Chapter 2: Semiconductor Fundamentals

2.1 Introduction
2.2 Crystals and Crystal Structures
2.3 Energy Bands
2.4 Density of States
2.5 Carrier Distribution Functions
2.6 Carrier Densities
2.7 Carrier Transport
2.8 Carrier Recombination and Generation
2.9 Continuity Equation
2.10 The Drift-Diffusion Model
2.11 Semiconductor Thermodynamics

Types of solid materials

- Amorphous material
  - No order
  - "Frozen" liquid state
- Poly-crystalline material
  - Short range order only
  - Small crystalline regions with random orientation
- Crystalline material
  - Both short and long range order

Chapter 2: Semiconductor Fundamentals

2.2 Crystals and Crystal Structures

- Bravais lattice
  - 2D
  - 3D
  - Miller indices
- Cubic crystals
  - Diamond lattice
  - Packing density
Bravais lattice construction

\[ \mathbf{r} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3 \]

Figure 2.2.1 The construction of lattice points using unit vectors

2D Bravais Lattices

(a) Square, (b) Rectangular, (c) Centered Rectangular, (d) Hexagonal, (e) Oblique

Figure 2.2.2 The five Bravais lattices of two-dimensional crystals:
(a) square, (b) rectangular, (c) centered rectangular, (d) hexagonal and (e) oblique

2D Bravais Lattices

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Bravais lattices</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1</td>
<td>( a_1 = a_2 = a_3, \alpha = 90^\circ )</td>
</tr>
<tr>
<td>Rectangular</td>
<td>2</td>
<td>( a_1 \neq a_2, a_2 = 90^\circ )</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>1</td>
<td>( a_1 = a_2, a_2 = 120^\circ )</td>
</tr>
<tr>
<td>Oblique</td>
<td>1</td>
<td>( a_1 \neq a_2, a_1 \neq 120^\circ, \alpha = 90^\circ )</td>
</tr>
</tbody>
</table>

Table 2.2.1 Bravais lattices of two-dimensional crystals

3D Bravais Lattices

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Bravais lattices</th>
<th>Conditions</th>
<th>Primitive</th>
<th>Base-centered</th>
<th>Body-centered</th>
<th>Face-centered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triclinic</td>
<td>1</td>
<td>( a_1 \neq a_2 \neq a_3, \alpha \neq \beta \neq \gamma )</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monoclinic</td>
<td>2</td>
<td>( a_1 \neq a_2 \neq a_3, \alpha = \beta = 90^\circ )</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>4</td>
<td>( a_1 \neq a_2 \neq a_3, \alpha = \beta = \gamma = 90^\circ )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Tetragonal</td>
<td>2</td>
<td>( a_1 = a_2 \neq a_3, \alpha = \beta = \gamma = 90^\circ )</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Cubic</td>
<td>3</td>
<td>( a_1 = a_2 = a_3, \alpha = \beta = \gamma = 90^\circ )</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Trigonal</td>
<td>1</td>
<td>( a_1 = a_2 = a_3, \alpha = \beta = \gamma = 120^\circ )</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Hexagonal</td>
<td>1</td>
<td>( a_1 = a_2 = a_3, \alpha = \beta = \gamma = 120^\circ )</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2.2 Bravais lattices of three-dimensional crystals (14 total)
3D Bravais Lattices

**Triclinic**
- \( a_1 \neq a_2 \neq a_3 \)
- \( \alpha \neq \beta \neq \gamma \)

**Monoclinic**
- \( a_1 \neq a_2 \neq a_3 \)
- \( \alpha = \beta = 90^\circ \neq \gamma \)

**Orthorhombic**
- \( a_1 \neq a_2 \neq a_3 \)
- \( \alpha = \beta = \gamma = 90^\circ \)

**Tetragonal**
- \( a_1 = a_2 \neq a_3 \)
- \( \alpha = \beta = \gamma = 90^\circ \)

**Cubic Crystals**

- \( a_1 = a_2 = a_3 \)
- \( \alpha = \beta = \gamma = 90^\circ \)

*Figure 2.2.3* The simple cubic (a), the body-centered cubic (b) and the face centered cubic (c) lattice
3D Bravais Lattices

Trigonal

\[ a_1 = a_2 = a_3 \]
\[ \alpha = \beta = \gamma < 120^\circ \neq 90^\circ \]

Hexagonal

\[ a_1 = a_2 
eq a_3 \]
\[ \alpha = \beta = 90^\circ \quad \gamma = 120^\circ \]

Miller Indices

Example

\[ p = 2, q = -3, r = 4 \]

Calculate inverses

\[ \frac{1}{2}, -\frac{1}{3}, \frac{1}{4} \]

Multiply with smallest common denominator, \( A = 12 \)

(6, 4, 3)

Minus signs are replaced with a bar above the number

Figure 2.2.4 Intersections of a plane and the \( x, y \) and \( z \) axes, as used to determine the Miller indices of the plane.

Diamond Lattice

The diamond lattice of silicon and germanium

The diamond lattice

Model detail

Front view
Chapter 2: Semiconductor Fundamentals

2.3 Energy Bands

- Free electron model
- Kronig-Penney model
- Real energy band diagrams
- Simplified energy band diagram
- Temperature and doping dependence
- Metals, insulators and semiconductors
- Effective mass

Packing Density of Cubic Crystals

\[
\text{Packing Density} = \frac{\text{Volume of atoms}}{\text{Volume of the unit cell}} = \frac{4}{3} \pi r^3
\]

<table>
<thead>
<tr>
<th>Radius</th>
<th>Atoms/unit cell</th>
<th>Packing density</th>
<th># Neighbors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple cubic</td>
<td>2</td>
<td>( \frac{4}{3} \pi r^3 )</td>
<td>6</td>
</tr>
<tr>
<td>Body centered cubic</td>
<td>( \frac{\sqrt{2}}{4} a )</td>
<td>( \frac{\sqrt{2}}{6} )</td>
<td>8</td>
</tr>
<tr>
<td>Face centered cubic</td>
<td>( \frac{\sqrt{2}}{4} a )</td>
<td>( \frac{\sqrt{2}}{6} )</td>
<td>12</td>
</tr>
<tr>
<td>Diamond</td>
<td>( \frac{\sqrt{2}}{8} a )</td>
<td>( \frac{\sqrt{2}}{16} )</td>
<td>4</td>
</tr>
</tbody>
</table>

Free electron model of a metal

Figure 2.3.1 The free electron model of a metal.
Energy levels versus lattice constant

Figure 2.3.2 Energy levels in a carbon crystal versus lattice constant, a.

Band diagrams of common semiconductors

Figure 2.3.7 Energy band diagram of (a) germanium, (b) silicon and (c) gallium arsenide

Simplified band diagram

Figure 2.3.8 A simplified energy band diagram used to describe semiconductors.

Temperature dependence of the energy bandgap

Figure 2.3.9 Temperature dependence of the energy bandgap of germanium (Ge), silicon (Si) and gallium arsenide (GaAs)
Doping dependence of the energy bandgap

\[ \Delta E_g(N) = -\frac{3q^2}{16\pi \varepsilon_0 \varepsilon_r kT} N \]

Figure 2.3.10 Doping dependence of the energy bandgap of silicon (Si), gallium arsenide (GaAs) and germanium (Ge)

Band Diagrams

Figure 2.3.11 Possible energy band diagrams of a crystal.

Electron and Hole transport

\[ \frac{dE}{dx} = q\varepsilon \]

Current in an almost filled band

\[ J_{vb} = \frac{1}{V} \sum_{\text{filled states}} (-q)\nu_i \]

\[ J_{vb} = \frac{1}{V} \sum_{\text{all states}} (-q)\nu_i - \sum_{\text{empty states}} (-q)\nu_i \]

\[ J_{vb} = \frac{1}{V} \sum_{\text{empty states}} (+q)\nu_i \]

Figure 2.3.12 Energy band diagram in the presence of a uniform electric field.

Effective Mass

Table 2.3.2 Effective mass of carriers in germanium, silicon and gallium arsenide (GaAs)

<table>
<thead>
<tr>
<th>Material</th>
<th>Germanium</th>
<th>Silicon</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest energy bandgap at 300 K (eV)</td>
<td>0.66</td>
<td>1.12</td>
<td>1.424</td>
</tr>
<tr>
<td>Electron effective mass for density of states calculations</td>
<td>( m^*_{\text{elec}} )</td>
<td>0.55</td>
<td>1.08</td>
</tr>
<tr>
<td>Hole effective mass for density of states calculations</td>
<td>( m^*_{\text{hole}} )</td>
<td>0.37</td>
<td>0.811</td>
</tr>
<tr>
<td>Electron effective mass for conductivity calculations</td>
<td>( m^*_{\text{elec}} )</td>
<td>0.12</td>
<td>0.26</td>
</tr>
<tr>
<td>Hole effective mass for conductivity calculations</td>
<td>( m^*_{\text{hole}} )</td>
<td>0.21</td>
<td>0.286</td>
</tr>
</tbody>
</table>
Chapter 2: Semiconductor Fundamentals

2.4 Density of States

Derivation of 3D density of states

Density of states derivation

Derivation (3 Dimensions)

\[ N = 2 \times \frac{1}{8} \times \left( \frac{L}{\pi} \right)^3 \times \frac{4}{3} \times \pi \times k^3 \]

\[ \frac{dN}{dE} = \frac{dN}{dk} \times \frac{dk}{dE} = \left( \frac{1}{2} \right)^3 \pi k^2 \frac{dk}{dE} \]

\[ E(k) = \frac{\hbar^2 k^2}{2m}, \text{ providing } \frac{dk}{dE} = \frac{m}{\hbar k} \text{ and } k = \sqrt{\frac{2mE}{\hbar}} \]

\[ g(E) = \frac{1}{L^3} \frac{dN}{dE} = \frac{8 \pi \sqrt{2}}{\hbar^3} \frac{m^{3/2}}{\sqrt{E}}, \text{ for } E \geq 0 \]

Conduction Band

\[ g_c(E) = \frac{8 \pi \sqrt{2}}{\hbar^3} m^{3/2} \sqrt{E-E_c}, \text{ for } E \geq E_c \]

\[ g_c(E) = 0, \text{ for } E < E_c \]
Statistical Thermodynamics

- Describes a large collection of particles
  - Parameters
    - Temperature, volume, number/type of particles
  - Statistical thermodynamics
    - All configurations are equally likely

- Needed concepts
  - Thermal equilibrium
  - Basic laws
  - Thermodynamic identity (energy conservation)
  - Fermi energy, $E_F$
  - Thermal voltage, $V_t$

Example

<table>
<thead>
<tr>
<th>$N$</th>
<th>$E$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>0K</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0K</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0K</td>
</tr>
</tbody>
</table>

Example (cont.)

The Fermi-Dirac distribution function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Figure 2.5.1 The Fermi function at three different temperatures
Example (cont.)

Figure 2.5.2 Eight of the 24 possible configurations in which 20 electrons can be placed having a total energy of 106 eV

Energy (eV) #1 #2 #3 #4 #5 #6 #7 #8
0 0 0 0 0 0 0 0
5 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0
15 0 0 0 0 0 0 0

Example (cont.)

All 24 possible configurations for \( N = 20 \) and \( E = 106 \text{ eV} \)

<table>
<thead>
<tr>
<th>Energy (eV)</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>10</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>15</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

Figure 2.5.3 Probability versus energy averaged over the 24 possible configurations (circles) fitted with a Fermi-Dirac function (solid line) using \( kT = 1.447 \text{ eV} \) and \( E_F = 9.998 \text{ eV} \)

Example (cont.)

Other distribution functions

Impurity distributions

- **Donors**
  \[ f_{\text{donor}(E_d)} = \frac{1}{1 + e^{(E_d - E_d, \text{F})/kT}} \]

- **Acceptors**
  \[ f_{\text{acceptor}(E_a)} = \frac{1}{1 + e^{(E_a - E_a, \text{F})/kT}} \]

Bose-Einstein distribution

For indistinguishable integer spin particles

\[ f_{BE}(E) = \frac{1}{e^{(E - E_F)/kT} - 1} \]

Maxwell-Boltzmann distribution

For distinguishable particles

\[ f_{MB}(E) = \frac{1}{\sqrt{2\pi kT}} e^{-(E - E_d, \text{F})^2/2kT} \]
Other distribution functions

Maxwell
Boltzmann
Fermi
Dirac
Bose
Einstein

Figure 2.5.4 Probability of occupancy versus energy of the Fermi-Dirac, the Bose-Einstein and the Maxwell-Boltzmann distribution

Chapter 2: Semiconductor Fundamentals
2.6 Carrier Densities

- Non-degenerate semiconductors
- Degenerate semiconductors
- Joyce-Dixon approximation
- Intrinsic semiconductors
- Mass action law
- Intrinsic material as a reference
- Doped semiconductors
- Donors and acceptors
- General solution
- Temperature dependence

Electron and hole densities per unit energy

Electrons

\[ n(E) = g_c(E) f(E) \]

\[ g_c(E) = \frac{8\sqrt{2}}{\pi^{1/2}} m^{3/2} E_c^{1/2} \quad \text{for} \quad E \geq E_c \]

Holes

\[ p(E) = g_v(E)(1 - f(E)) \]

\[ g_v(E) = \frac{8\sqrt{2}}{\pi^{1/2}} m^{3/2} (E - E_v) \quad \text{for} \quad E \leq E_v \]

Carrier density integral

\[
\begin{aligned}
\int_{E_c}^{\text{top of the conduction band}} n(E)dE &= \int_{E_c}^{\text{top of the conduction band}} g_v(E)f(E)dE \\
&= \int_{E_v}^{\text{top of the valence band}} g_v(E)dE
\end{aligned}
\]

Figure 2.6.1 The carrier density integral. Shown are the density of states, \( g_v(E) \), the density per unit energy, \( n(E) \), and the probability of occupancy, \( f(E) \).
Calculation of the electron and hole density

The density of states and carrier densities in the conduction and valence band.

Solution to the Fermi integral

Non-degenerate semiconductors:

- Electrons:
  \[ n_e = \frac{8\pi \sqrt{2}}{\hbar^2} m_n^{-3/2} \int_{E_F}^{\infty} \frac{1}{1 + e^{(E - E_F)/kT}} dE \]

- Holes:
  \[ p_h = \frac{8\pi \sqrt{2}}{\hbar^2} m_p^{-3/2} \int_{-\infty}^{E_F} \frac{1}{1 + e^{(E_F - E)/kT}} dE \]

Zero Kelvin solution

- Electrons:
  \[ n_e = \frac{2}{\pi^2} \frac{\sqrt{2}}{\hbar^2} (E_F - E_C)^{3/2}, \text{ for } E_F \geq E_C \]

- Holes:
  \[ p_h = \frac{2}{\pi^2} \frac{\sqrt{2}}{\hbar^2} (E_F - E_V)^{3/2}, \text{ for } E_F \geq E_V \]

Joyce-Dixon approximation (Degenerate semiconductors)

- Electrons:
  \[ E_F - E_C \geq \frac{kT}{\ln \left( \frac{N_C}{n_e} \right), \frac{1}{\sqrt{N_C}} \frac{1}{N_e}}, \text{ for } \frac{E_F}{N_C} \geq \frac{1}{\sqrt{N_C}} \frac{1}{N_e}, \text{ and } \frac{1}{\sqrt{N_C}} \frac{1}{N_e} \]

- Holes:
  \[ E_F - E_V \geq \frac{kT}{\ln \left( \frac{N_P}{p_h} \right), \frac{1}{\sqrt{N_P}} \frac{1}{N_p}}, \text{ for } \frac{E_F}{N_P} \geq \frac{1}{\sqrt{N_P}} \frac{1}{N_p}, \text{ and } \frac{1}{\sqrt{N_P}} \frac{1}{N_p} \]

Fermi integral comparison

Figure 2.6.3 Carrier (electron) density versus Fermi energy, for \( E_F = 0 \) eV.
Intrinsic semiconductors

Carrier density for non-degenerate semiconductors

\[ n_i = N_e e^{\frac{E_F}{kT}} \]
\[ p_i = N_e e^{\frac{E_F}{kT}} \]
\[ N_e = \frac{2\pi m^*}{h^2} \]
\[ N_i = \frac{2\pi m^*}{h^2} \]

Intrinsic carrier density, \( n_i \)

\[ n_i = \sqrt{N_e N_v} e^{-E_F/2kT} \]

Intrinsic Fermi energy

\[ E_F = \frac{E_v + E_c}{2} - \frac{1}{2} kT \ln \left( \frac{N_i}{N_c} \right) \]
\[ E_i = \frac{E_v + E_c}{2} - \frac{3}{4} kT \ln \left( \frac{m_e^*}{m_c} \right) \]

Intrinsic material as a reference

\[ n_i = N_e e^{\frac{E_F}{kT}} \]
\[ p_i = N_e e^{\frac{E_F}{kT}} \]
\[ n_i = \sqrt{N_e N_v} e^{-E_F/2kT} \]

Mass action law

Electron, hole and intrinsic carrier density

\[ n_i = N_e e^{\frac{E_F}{kT}} \]
\[ p_i = N_e e^{\frac{E_F}{kT}} \]
\[ n_i = \sqrt{N_e N_v} e^{-E_F/2kT} \]

Product of electrons and holes

\[ n_i \cdot p_i = N_e N_v e^{(E_F - E_v)/kT} = n_i^2 \]

Mass action law: Product of electrons and holes is constant

\[ n_i \cdot p_i = n_i^2 \]

Figure 2.6.4 Intrinsic carrier density versus temperature in gallium arsenide (GaAs), silicon and germanium.
2D representation of the diamond lattice

Donors and acceptors in silicon:

Donor

Acceptor

Donors and acceptors in silicon:

Donor

Acceptor

Donor ion

Acceptor ion

Ionization of donors and acceptors

Shallow impurities

\[ n_0 \approx N_s = N_i \]

\[ p_0 \approx N_p = N_i \]

Ionization energy model

\[ E_c - E_d = 13.6 \frac{m^*_{\text{free}}}{m_e} eV \]

Figure 2.6.5 Ionization of a) a shallow donor and b) a shallow acceptor

Figure 2.6.6 Trajectory of an electron bound to a donor ion within a semiconductor crystal. A 2-D square lattice is used for ease of illustration.
Compensated material

\[ n - p = N_d^+ - N_a^- \]

\[ \frac{E_c}{E_d} \]

\[ \frac{E_a}{E_v} \]

Range of doping densities

<table>
<thead>
<tr>
<th>Density</th>
<th>Intrinsic</th>
<th>Low density</th>
<th>High density</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>(n_i) (300K)</td>
<td>(cm(^-3))</td>
<td>(cm(^-3))</td>
<td>(n_{Si}) (cm(^-3))</td>
</tr>
<tr>
<td>Density</td>
<td>(10^{10})</td>
<td>(10^{13})</td>
<td>(10^{20})</td>
<td>(10^{23})</td>
</tr>
</tbody>
</table>

Analysis for non-degenerate semiconductors

Charge neutrality

\[ \rho = q(n_d - n_a + N_d^+ - N_a^-) = 0 \]

Mass action law

\[ n_a = \frac{n_i^2}{n_d} + N_d^+ - N_a^- \]

Solution to the quadratic equation

\[ n_i = \frac{N_d^+ - N_a^-}{2} \pm \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2 + \frac{n_i^2}{2}} \]

Calculation of the Fermi energy

\[ E_F = E_i + kT \ln \frac{p_i}{n_i} \]

\[ E_F = E_i - kT \ln \frac{p_i}{n_i} \]

\[ p = \frac{n_i^2}{(N_d^+ - N_a^-)} \]

\[ n = \frac{N_d^+ - N_a^-}{2} + \frac{(N_d^+ - N_a^-)^2 + n_i^2}{2} \]

\[ N_d^+ = 10^{17} \text{ cm}^{-3} \]

\[ N_a^- = 5 \times 10^{16} \text{ cm}^{-3} \]

\[ n_i = 10^{10} \text{ cm}^{-3} \]

\[ n = 5 \times 10^{16} \text{ cm}^{-3} \]

\[ p = 2 \times 10^3 \text{ cm}^{-3} \]

\[ n_{Si} = 10^{23} \text{ cm}^{-3} \]
General analysis

Figure 2.6.8 Graphical solution of the Fermi energy based on the general analysis.

Temperature dependence

Figure 2.6.9 Electron density as a function of temperature

Non-equilibrium carrier densities

Quasi-Fermi energies: \( F_n \) and \( F_p \)

\[
n = n_i + \delta n = n_i \exp\left(\frac{F_n - E_F}{kT}\right)
\]

\[
p = p_i + \delta p = p_i \exp\left(\frac{F_p - E_F}{kT}\right)
\]

Chapter 2: Semiconductor Fundamentals

2.7 Carrier Transport

- Carrier drift
- Mobility
- Doping and temperature dependence
- Resistivity
- Velocity saturation
- Diffusion
- Diffusion current derivation
- Einstein relation
- Total current
- Hall effect
Carrier drift

\[ I = \frac{Q}{t} = \frac{Q}{L/V} \]

\[ j = \frac{\vec{i}}{A} = \frac{\vec{Q}}{AL} = \rho \vec{v} \]

Electrons: \[ J = -q\vec{v}_n \]
Holes: \[ J = q\vec{v}_p \]

Figure 2.7.1 Drift of a carrier due to an applied electric field.

Random carrier motion

\[ \vec{\varepsilon} = 0 \quad \vec{\varepsilon} \neq 0 \]

Figure 2.7.2 Random motion of carriers in a semiconductor with and without an applied electric field.

Mobility vs. doping density

\[ \mu = \mu_{\text{min}} + \frac{\mu_{\text{max}} - \mu_{\text{min}}}{1 + (\frac{N}{N_r})^\alpha} \]

<table>
<thead>
<tr>
<th>Doping density (cm(^{-3}))</th>
<th>Mobility (cm(^2)/V-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^{14})</td>
<td>200</td>
</tr>
<tr>
<td>10(^{15})</td>
<td>400</td>
</tr>
<tr>
<td>10(^{16})</td>
<td>600</td>
</tr>
<tr>
<td>10(^{17})</td>
<td>800</td>
</tr>
<tr>
<td>10(^{18})</td>
<td>1000</td>
</tr>
</tbody>
</table>

Electrons

<table>
<thead>
<tr>
<th>Arsenic</th>
<th>Phosphorus</th>
<th>Boron</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_{\text{min}}) (cm(^2)/V-s)</td>
<td>52.2</td>
<td>68.5</td>
</tr>
<tr>
<td>(\mu_{\text{max}}) (cm(^2)/V-s)</td>
<td>1417</td>
<td>1414</td>
</tr>
<tr>
<td>(N_r) (cm(^{-3}))</td>
<td>9.68 \times 10(^{16})</td>
<td>9.20 \times 10(^{16})</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.68</td>
<td>0.711</td>
</tr>
</tbody>
</table>

Figure 2.7.3 Electron and hole mobility versus doping density for silicon.

Doping dependent mobility

\[ \mu = \mu_{\text{min}} + \frac{\mu_{\text{max}} - \mu_{\text{min}}}{1 + (\frac{N}{N_r})^\alpha} \quad \text{with} \quad N = N_d^+ + N_a^- \]

<table>
<thead>
<tr>
<th>Doping density (cm(^{-3}))</th>
<th>Mobility (cm(^2)/V-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^{14})</td>
<td>200</td>
</tr>
<tr>
<td>10(^{15})</td>
<td>400</td>
</tr>
<tr>
<td>10(^{16})</td>
<td>600</td>
</tr>
<tr>
<td>10(^{17})</td>
<td>800</td>
</tr>
</tbody>
</table>

Electrons

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<tr>
<td>(\alpha)</td>
<td>0.68</td>
<td>0.711</td>
</tr>
</tbody>
</table>

Figure 2.7.4 Electron and hole mobility versus doping density for silicon with doping-dependent mobility.
Resistivity of silicon

\[ \rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)} \]

Figure 2.7.4 Resistivity of n-type and p-type silicon versus doping density.

Sheet resistance: Uniform doping

Resistance of a uniformly doped semiconductor

\[ R = \frac{\rho L}{W t} \]

Sheet resistance

\[ R_s = \frac{\rho}{t} \quad \text{Units: Ohm/square} \]

\[ R = R_s \frac{L}{W} = R_s \times \text{number of squares} \]

Figure 2.7.5 Sheet resistance of a 14 mil thick n-type and p-type silicon wafer versus doping density.

Non-uniform doping

\[ dR = \frac{dR}{dx} = \frac{1}{q\mu_n(x)p(x)} \]

\[ R = \frac{1}{q\int \mu_n(x)p(x)dx} \]

\[ R = \frac{1}{q\int \mu_p(x)p(x)dx} \]

Figure 2.7.6 Sheet resistance calculation of a non-uniform doping distribution.
Velocity saturation

\[ v = \mu \frac{E}{V_{sat}} \]

\[ v_{peak} = \frac{\mu E}{pV_{sat}} \]

Figure 2.7.7 Velocity-field relation for a) materials without accessible higher bands and b) materials with an accessible higher band.

Diffusion current derivation

\[ J_n = q \Phi_n = q \nu_{th} \frac{dn}{dx} \]

\[ J_p = -q \Phi_p = -q \nu_{th} \frac{dp}{dx} \]

Einstein relation

\[ D_n = \mu_n \frac{kT}{q} = \mu_n \mathcal{V}_l \]

\[ D_p = \mu_p \frac{kT}{q} = \mu_p \mathcal{V}_l \]

\[ \nu_{th} = \frac{1}{\tau} \]

\[ \nu_{th} = \frac{kT}{2 \mu_V^2} \frac{V_T}{m} = \frac{kT}{2 \mu_p} \]

Figure 2.7.8 Carrier density profile used to derive the diffusion current expression.

Total current

Total current = Drift current + Diffusion current

\[ J_n = q \nu_n \mathcal{E} + q D_n \frac{dn}{dx} \]

\[ J_p = q \nu_p \mathcal{E} - q D_p \frac{dp}{dx} \]

Total current = electron current + hole current

\[ I_{total} = A(J_n + J_p) \]
The Hall effect

$$R_H = \frac{L}{W} = \frac{1}{q\mu_y}$$

Figure 2.7.9 Hall setup and carrier motion for a) holes and b) electrons.

Chapter 2: Semiconductor Fundamentals

2.8 Carrier Recombination and Generation

- Recombination/generation mechanisms
- Other generation mechanisms
  - Simple R/G model
  - Band-to-band recombination
  - SHR recombination
  - Surface recombination
  - Auger recombination

Carrier recombination and generation

- Band-to-band recombination
- Trap-assisted recombination
- Auger recombination

Figure 2.8.1 Carrier recombination mechanisms in semiconductors

Light absorption and ionization

- Generation due to light absorption
- Ionization due to charged high-energy particle beams

Figure 2.8.2 Carrier generation due to: a) light absorption and b) ionization due to high-energy particle beams
**Avalanche multiplication**

![Avalanche multiplication diagram](image)

Figure 2.8.3 Impact ionization and avalanche multiplication of electrons and holes in the presence of a large electric field.

**Carrier recombination and generation**

![Carrier recombination diagram](image)

Figure 2.8.1 Carrier recombination mechanisms in semiconductors

**Simple recombination model**

\[
U_n = G_n - R_n = \frac{n_n - n_p n_0}{\tau_n}
\]

\[
U_p = G_p - R_p = \frac{p_n - p_p n_0}{\tau_p}
\]

**Light absorption**

![Light absorption diagram](image)

Figure 2.8.2 Carrier generation due to: a) light absorption

\[
\frac{dP_{opt}(x)}{dx} = -\alpha P_{opt}(x)
\]

\[
G_{opt} = \frac{P_{opt}(x)}{E_{opt} A}
\]
Chapter 2: Semiconductor Fundamentals

2.9 Continuity Equation

- Derivation of the continuity equation
- Diffusion equation
- Solution for quasi-neutral semiconductors

Derivation of the continuity equation

\[
\frac{\partial n(x,t)}{\partial t} \ dx = \frac{J_n(x) - J_n(x + dx)}{dx} + (G_n(x,t) - R_n(x,t)) \ dx
\]

\[
J_n(x + dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x} \ dx
\]

Alternate continuity equations

Substitution of the current density

3-Dimensional continuity equation

\[
\frac{\partial n(x,y,z,t)}{\partial t} = \frac{1}{2} \nabla \cdot \vec{J}_n(x,y,z,t) + G_n(x,y,z,t) - R_n(x,y,z,t)
\]

\[
\frac{\partial p(x,y,z,t)}{\partial t} = -\frac{1}{2} \nabla \cdot \vec{J}_p(x,y,z,t) + G_p(x,y,z,t) - R_p(x,y,z,t)
\]
Diffusion equations

Continuity equations applied to a quasi-neutral region

Assumptions:
- Zero electric field
- Simple recombination/generation model
- Valid for minority carriers only

\[ n \:
\frac{\partial (n(x,t))}{\partial t} = D_n \frac{\partial^2 n(x,t)}{\partial x^2} + \frac{n(x,t) - n_0}{\tau_n} \]

\[ p \:
\frac{\partial (p(x,t))}{\partial t} = D_p \frac{\partial^2 p(x,t)}{\partial x^2} + \frac{p(x,t) - p_0}{\tau_p} \]

Steady state solution to the diffusion equations

Steady state equations

\[ 0 = D_p \frac{d^2 p_n(x)}{dx^2} - \frac{p_n(x) - p_0}{\tau_p} \]

General solutions

\[ p_n(x) = p_{n0} + Ae^{-\frac{x}{L_n}} + Be^{\frac{x}{L_n}} \]

\[ n_n(x) = n_{n0} + C \text{e}^{-\frac{x}{L_n}} + D \text{e}^{\frac{x}{L_n}} \]

Formulation as a function of excess carrier densities

Excess minority carrier densities

\[ n = n_0 + \delta n \quad p = p_0 + \delta p \]

Diffusion equations

\[ 0 = \frac{d^2 (\delta n)}{dx^2} - \frac{\delta n}{L_n} \]

\[ 0 = \frac{d^2 (\delta p)}{dx^2} - \frac{\delta p}{L_p} \]

Chapter 2: Semiconductor Fundamentals

2.10 The Drift-Diffusion Model

- Assumptions
- 10 equations, 10 unknowns
Drift-diffusion model assumptions

**Full ionization:**
all dopants are assumed to be ionized (shallow dopants)

**Non-degenerate:**
the Fermi energy is assumed to be at least $3kT$
below/above the conduction/valence band edge.

**Steady state:**
All variables are independent of time

**Constant temperature:**
The temperature is constant throughout the device.

Drift-diffusion model variables

$\rho$ the charge density

$n$ the electron density

$p$ the hole density

$E$ the electric field

$\phi$ the potential

$E_i$ the intrinsic energy

$F_n$ the electron quasi-Fermi energy

$F_p$ the hole quasi-Fermi energy

$J_n$ the electron current density

$J_p$ the hole current density

Drift-diffusion model equations

\[
\begin{align*}
\rho &= q(p - n + N_d^+ - N_a^-) \\
J_n &= qn\mu_n E + qD_n \frac{dn}{dx} \\
\frac{dE}{dx} &= \frac{P}{\varepsilon} \\
J_p &= qp\mu_p E - qD_p \frac{dp}{dx} \\
\frac{d\phi}{dx} &= -E \\
\frac{dE_i}{dx} &= qE \\
n &= n_i e^{(F_n - E_i)/kT} \\
p &= n_i e^{(E_i - F_p)/kT}
\end{align*}
\]