f_i - V_a \tag{4.2.2}$$

4.3. Electrostatic analysis of a p-n diode

$$\frac{d^2 f}{dx^2} = -\frac{r}{e_s} = -\frac{q}{e_s}(p - n + N_d^+ - N_a^-) \tag{4.3.1}$$

$$\frac{d^2 f}{dx^2} = \frac{2qn_i}{e_s}(\sinh \frac{f - f_F}{V_t} + \sinh \frac{f_F}{V_t}) \tag{4.3.2}$$

$$\sinh \frac{f_F}{V_t} = \frac{N_a^- - N_d^+}{2n_i} \tag{4.3.3}$$

$$x_d = x_n + x_p \tag{4.3.4}$$

$$r = q(p - n + N_d^+ - N_a^-) \cong q(N_d^+ - N_a^-), \text{for } -x_p \le x \le x_n \tag{4.3.5}$$

$$\begin{aligned} r(x) &= 0 && \text{, for } x \le -x_p \\ r(x) &= -qN_a && \text{, for } -x_p \le x \le 0 \\ r(x) &= qN_d && \text{, for } 0 \le x \le x_n \\ r(x) &= 0 && \text{, for } x_n \le x \end{aligned} \tag{4.3.6}$$

$$Q_n = qN_d x_n \tag{4.3.7}$$

$$Q_p = -qN_a x_p \tag{4.3.8}$$

$$\frac{dE(x)}{dx} = \frac{r(x)}{e_s} \cong \frac{q}{e_s}(N_d^+(x) - N_a^-(x)), \text{for } -x_p \le x \le x_n \tag{4.3.9}$$

$$E(x) = 0 \quad \text{, for } x \le -x_p \tag{4.3.10}$$

$$E(x) = -\frac{qN_a(x + x_p)}{e_s} \quad \text{, for } -x_p \le x \le 0$$

$$E(x) = \frac{qN_d(x - x_n)}{e_s} \quad \text{, for } 0 \le x \le x_n$$

$$E(x) = 0 \quad \text{, for } x_n \le x$$

$$E(x = 0) = -\frac{qN_a x_p}{e_s} = -\frac{qN_d x_n}{e_s} \tag{4.3.11}$$

$$N_d x_n = N_a x_p \tag{4.3.12}$$

$$x_n = x_d \frac{N_a}{N_a + N_d} \tag{4.3.13}$$

$$x_p = x_d \frac{N_d}{N_a + N_d} \tag{4.3.14}$$

$$\frac{df(x)}{dx} = -E(x) \tag{4.3.15}$$

$$f_i - V_a = \frac{qN_d x_n^2}{2e_s} + \frac{qN_a x_p^2}{2e_s} \tag{4.3.16}$$

$$x_d = \sqrt{\frac{2e_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(f_i - V_a)} \tag{4.3.17}$$

$$x_n = \sqrt{\frac{2e_s}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}(f_i - V_a)} \tag{4.3.18}$$

$$x_p = \sqrt{\frac{2e_s}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}(f_i - V_a)} \tag{4.3.19}$$

$$C(V_a) \overset{\Delta}{=} \left|\frac{dQ(V_a)}{dV_a}\right| \tag{4.3.20}$$

$$C_j = \sqrt{\frac{qe_s}{2(f_i - V_a)}\frac{N_a N_d}{N_a + N_d}} \tag{4.3.21}$$

$$C_j = \frac{e_s}{x_d} \tag{4.3.22}$$

$$\frac{1}{C_j^2} = \frac{2}{qe_s}\frac{N_a + N_d}{N_a N_d}(f_i - V_a) \tag{4.3.23}$$

$$\frac{d(1/C_j^2)}{dV_a} = -\frac{2}{qe_s}\frac{N_a + N_d}{N_a N_d} \tag{4.3.24}$$

$$N_d = -\frac{2}{qe_s}\frac{1}{\frac{d(1/C_j^2)}{dV_a}}, \text{ if } N_a >> N_d \tag{4.3.25}$$

4.4. The p-n diode current

$$p_n(x = x_n) = p_{n0}e^{V_a/V_t} \tag{4.4.1}$$

$$n_p(x = -x_p) = n_{p0}e^{V_a/V_t} \tag{4.4.2}$$

$$p_n(x = w_n) = p_{n0} \tag{4.4.3}$$

$$n_p(x = -w_p) = n_{p0} \tag{4.4.4}$$

$$p_n(x \geq x_n) = p_{n0} + Ae^{-(x-x_n)/L_p} + Be^{(x-x_n)/L_p} \tag{2.9.13}$$

$$n_p(x \leq -x_p) = n_{p0} + Ce^{-(x+x_p)/L_p} + De^{(x+x_p)/L_p} \tag{2.9.14}$$

$$p_n(x \geq x_n) = p_{n0} + A^*\cosh\frac{x - x_n}{L_p} + B^*\sinh\frac{x - x_n}{L_p} \tag{4.4.5}$$

$$n_p(x \leq -x_p) = n_{p0} + C^*\cosh\frac{x + x_p}{L_n} + D^*\sinh\frac{x + x_p}{L_n} \tag{4.4.6}$$

$$p_n(x \geq x_n) = p_{n0} + p_{n0}(e^{V_a/V_t} - 1)[\cosh\frac{x - x_n}{L_p} - \coth\frac{w_n'}{L_p}\sinh\frac{x - x_n}{L_p}] \tag{4.4.7}$$

$$n_p(x \leq -x_p) = n_{p0} + n_{p0}(e^{V_a/V_t} - 1)[\cosh\frac{x + x_p}{L_n} + \coth\frac{w_p'}{L_n}\sinh\frac{x + x_p}{L_n}] \tag{4.4.8}$$

$$w_n' = w_n - x_n \tag{4.4.9}$$

$$w_p' = w_p - x_p \tag{4.4.10}$$

$$J_p(x \geq x_n) = -qD_p \frac{dp}{dx} = -\frac{qD_p p_{n0}}{L_p}(e^{V_a/V_t} - 1)[\sinh \frac{x - x_n}{L_p} - \coth \frac{w_n'}{L_p} \cosh \frac{x - x_n}{L_p}] \tag{4.4.11}$$

$$J_n(x \leq -x_p) = qD_n \frac{dn}{dx} = \frac{qD_n n_{p0}}{L_n}(e^{V_a/V_t} - 1)[\sinh \frac{x + x_p}{L_n} + \coth \frac{w_p'}{L_n} \cosh \frac{x + x_p}{L_n}] \tag{4.4.12}$$

$$I = A[J_n(x = -x_p) + J_p(x = x_n) + J_r] \cong I_s(e^{V_a/V_t} - 1) \tag{4.4.13}$$

$$I_s = qA[\frac{D_n n_{p0}}{L_n} \coth(\frac{w_p'}{L_n}) + \frac{D_p p_{n0}}{L_p} \coth(\frac{w_n'}{L_p})] \tag{4.4.14}$$

$$\coth x = \frac{1}{\tanh x} \cong \frac{1}{x} \text{ , for } x << 1 \tag{4.4.15}$$

$$p_n(x \geq x_n) = p_{n0} + p_{n0}(e^{V_a/V_t} - 1) \exp \frac{-(x - x_n)}{L_p} \tag{4.4.16}$$

$$n_p(x \leq -x_p) = n_{p0} + n_{p0}(e^{V_a/V_t} - 1) \exp \frac{x + x_p}{L_n} \tag{4.4.17}$$

$$J_p(x \geq x_n) = \frac{qD_p p_{n0}}{L_p}(e^{V_a/V_t} - 1) \exp \frac{-(x - x_n)}{L_p} \tag{4.4.18}$$

$$J_n(x \leq -x_p) = \frac{qD_n n_{p0}}{L_n}(e^{V_a/V_t} - 1) \exp \frac{x + x_p}{L_n} \tag{4.4.19}$$

$$I_s = qA[\frac{D_n n_{p0}}{L_n} + \frac{D_p p_{n0}}{L_p}] = qA[\frac{n_{p0} L_n}{t_n} + \frac{p_{n0} L_p}{t_p}] \tag{4.4.20}$$

$$I_r \geq qA \frac{p_{n0} x_n}{t_p}(e^{V_a/V_t} - 1) \tag{4.4.21}$$

$$x_n << L_p \tag{4.4.22}$$

$$0 = D_n \frac{d^2 n_p}{dx^2}, \text{ and } 0 = D_p \frac{d^2 p_n}{dx^2} \tag{4.4.23}$$

$$n_p = A + Bx\,, \text{ and } \quad p_n = C + Dx \tag{4.4.24}$$

$$p_n = p_{n0} + p_{n0}(\mathrm{e}^{V_a/V_t} - 1)\left(1 - \frac{x - x_n}{w_n'}\right) \tag{4.4.25}$$

$$n_p = n_{p0} + n_{p0}(\mathrm{e}^{V_a/V_t} - 1)\left(1 + \frac{x + x_p}{w_p'}\right) \tag{4.4.26}$$

$$I = A[J_n(x = -x_p) + J_p(x = x_n) + J_r] \cong I_s(e^{V_a/V_t} - 1) \tag{4.4.27}$$

$$I_s = qA[\frac{D_n n_{p0}}{w_p'} + \frac{D_p p_{n0}}{w_n'}] \tag{4.4.28}$$

$$J_{b-b} = q\int_{-x_p}^{x_n} U_{b-b}\,dx \tag{4.4.29}$$

$$U_{b-b} = b(np - n_i^2) \tag{4.4.30}$$

$$n\,p = n_i e^{(E_{Fn} - E_i)/kT} n_i e^{(E_i - E_{Fp})/kT} = n_i^2 e^{V_a/V_t} \tag{4.4.31}$$

$$J_{b-b} = q\int_{-x_p}^{x_n} n_i^2(e^{V_a/V_t} - 1)dx = qn_i^2 b\, x_d\,(e^{V_a/V_t} - 1) \tag{4.4.32}$$

$$J_{SHR} = q\int_{-x_p}^{x_n} U_{SHR}\,dx \tag{4.4.33}$$

$$J_{SHR} = q\int_{-x_p}^{x_n} \frac{1}{t}\,\frac{n_i^2(e^{V_a/V_t} - 1)}{n + p + 2n_i\cosh(\frac{E_t - E_i}{kT})}dx \tag{4.4.34}$$

$$n\,p = n_i e^{(F_n - E_i)/kT} n_i e^{(E_i - F_p)/kT} = n_i^2 e^{V_a/V_t} \tag{4.4.35}$$

$$U_{SHR,\max} = \mathrm{MAX}\,(\frac{1}{t}\,\frac{n_i^2(e^{V_a/V_t} - 1)}{n + p + 2n_i\cosh(\frac{E_t - E_i}{kT})}) \cong \frac{n_i}{2t}(e^{V_a/2V_t} - 1) \tag{4.4.36}$$

$$x' = \frac{\displaystyle\int_{-x_p}^{x_n} \frac{1}{t}\,\frac{n_i^2(e^{V_a/V_t} - 1)}{n + p + 2n_i\cosh(\frac{E_t - E_i}{kT})}dx}{U_{SHR,\max}} \tag{4.4.37}$$

$$J_{SHR} = \frac{qn_i x'}{2t}(e^{V_a / 2V_t} - 1) \quad (4.4.38)$$

$$J = J_s e^{V_a / hV_t} \quad (4.4.39)$$

$$h = \frac{\log(e)}{V_t \text{ slope}} = \frac{\frac{1}{\text{slope}}}{59.6 \text{ mV/decade}} \quad (4.4.40)$$

$$V_a = 2V_t \ln \frac{N_d}{n_i} \quad (4.4.41)$$

$$V_a^* = V_a + IR_s \quad (4.4.42)$$

$$C = \frac{d\Delta Q}{dV_a} \quad (4.4.43)$$

$$\Delta Q_p = \int_{x_n}^{w_n} qA(p_n - p_{n0})\, dx \quad (4.4.44)$$

$$\Delta Q_p = q A p_{n0}(\mathrm{e}^{V_a / V_t} - 1) L_p = I_{s,p}(\mathrm{e}^{V_a / V_t} - 1) t_p \quad (4.4.45)$$

$$I_{s,p} = q \frac{A p_{n0} D_p}{L_p} \quad (4.4.46)$$

$$C_{d,p} = \frac{d(I_{s,p}(\mathrm{e}^{V_a / V_t} - 1) t_p)}{dV_a} = \frac{I_{s,p} \mathrm{e}^{V_a / V_t} t_p}{V_t} \quad (4.4.47)$$

$$C_{d,p} = \frac{I_{s,p} \mathrm{e}^{V_a / V_t} t_{r,p}}{V_t} \quad (4.4.48)$$

$$t_{r,p} = \frac{w_p'^2}{2D_p} \quad (4.4.49)$$

4.5. Reverse bias breakdown

$$|E_{br}| = \frac{4x10^5}{1 - \frac{1}{3}\log(N/10^{16})} \text{ V/cm} \quad (4.5.1)$$

$$|V_{br}| = -f_i + \frac{|E_{br}|^2 e_s}{2qN} \quad (4.5.2)$$

$$x_{d,br} = \frac{|E_{br}| e_s}{qN} \tag{4.5.3}$$

$$dn = a_n n\, dx \tag{4.5.4}$$

$$M = \frac{1}{1 - \int_{x_1}^{x_2} a \; dx} \tag{4.5.5}$$

$$M = \frac{1}{1 - \left|\frac{V_a}{V_{br}}\right|^n}, \text{ where } 2 < n < 6 \tag{4.5.6}$$

$$\Theta = \exp\left(-\frac{4}{3} \frac{\sqrt{2m^*}}{q\hbar} \frac{E_g^{3/2}}{E} \right) \tag{4.5.7}$$

$$J_n = q\, v_R\, n\, \Theta \tag{4.5.8}$$

4.6. Optoelectronic devices

$$I = I_s (\mathrm{e}^{V_a / V_t} - 1) - I_{ph} \tag{4.6.1}$$

$$I_{ph,\max} = \frac{q}{hn} P_{in} \tag{4.6.2}$$

$$I_{ph} = (1 - R)(1 - \mathrm{e}^{-a\, d}) \frac{qP_{in}}{hn} \tag{4.6.3}$$

$$< i^2 > = 2\, q\, I\, \Delta f \tag{4.6.4}$$

$$\text{Fill Factor} = \frac{I_m V_m}{I_{sc} V_{oc}} \tag{4.6.5}$$

$$\text{Roundtrip amplification} = \mathrm{e}^{2gL} R_1 R_2 = 1 \tag{4.6.6}$$

$$g = \frac{1}{2L} \ln \frac{1}{R_1 R_2} \tag{4.6.7}$$

$$P_{out} = h \frac{hn}{q} (I - I_{th}) \tag{4.6.8}$$