Problem Set 1 Solutions

1. 1.7

**Problem 1–7.** The net positive charge flowing through a device is \( q(t) = 20 + 4t \) mC. Find the current through the device.

The current through the device is the derivative of the charge, \( i = dq/dt \).

\[
i = \frac{dq}{dt} = \frac{d}{dt}(20 + 4t) \text{ mC} = 4 \text{ mA}
\]

The following MATLAB code calculates the same answer.

```matlab
syms t real
qt = 20 + 4*t;
it = diff(qt,t)
```

2. EK1-1

**EK1-1** The battery can deliver an energy of

\[
w = \int_0^T ui \, dt = u \int_0^T i \, dt
\]

\[
= (12 \text{V})(15 \text{ A-h}) = 180 \text{ Watt-hours (Wh)}
\]

Since \( ui = 50 \text{ W} \), we also have

\[
w = \int_0^T (50 \text{W}) \, dt = (50 \text{W}) T
\]

So

\[
T = \frac{180 \text{ Wh}}{50 \text{ W}} = 3.6 \text{ hours}
\]

3. EK1-2
4. **EK1-3**

**EK1.3**) To add positive charge to the upper reservoir, current must flow up through the wire. Since this is opposite to the reference direction given for $i(t)$, 

$$i(t) = -\frac{dQ}{dt} = \begin{cases} -\frac{1 \mu C}{2\text{ms}} = -0.5 \text{ mA} & (0 < t < 2\text{ms}) \\ 0 & (2\text{ms} < t < 7\text{ms}) \\ \frac{1 \mu C}{1\text{ms}} = +1 \text{ mA} & (7\text{ms} < t < 8\text{ms}) \end{cases}$$

The plot looks like this:

![Image of the plot](image-url)
Problem 1–20. Traffic lights are being converted from incandescent bulbs to LED arrays to save operating and maintenance costs. Typically each incandescent light uses three 100-W bulbs, one for each color R, Y, G. A competing LED array consists of 61 LEDs with each LED requiring 9 V and drawing 20 mA of current. There are three arrays per light - R, Y, G. A small city has 1560 traffic signals. Since one light is always on 24/7, how much can a city save in one year if the city buys their electricity at 7.2 cents per kWh?

To solve this problem, compare two lights and then scale the problem to the number of lights in the city. The incandescent light always has one 100-W bulb operating 24 hours per day for 365 days, which yields a total of 876 kWh at a cost of $63.072. The LED light always has one array, using a total power of \((61)(9 \text{ V})(20 \text{ mA}) = 10.98 \text{ W}\). Over one year, the LED lights uses 96.185 kWh of energy at a cost of $6.9253. The savings per light per year is $56.147, which, for a total of 1560 lights, translates into a city-wide saving of $87,589 per year. The following MATLAB code calculates the same answer.

```matlab
% Incandescent power
p_incand = 100/1000;  
% Dollars per kWh
rate = 0.072;  
% Hours per year
hours = 24*365;

% Incandescent cost
cost_incand = p_incand*hours*rate;

% LED voltage
v_led = 9;
% LED current
i_led = 20e-3;
% Number of LED lights per array
n_led = 61;

% Power in array
p_led = n_led*vLed + iLed; 
% Power in kWh
costLed = pLed*hours*rate;

savings_light = cost_incand - costLed;
% Savings per year
savings_year = lights*savings_light;
```

6. 1.22

Problem 1–22. Figure P1–22 shows an electric circuit with a voltage and a current variable assigned to each of the six devices. The device voltages and currents are observed to be

<table>
<thead>
<tr>
<th>Device</th>
<th>(v) (V)</th>
<th>(i) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-10</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>-3</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the power associated with each device and state whether the device is absorbing or delivering power. Use the power balance to check your work.

The power associated with each device is the product of the voltage and current, \(p = vi\). If the power is positive, the device is absorbing power. If the power is negative, the device is delivering power. The original table is expanded below to include power and the direction of power flow.
### 7. 2.3

**Problem 2–3.** A 100-kΩ resistor dissipates 100 mW. Find the current through the resistor.

The power dissipated by a resistor is $p = i^2R$. Solving for current, we have $i = \sqrt{p/R} = \sqrt{(10^{-1})/(10^5)} = 1$ mA.

### 8. 2.5

**Problem 2–5.** In Figure P2–5 the resistor dissipates 25 mW. Find $R_x$.

The power dissipated by a resistor can be written as $p = v^2/R$. Solving for the resistance, we have $R_x = v^2/p = (15^2)/(25 \times 10^{-3}) = 9$ kΩ.

### 9. 2.6

**Problem 2–6.** In Figure P2–6 find $R_x$ and the power delivered to the resistor.

Using Ohm’s law to solve for resistance, we have $R_x = v/i = 100/(10^{-2}) = 10$ kΩ. The power delivered to the resistor is $p = vi = (100)(10^{-2}) = 1$ W.

### 10. 2.12

**Problem 2–12.** A thermistor is a temperature-sensing element composed of a semiconductor material which exhibits a large change in resistance proportional to a small change in temperature. A particular thermistor has a resistance of 5 kΩ at 25°C. Its resistance is 340 Ω at 100°C. Assuming a straight-line relationship between these two values, at what temperature will the thermistor’s resistance equal 1 kΩ?

Find the rate at which the resistance changes for each degree of temperature.

$$\Delta\Omega = \frac{5000 - 340}{25 - 100} = \frac{4660}{-75} = -62.13 \, \Omega/°C$$

To go from 5 kΩ to 1 kΩ, the resistance changes by −4 kΩ, which means the temperature change is $-4000/(-62.13) = 64.38°C$. The final temperature is $25 + 64.38 = 89.38°C$. 

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The following MATLAB code calculates the same results.

```matlab
v = [15 5 10 -10 20 20];
i = [-1 1 2 -1 -3 2];
p = v.*i
Absorbing = p>0
Balance = sum(p)
Results = [v’ i’ p’ Absorbing’]
```